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Introduction to Logarithms

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Overview

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This presentation will cover:

- what are logarithms are;
- what are the logarithms rules; and
- using the logarithm rules to solve problems.

What are Logarithms?

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We know:

$$\begin{aligned} 10^2 &= 100 \\ 10^3 &= 1\,000 \\ 10^4 &= 10\,000 \\ 10^5 &= 100\,000 \end{aligned}$$

Now calculate:

$$\begin{aligned} \log_{10} 100 &= 2 \\ \log_{10} 1\,000 &= 3 \\ \log_{10} 10\,000 &= 4 \\ \log_{10} 100\,000 &= 5 \end{aligned}$$

Notice the power you are raising the 10 to is same as the answer when we take the logarithm.

Therefore, **logarithms are powers in another form**.

For example,

$$\log_{10} 2.36 \approx 0.3729 \qquad \longrightarrow \qquad 10^{0.3729} \approx 2.36$$

Logarithms of different bases

Exponential form		Logarithmic form
$2^5 = 32$	$\longrightarrow$	$\log_2 32 = 5$
$10^0 = 1$	$\longrightarrow$	$\log_{10} 1 = 0$
$4^{-1} = \frac{1}{4} = 0.25$	$\longrightarrow$	$\log_4 0.25 = -1$
$8^{\frac{1}{3}} = 2$	$\longrightarrow$	$\log_8 2 = \frac{1}{3}$
$e^2 \approx 7.389$	$\longrightarrow$	$\log_e 7.389 = \ln 7.389 \approx 2$
$y = b^x$	$\longrightarrow$	$\log_b y = x$

Exponential functions and logarithmic functions are **inverse functions**, meaning that they undo each other.

Logarithmic laws

$$\begin{aligned}
\log_b(x \times y) &= \log_b(x) + \log_b(y) \\
\log_b\left(\frac{x}{y}\right) &= \log_b(x) - \log_b(y) \\
\log_b(x^y) &= y \log_b(x) \\
\log_b(b) &= 1 \\
\log_b 1 &= 0
\end{aligned}$$

Example 1

Evaluate the logarithm of 3×10^{-9}.

We have not been explicitly told which logarithm, so in this case it would be advisable to choose to take the logarithm to the base of 10 to see how the logarithms laws work.

$$\begin{aligned}
& \log\left(3 \times 10^{-9}\right) \\
= \;& \log 3 + \log 10^{-9} && \text{Using the Law: } \log(x \times y) = \log x + \log y \\
= \;& \log 3 + (-9) \log 10 && \text{Using the Law: } \log(x^y) = y \log x \\
= \;& \log 3 + (-9) && \text{Using the Law: } \log_b(b) = 1 \\
\approx \;& 0.47712 - 9 \approx -8.52\,.
\end{aligned}$$

We could also just evaluate this on out calculator: $\log\left(3 \times 10^{-9}\right) \approx -8.52\,.$

Example 2:

Solve for x in the following equations:

1. $\log_{10} x = 1$

   $\log_{10} x = 1 \qquad 10^1 = x \qquad x = 10$

2. $\log_{10} x = 6.80$

   $\log_{10} x = 6.80 \qquad 10^{6.80} = x \qquad x \approx 6\,309\,573$

3. $\log_{10} x = -0.8$

   $\log_{10} x = -0.8 \qquad 10^{-0.8} = x \qquad x \approx 0.158$

Example 3: simplify the expression:

$$
\begin{aligned}
& \log_3 81 + \log_3\left(\frac{1}{9}\right) && \text{check that the bases are the same,} \\
= \;& \log_3\left(81 \times \frac{1}{9}\right) && \text{using the law: } \log(x \times y) = \log x + \log y\,, \\
= \;& \log_3 9 && \\
= \;& \log_3\left(3^2\right) && \\
= \;& 2 \times \log_3 3 && \text{using the law: } \log(x^y) = y \log x\,, \\
= \;& 2 \times 1 && \text{using the law: } \log_a a = 1\,, \\
= \;& 2\,. &&
\end{aligned}
$$

Example 4: Simplify

$$
\begin{aligned}
& \ln 25 + 2 \ln 0.2 && \text{Check that the logarithms have the same base} \\
= \;& \ln 25 + \ln 0.2^2 && \text{using the law: } n \log a = \log a^n\,, \\
= \;& \ln\left(25 \times 0.2^2\right) && \text{using the law: } \log a + \log b = \log(a \times b)\,, \\
= \;& \ln 1 && \\
= \;& 0 && \text{using the law: } \log_a 1 = 0\,.
\end{aligned}
$$

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