UNIVERSITY OF SOUTHERN QUEENSLAND

fulfilling lives

Success in maths for statistics

The **L**earning **C**entres

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Success in maths for statistics

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Module 1

Arithmetic and the calculator

Aims

Many students find Data Analysis a difficult subject. It requires learning difficult concepts but also an understanding of the underlying mathematics. This module aims to review some basic arithmetic calculations and look at how these are performed on the calculator.

On successful completion of this module you should be able to:

- give examples of different types of numbers;
- perform the operations of addition, subtraction, multiplication and division on all of the above numbers;
- perform complex calculations using order of operation;
- use techniques of estimation to aid in calculation;
- use percentages; and
- use a calculator for calculations.

1.1 Our number system

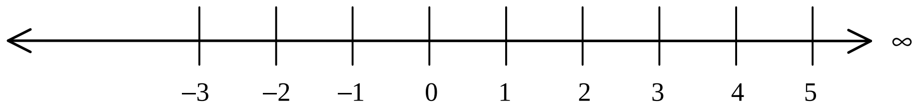
The numbers and symbols that we use in mathematics are a part of our **language**. If we have a common understanding of what numbers mean and say, we can be assured that the message we want to send will be received as it was intended. When we communicate with numbers, just as with words, we have to be aware of the **purpose** of the message as well as the **audience** with whom we are communicating.

The number system we use today has developed over many hundreds of years. Early civilisations only counted using fingers and toes where necessary. When it became necessary for additional representation of number, piles of stones were used. Eventually these needed to be grouped to make counting easier. Although we now group our numbers in lots of ten, early civilisations used other groupings for their number systems. For example some grouped in lots of five because they were used to seeing five fingers and five toes on each hand and foot. Another grouping that is still in use today is 60. The Babylonian civilisations used this for counting and we continue with its use for measurement of time (60 minutes in 1 hour, 60 seconds in 1 minute).

The system of numbers we use today is called the Hindu-Arabic system. It consists of the **digits** 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 believed to have been used by the Arabs before being adopted by other countries. This system differed greatly from other systems because of the inclusion of a symbol for zero. Our system of numbers is made up of many different types of numbers – whole numbers, negative numbers, fractions, decimals etc. and you need to know how to manipulate these types of numbers in statistics.

1.2 Positioning numbers

Let's picture numbers on the **number line**. This allows us to see these numbers in order, moving from small to large. To draw a number line we draw a line and choose any point to represent zero (0). With a ruler we then mark off even spaces along the line to the right.



The arrows at the end of the line means that the numbers keep going, getting larger and larger (forever!!) or smaller and smaller. We call the imaginary point that we never reach at the end of this number line, **infinity** or **negative infinity**. It is given the symbol ∞ or $-\infty$. Over history there has been much discussion about the concept of infinity. It is still a concept that is very hard to grasp. We do know that the first person to use the symbol ∞ for infinity was Englishman John Wallis in a publication in 1655.

Let's look at some features of the number line.

You know that 3 is a number less than 5 and we can represent this as $3 < 5$. The symbol '<' means **less than**.

If you look at the number line above you will see that the number 3 is to the left of 5. This also indicates that $3 < 5$.

The opposite symbol to less than is **greater than** represented by the symbol '>'.

An example of this would be to say that $4.8 > 4.2$ or *4.8 is greater than* 4.2. You can imagine that 4.8 is further along the number line than 4.2 and hence is greater than 4.2.

You should note that the 'point' of the < or > sign will always point to the smaller number while the 'open side' will always point to the larger number.

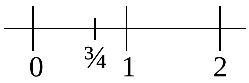
The following two symbols are an extension of the above:

$\leq$ which means less than or equal to; and
$\geq$ which means greater than or equal to

For example 3, 7 and 9 are all less than or equal to 9, while 12, 17 and 98 are all greater than or equal to 12.

1.2.1 Fractions

You will also be dealing with some fractions in statistics. Recall that ¾ the 3 is called the numerator and the 4 is called the denominator. This number can be placed on the number line between the 0 and 1



You can also think of ¾ as 3 divided by 4 so ¾ = 0.75. Try using your fraction key on the calculator.

1.2.2 Absolute value

Sometimes we only want to know the size of the number not its sign. This is called the **absolute value**. The absolute value of –4 is 4. It is written as | –4 | = 4. | 7 | = 7 We can also think of the distance of a number from 0, as the absolute value. So –4 is 4 units away from 0; 6 is 6 units away from 0.

Activity 1.1

1. Determine whether the following statements are true or false

   (a) –6 < –9

   (b) 5.4 > 6.3

   (c) 2½ < 19½

   (d) 56½ > 56¼

   (e) 0.01 < 0.001

   (f) | –1 | = 1

   (g) | 4 | = | –4 |

2. Insert the correct symbol between these two numbers to make the statement true.

   (a) 6 $\square$ 12

   (b) 0.6 $\square$ 0.12

   (c) –6 $\square$ –12

   (d) 17 $\square$ 0

   (e) 41 $\square$ 41

Before moving to the next section be sure to check your answers using the solutions at the end of this module.

1.3 Rounding numbers

When performing calculations, particularly on the calculator, you are going to get answers that have many decimal places. It is not normally necessary to report all of these places. You should round your answer.

It is important that you never round off before you get to the final answer.

You may be asked to round in a number of ways.

You could be asked to round a number to a specific place value or to a given number of decimal places. The following method is applied in both of these cases.

To round numbers to a particular place value, investigate the digit immediately to the right of it.

- If the digit immediately to the right is greater than or equal to 5 (i.e. ≥ 5, which includes the digits 5, 6, 7, 8 or 9), then increase the required place value by 1.
  We say the number has been rounded up.
- If the digit to the right is less than 5 (i.e. < 5 which includes the digits 0, 1, 2, 3 or 4), then the required place value remains the same.
  We say the number has been rounded down.
- All digits to the right of the round off place are replaced by zeros.

Let's look at some examples:

Example

Round 572 to the nearest hundred.

572

hundreds position (the 5)

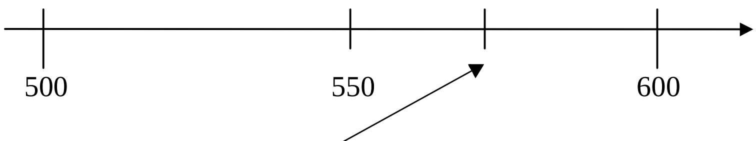
look at the digit immediately to the right (the 7)

The digit immediately to the right of the hundreds column is greater than or equal to 5 so the digit in the hundreds column is increased by 1. The 5 in the hundreds place will increase to 6

All place values to the right of the required position are then filled with zeros.

So 572 rounded to the nearest hundred becomes 600.

We could picture this on the number line.



We can see that 572 is closer to 600 than to 500.

Example

Round 674 927 to the nearest ten thousand.

674 927

ten thousands column (the 7)

look at the digit immediately to the right (the 4)

The digit immediately to the right of the ten thousands column is less than 5 so the digit in the ten thousands column remains the same.

All place values to the right of the required position are then filled with zeros.

So 674 927 rounded to the nearest ten thousand becomes 670 000

Activity 1.2

Complete the following table by rounding each number as indicated.

	Number	To nearest thousand	To nearest hundred	To nearest ten
(a)	2 575			
(b)	324			
(c)	105			
(d)	26 897			
(e)	5 502 471			

When rounding decimal numbers we use much the same method as we used with whole numbers. The only difference is that when the round-off position is after the decimal point, we do not fill any spaces with zeros.

Example

Round 3.648 to the nearest tenth

As before, look at the digit to the right of the tenth position, in this case the 4. It is less than 5 so we leave the digit in the tenths position as it is.

3.648 ≈ 3.6 rounded to the nearest tenth.

The symbol ≈ means **approximately equal to**. So we would read the above statement as *379 is approximately equal to 400*.

You may also see the symbols ≈ or ≅ used to represent ‘approximately equal to’ but in these modules we will continue to use ≈

Example

Round 3 476.638 to two decimal places.

To end up with two decimal places we are rounding to the nearest hundredth. Look at the digit to the right of the hundredth place, in this case the 8. It is greater than or equal to 5 so we take the round-off place up by one.

3 476.638 ≈ 3 476.64 rounded to two decimal places.

Activity 1.3

1. Round these numbers to the place value indicated.

(a)	576.205	to the nearest hundredth
(b)	75 201.3	to the nearest unit
(c)	–0.008	to the nearest hundredth
(d)	67.345 67	to one decimal place
(e)	–6 399.998	to two decimal places

In any calculations that follow it is always good practice to quickly estimate the answer before you calculate. Having done this you can be sure that your answer is ‘close to the mark’. We would expect you to follow this practice throughout the rest of these modules wherever possible.

1.4 Working with numbers

You will probably remember all or parts of the following from your previous studies. Although everybody should work through all sections, some people may move more quickly than others. If you are unsure, take your time, this groundwork is very important.

In mathematics there are four basic operations to perform: addition, subtraction, multiplication and division. We will deal with each of these in turn.

1.4.1 Addition

Example

On a recent weekend trip I travelled the following distances each day.

Friday	Saturday	Sunday	Monday
356.3 km	126.9 km	91.1 km	402 km

To find the total distance travelled on this weekend trip I need to find the **sum** of the distances for each day. That is, I must **add** together each of the above distances.

Before doing any calculation we should estimate the answer. It is easy on the calculator to press the wrong key and end up with an incorrect answer. If we estimate the answer first, we should then be aware that our answer may be wrong, if the estimated and calculated answers differ by a large amount.

The most convenient way to estimate is to round each number to its leading digit.

$$\begin{aligned} 356.3 &\approx 400 \\ 126.9 &\approx 100 \\ 91.1 &\approx 90 \\ 402 &\approx 400 \end{aligned}$$

The total distance travelled then would be approximately equal to

400 + 100 + 90 + 400 = 990 km

Example

The following showed the profit of my son's 'business' over 4 weeks:

$6 $–3 $5 $–7

What was his total profit for the month?

To answer this we must add 6, –3, 5, and –7.

To do this you must find the +/– key or the (–) key on your calculator.

His profit for the 4 weeks was $1!

Find how to add numbers on your calculator, including how to use:

- the cancel keys
- fraction keys
- the negative key

Activity 1.4

Find the following answers **without** using a calculator. Estimate your answer before commencing. Check your answer by using a calculator.

1. (a) 58 + 61
   (b) 25.01 + 956.5+ 0.32
   (c) 0.750 + 0.023 05 +0.10 + 0.9 + 0.003 15
   (d) 658 + 0

2. The children's ward of the local hospital admitted 5 children on Sunday, 3 on Monday, 2 on Tuesday, none on Wednesday and Thursday but 6 on Friday and 9 on Saturday. How many children were admitted to the children's ward for the week?

3. The attendances for the Bold Batters Baseball Club's first four home games were 10 428, 8 922, 7 431 and 9 647. How many people came to the first four home games?

4. David's lunch consisted of a triple hamburger that contained 2 103 kJ (kilojoules, the measure of energy in food), hot potato chips containing 1 714 kJ, an apple pie with 1 148 kJ and a diet soft drink containing just 18 kJ. How many kilojoules did David consume for lunch?

5. (a) $\frac{1}{5}+\frac{3}{4}$

   (b) $3\frac{3}{4}+1\frac{1}{2}$

6. Complete these **without** a calculator and then check your answers with the calculator.

   (a) –5 + 3

   (b) –5.6 + 78

   (c) 0.456 + –67

   (d) 89 + –567

   (e) –1.789 + 1.674

7. Translate each of the following questions into an expression involving a sum and then solve. A diagram may sometimes help you to estimate whether the answer will be positive or negative.

   (a) Naomi owes $25. If she is able to pay back $5, how much does she still owe?

   (b) A submarine dived 37 metres and then rose 23 metres. What is its new depth?

   (c) Australia started their second innings in the cricket, 134 runs behind South Africa's first innings total.

      (i) If the opening partnership scored 76 runs, what is Australia's position?

      (ii) If Australia scored 475 runs in their second innings, what is their position at the end of the innings?

   (d) From a certain floor in a building the lift descended 3 floors. It then rose 6 floors before descending 8 floors. What is the lift's final position relative to its starting position?

1.4.2 Subtraction

Example

On my previously mentioned weekend escape, I took $300. My expenses for the weekend totalled $121.

To find the amount of money I came home with I must **subtract** these expenses from the original amount of money. We call this the **difference** between the amount of money I took with me and the expenses.

That is, $300 – $121

Firstly, estimate the remaining amount.

300 = 300
121 ≈ 100

300 – 100 = 200 The remaining money is about $200

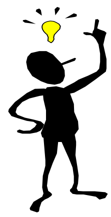
We can now do this **subtraction** on the calculator.

Find the – key on your calculator.

Key in 300 – 121. The display should read 179

$179 is close to our estimate of $200 so is a reasonable answer. The amount of money I had left after the weekend escape was $179.

Example



One concept that you will use in statistics is the difference between the average of a set of scores and a particular score. Look at the following examples and pay particular attention to the negative sign. We will look at these examples in a later module on formula.

The average take home pay for a group of people was $872. If you earned $697, how far from the average were you?

697 – 872 = –175

So you were $175 below the average.

The average temperature in December in a Northern Hemisphere country is –10° C. If the temperature one December day was 5°. How far from the average was this temperature?

5 – $^{-}$10 = 5 + 10 = 15

So the temperature was 15° above normal. Note here subtracting a negative is the same as adding.

Activity 1.5

Find the following answers **without** using a calculator. Estimate your answers before you commence. Check your answers with a calculator.

1. (a) 759 – 326

   (b) 39.2 – 14.125

   (c) 1.264 30 – 247.2

   (d) 3.7 – 0

   (e) $7 - {}^{-}2$

2. The average profit for a company over the last 4 years was \$–5 000. If this year's profit was \$8 000, how far from the average was this years profit?

3. A cleaner is asked to dilute 542 mL of disinfectant concentrate by making it into 3 500 mL of diluted solution. How much distilled water will they need to add? A diagram may help you 'picture' this situation.

4. The following table shows the weekly pay schedule for a number of employees of a small company. Gross pay is the amount of money you receive before any deductions. Deductions for these employees include taxation, superannuation and union fees. After these deductions have been made, the amount remaining is the take home pay.

Name	Gross Pay	Tax	Superannuation	Union Fees	Take Home Pay
Adams J	\$500	\$105	\$30	\$2	
Bull P	\$1 200	\$407	\$74	\$2	
Filbee Y	\$678	\$169	\$41	\$2	
Hand I	\$893	\$261	\$54	\$2	
Ruse K	\$560	\$127	\$34	\$2	
Totals					

   (a) For each employee calculate the take home pay.

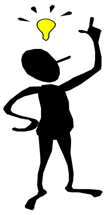
   (b) What is the total wages bill for these employees (Gross Pay)?

   (c) What amount of money will be sent to the taxation office on behalf of these employees?

5. (a) $2\frac{6}{7} - 5\frac{1}{12}$

   (b) $\frac{2}{123} - \frac{1}{23}$

Statistics deals a lot with uncertainty. We often talk about a range of values instead of an exact amount. For example the profit for next year will be one million dollars plus or minus a few hundred thousand. We can express symbolically as:



$1 000 000 ± $200 000

This means the profit could be as high as $1 200 000 or as low as $800 000.

Activity 1.6

1. Interpret the following:
   (a) 7.34 ± 0.06
   (b) 0 ± 15
   (c) $21 000 ± $1 000

1.4.3 Multiplication

Example

Suppose a builder is prepared to employ you as a casual labourer for $80 a day.

Your total weekly pay would be: $80 + $80 + $80 + $80 + $80 = $400

This is the same as saying 5 lots of $80 added together.

A short hand way to write 5 lots of $80 added together is to use **multiplication**.

That is, $5 \times 80 = 400$

We say *five multiplied by eighty equals four hundred.*

We call 5 and 80 **factors** of 400 and say that 400 is the **product** of 5 and 80.

You may be familiar with these terms in everyday language. For example, 'one of the **factors** leading to an early arrest was the accurate description given to police by the witness'. This simply means a part of the reason for the early arrest was the good description. We could also say 'the reason for an early arrest was a **product** of good police work and an observant witness'. We are saying that the early arrest was a combination of these two factors.

Multiplying numbers on the calculator

Find the × key on your calculator

Multiply 5 by 80

The display should read 400

The multiplication of decimals and fractions less than one is a bit more difficult to understand. For example when multiplying one half by a half the answer is a quarter. If you had half a cup of water and someone wanted half of it, they would get a quarter of a cup, i.e.:

Or we could say $0.5 \times 0.5 = 0.25$. Remember when you are multiplying by fractions or decimals between 0 and 1 your answer is going to be less than your original numbers.

To multiply numbers involving negatives remember:

- Positive multiplied by Negative is negative, e.g. $7 \times -4 = -28$
- Negative multiplied by Negative is positive, e.g. $-7 \times -4 = 28$

Activity 1.7

Find the following answers **without** using a calculator. Estimate your answers before you commence. Check your answer with a calculator.

1. (a) 9×45

   (b) 0.9×7

   (c) 1.95×0.2

   (d) $2\,000 \times 346$

   (e) 4.2×0.38

   (f) $^{-}500 \times 4\,000$

   (g) $^{-}52 \times {}^{-}21$

2. Sally earns $14.80 per hour and works for 38 hours in a week. How much does Sally earn?

3. Your blood alcohol concentration (BAC) is measured in grams of alcohol per 100 mL of blood. If 50 mL of blood was drawn from a driver and 0.02 grams of alcohol was measured, what would the BAC of the driver be?

4. Jeremy sees a shirt normally priced at $45 on sale for only $26.

   (a) How much would Jeremy save if he purchased a shirt at the sale price?

   (b) Jeremy decides to buy 5 shirts in different colours. How much does he save by buying them all at the sale price?

5. An express coach from Brisbane to Melbourne stops to load and unload passengers only in Sydney.

	Number of passengers	
City	**Loaded**	**Unloaded**
Brisbane	36	
Sydney	23	14
Melbourne		

(a) How many passengers travelled all the way from Brisbane to Melbourne?

(b) How many passengers got off the coach in Melbourne?

6. The average winter temperature in a city if –5ºC. Another city is twice as cold. What is the average winter temperature in this city?

7. Calculate the following:

(a) $\frac{2}{7} \times \frac{1}{3}$

(b) $0.24 \times \frac{1}{10}$

(c) 0.4×0.37

(d) 4.2×0.5

Power notation

Often you will see different symbols being used as ways to reduce the writing out of a long process. So far we have used multiplication as a short hand way of writing repeated additions. **Power notation** is a shorthand way of representing the same numbers being multiplied together several times.

Consider 6×6. This could be written 6^2. We would say *six squared or six to the power two.*

6^2 indicates that we are multiplying together two numbers which are both 6.

$6 \times 6 \times 6$ then would be written 6^3 and would mean that we are multiplying three numbers together which are all 6. We say *six to the power three or six cubed.*

See if you can write out and describe in words what is meant by 6^5

....

Did you say something like 6^5 means 5 numbers which are all 6 are multiplied together to give $6 \times 6 \times 6 \times 6 \times 6$?

What if the numbers are negative?
$-6 \times -6 = 36$ so $(-6)^2 = 36$.

Any number squared will be positive.

To evaluate powers on the calculator

There are two buttons on your calculator that we can use to evaluate powers

Find the x^2 key

and the x^y key

The x^2 key is only for finding powers of 2.

Evaluate 6^2

The display should read 36.

This key is very useful when finding powers of two but the key for finding all powers including 2 is the x^y key.

Evaluate 5^3

The display should read 125

You may not have had to use these two new keys before so here is an activity if you feel you would like more practice.

Activity 1.8

1. Evaluate the following without a calculator. Check your answers with your calculator.

   (a) 3^3

   (b) 2^4

   (c) $(-5)^4$

   (d) 4^1

   (e) 0.9^4

   (f) 0.06^5

2. Complete:

   (a) $4^{\square} = 16$ (How many times is 4 multiplied together to give 16?)

   (b) $2^{\square} = 32$

   (c) $5^{\square} = 25$

   (d) $3^{\square} = 81$

3. The Heptane family has seven children. Each of the seven children spends 7 minutes per day reading. What is the total time the children spend on reading in one week?

4. In statistics you will find a number called the variance. It is the square of another important concept in statistics called the standard deviation. If the standard deviation is 0.7, what is the variance?

The square root

What if a question on powers asked you to find a **positive number** that when raised to the power 2 gave you 49 as the answer.

That is $\boxed{}^2 = 49$

Did you get 7 for this answer?

We have a shorthand way to express this type of question.

We could write √49 and it is understood that we are meant to find the two identical numbers that multiply to give 49.

We would read √49 as *the square root of 49*

The symbol we use for the **square root** is √

To evaluate square roots on the calculator

Find the √ key on your calculator

Evaluate √49.

The display should read 7

Find the square root of 0.25

√0.25 = 0.5

Notice the square root of a number between 0 and 1 gives a larger answer. This is because 0.5 × 0.5 = 0.25 (recall page 11).

If we multiply 7 × 7, we get 49. But if we multiply –7 × –7 we also get 49. So we can say √49 = 7 or –7. This is sometimes written as ±7 (read this as plus or minus 7). Notice your calculator will only give you the positive answer. You will have to decide for yourself if you need to consider the negative answer.

Activity 1.9

1. Find a positive answer for the following. Check your answers on the calculator.

   (a) √81 Remember, we are asking what two identical numbers multiply to give 81.

   (b) √36

   (c) √25

   (d) √64

   (e) √100

   (f) √0.81

2. Evaluate the following on your calculator – give two possible answers.

   (a) √4.2

   (b) √0.05

   (c) $\sqrt{25.3+2.7}$

3. In statistics the standard deviation of a number is the square root of the variance. If the variance is 8.4, what is the standard deviation?

1.4.4 Division

Recall that multiplication is a shorthand way of adding the same number many times. Similarly, division is a shorthand way of subtracting the same number several times.

For example, to determine how many 5's are contained in the number 20, you could use subtraction as follows:

20 – 5 = 15	one step
15 – 5 = 10	two steps
10 – 5 = 5	three steps
5 – 5 = 0	four steps

Using subtraction it has taken **four** steps to determine how many 5's there are in 20. A shorthand way of writing this using division would be

$$20 \div 5 = 4$$

We would say this as *twenty divided by five equals four.*

We call the result of division a **quotient.** In our example above the quotient is 4.

It is interesting to note that the word quotient comes from the Latin word *quotiens* meaning 'how often' or 'how many times'.

Dividing by a negative or into a negative number gives a negative. For example

$$-21 \div 3 = -7$$
$$21 \div -3 = -7$$

Dividing Numbers on the Calculator

Find the ÷ key on your calculator.

Divide 20 by 5

The display should read 4

We can say that 20 ÷ 5 = 4 because there are 4 lots of 5 in 20

i.e. $4 \times 5 = 20$

Can you see that division is the opposite of multiplication. Knowing your multiplication tables will be a help when doing division as well as multiplication.

For example:

72 ÷ 8 = 9	since 9 × 8 = 72
45 ÷ 9 = 5	since 5 × 9 = 45

Before we move on to the next section let's consider divisions involving zero and divisions involving decimals and fractions.

Zero

$0 \div 6 = 0$ since $0 \times 6 = 0$ No cake divided among 6 children!
$0 \div 342 = 0$ since $0 \times 342 = 0$

What happens when it is zero that we are dividing by?

For example, $7 \div 0 = ?$ since $? \times 0 = 7$

We know that anything multiplied by zero equals zero so there is **no** number that we can write instead of the ? sign.

We say that division by zero is undefined. That is, it is impossible to divide by zero.

Try $7 \div 0$ on your calculator. You should get an error message (– *E* –)

Decimals & fractions

When dividing by numbers between 0 and 1, our answers are going to be larger than our original. Look at the following examples:

There is 21mL of medicine to be put into ampoules that will contain only 0.2mL. How many ampoules should there be?

$21 \div 0.2 = 105$

So we need 105 ampoules.

Example

In a kindergarten, there are 25 children and each child has an apple. Apples have been divided into quarters. How many quarters are there?

$25 \div \frac{1}{4} = 100$

So there are 100 quarters.

Activity 1.10

1. Complete the following **without** using a calculator. Estimate your answer before you begin. Check your answer with the calculator.

   (a) $1\,204 \div 4$

   (b) $432 \div 12$

   (c) $10\,608 \div 26$

   (d) $3612 \div 42$

2. Three children were left a \$4 500 inheritance. How much will each child receive?

3. Joseph purchased 2 700 grams of minced steak to be used for 6 family meals. He wishes to freeze the meat in meal size portions. What amount of mince should he use for each meal?

4. If you receive and annual salary of \$31 025, what is your weekly wage? Hint: you will need to work out the daily rate (365 days in a year) and then the weekly rate.

5. A merry-go-round at the local show revolves once every 32 seconds. Keely's ride lasts 8 minutes, how many times did the merry-go-round revolve? (Hint: 8 minutes equals $8 \times 60 = 480$ seconds)

6. Calculate the following:

   (a) $2.8 \div 0.4$

   (b) $2\,600 \div {}^{-}0.07$

   (c) $23 \div \dfrac{2}{9}$

1.5 Order of calculation

So far we have been dealing with expressions involving just one operation, for example

$-12 \times 6 =$

However, in reality, calculations usually involve performing more than one operation.

Consider the following situation. We are asked to calculate the amount earned by a person working for 12 hours if the pay rate is \$14 per hour for the first 8 hours and \$17 for each hour after that.

This means that the person is working 8 hours at \$14 per hour and 4 hours at \$17 per hour. We can express this mathematically as

$8 \times \$14 + 4 \times \$17$

Look carefully at this expression. Note that there are **three** operations: a multiplication, an addition and another multiplication.

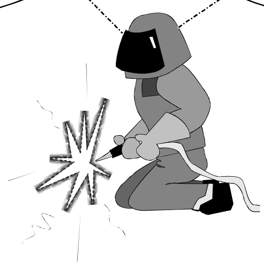
The order in which the operations are performed will determine our final result. Two possibilities are presented below.

Possibility 1

$8 \times \$14 + 4 \times \$17$	
$= \$112 + 4 \times \$17$	Evaluate Multiplication
$= \$116 \times \$17$	Evaluate Addition
$= \$1\,972$	Evaluate Multiplication

Possibility 2

$8 \times \$14 + 4 \times \$17$	
$= \$112 + 4 \times \$17$	Evaluate Multiplication
$= \$112 \times \$68$	Evaluate Multiplication
$= \$180$	Evaluate Addition



Obviously only one answer can be right! Unfortunately for the person involved, **Possibility 2** is the correct answer and the amount earned is \$180 for the 12 hours work.

Mathematical expressions are written to convey specific information, therefore everyone reading them needs to interpret them the same way. For this reason, mathematicians have established a **convention** (an accepted method) that specifies the order in which operations are to be performed.

This **order of operations convention** can be stated as:

When **working from left to right**

Step 1 Evaluate any expressions in brackets* first.
Step 2 Evaluate any powers and roots
Step 3 Evaluate any multiplications or divisions
Step 4 Evaluate any additions or subtractions

* If there are brackets inside another set of brackets, do the inside brackets first.

If you check Possibility 2 above, you will see that it follows the convention for the order of operations, whereas Possibility 1 does not.

This is a very important aspect of mathematics that you must ensure that you understand.

Follow through the examples below carefully.

Example

Evaluate $12 + 2 \times 3$

$12 + 2 \times 3$	Evaluate the multiplication first
$= 12 + 6$	Finally evaluate the addition
$= 18$	

Let's check this answer on the calculator. If you have a scientific calculator it will **automatically** apply the order of operation convention.

On your calculator press 1 2 + 2 × 3 = The display should read 18

Example

Evaluate $8 - 12 \div (7 - 4)$

$8 - 12 \div (7 - 4)$	Evaluate the brackets first
$= 8 - 12 \div 3$	Next do the division
$= 8 - 4$	Finally the subtraction
$= 4$	

To check this on the calculator you will need to find the keys for brackets on your calculator. Notice that there are opening brackets as well as closing brackets.

Example

Evaluate $12 \div -3 \times 5 + 6 - (5 - 8)$ by hand then use the calculator.

$12 \div -3 \times 5 + 6 - (5 - 8)$	Evaluate bracket first
$= 12 \div -3 \times 5 + 6 - -3$	Rewrite $6 - -3$ as $6 + 3$
$= 12 \div -3 \times 5 + 6 + 3$	Evaluate multiplication and division left to right
$= -4 \times 5 + 6 + 3$	
$= -20 + 6 + 3$	Evaluate additions
$= -14 + 3$	
$= -11$	

Now check this on your calculator.

Example

Evaluate $5^3 \times 2 - (\sqrt{81} - (7 - 3)^2 + 43)$

$5^3 \times 2 - (\sqrt{81} - (7 - 3)^2 + 43)$	Evaluate the inside bracket first
$= 5^3 \times 2 - (\sqrt{81} - 4^2 + 43)$	Evaluate the last bracket. Powers and roots firstly.
$= 5^3 \times 2 - (9 - 16 + 43)$	Now addition and subtraction left to right
$= 5^3 \times 2 - (-7 + 43)$	
$= 5^3 \times 2 - 36$	Finished brackets. Evaluate powers and roots.
$= 125 \times 2 - 36$	Evaluate multiplication and division
$= 250 - 36$	Finally addition and subtraction
$= 214$	

Now check this on your calculator.

You must be careful with negatives involving powers. Notice the following

$-7^2 = -49$
$(-7)^2 = 49.$

Can you explain the difference?

In the first instance the 7 only is squared, and this number is negative.

In the second instance the (-7) is squared, so it's -7×-7 which is positive 49.

Activity 1.11

1. Evaluate the following **without** a calculator. Check your results on the calculator.

   (a) $7 \times 5 + 4$

   (b) $10 - 6 \times 7$

   (c) $6 + (3 - 9)$

   (d) $-2 - 2 \times -3$

   (e) $9 \div 3 \times 7 + 3$

   (f) $-3 \div -1 + 9^2 \times -2$

   (g) $27 - \sqrt{9} + 21 \div -3$

   (h) $4 \times (2 + 5) \div (3 + 1)$

   (i) $(17 - 5^3) \div -3 + 9 \times -5$

   (j) $(-5 \times -4)^2 - 6 \div (-3 + \sqrt{4})$

2. Evaluate the following **without** a calculator. Estimate your answer before calculating. Check your results on the calculator.

   (a) $765 \div 15 + 822$

   (b) $89 + 21 - 48 \times 23$

   (c) $591 + 37^2 \times \sqrt{49}$

   (d) $4\,763 + 395 \div 5 \times 16$

   (e) $(62 - 24^2 + (7 + 3 \times 81) - \sqrt{169}) + 61 \times 453$

3. At Andy's Engineering Works the hourly rate for workers is \$14. Overtime is paid at \$21 per hour, while working on a public holiday pays \$28 per hour.

   (a) Ahmid works as a spray painter. In one busy week Ahmid worked 32 normal hours and 8 hours on Monday which was a public holiday. How much money did Ahmid earn in this week.

   (b) Mary works as an electrician. To catch up on outstanding work Mary agreed to work 8 hours on Show Day, a public holiday. She worked the other four days of the week at 8 hours of normal time and 2 hours of overtime each day. How much money did Mary earn in this week.

4. A coach can carry a maximum of 43 passengers. If the average passenger weighs 54 kg and carries luggage weighing 12 kg what is the usual 'load' for the full coach carries.

5. An express coach from Brisbane to Melbourne stops to load and unload passengers only in Sydney. What was the total amount paid by people using this coach?

Single Fares:

Brisbane – Melbourne	\$120
Brisbane – Sydney	\$75
Sydney – Melbourne	\$68

	Number of Passengers	
City	**Loaded**	**Unloaded**
Brisbane	36	
Sydney	23	14
Melbourne		

1.6 Percentages

Percentages were first used in the fifteenth century for calculating interest, profits and losses. Currently they have a much broader application as indicated in the newspaper items below.

...maths scores of the children who had learnt to play the piano leapt by 34 %

6% p a
Term Deposit

Surfboards, bodyboards and surfskis caused 61% of injuries....

Clearance
10 to 50% off everything.

When you insure with NRMA you receive a 10% discount on each policy

1.6.1 Converting from a fraction to a percentage

Let's consider a person with the following marks over their first three assignments for a particular subject.

Assignment 1	Assignment 2	Assignment 3
17/20	8/10	21/25

Because the assignments are all out of different marks, it is very hard just looking at these figures to decide which assignment has given the student the best result. A very convenient method of comparing these results is to make them all out of the same mark....100.

Assignment 1 $\quad \dfrac{17}{20} = \dfrac{17 \times 5}{20 \times 5} = \dfrac{85}{100}$

Assignment 2 $\quad \dfrac{8}{10} = \dfrac{8 \times 10}{10 \times 10} = \dfrac{80}{100}$

Assignment 3 $\quad \dfrac{21}{25} = \dfrac{21 \times 4}{25 \times 4} = \dfrac{84}{100}$

It is now quite easy to compare results and see that the first assignment gave the student the best result. We call the resulting comparison with 100 a **percentage** (from *per cent* meaning out of one hundred). We represent a percentage by the symbol %.

The percentage sign is thought to have been derived as an economy measure when recording in the old counting houses; writing in the fraction 25/100 of a cargo would take two lines of parchment, and hence the 100 denominator was put alongside the 25 and rearranged to become %.

Back to the above student's assignment marks expressed as percentages:

Assignment 1 85% Say 85 percent, meaning 85 out of 100.
Assignment 2 80%
Assignment 3 84%

Rather than writing the assignment mark as a mark out of 100 we can simply multiply the fraction by 100 to get the value of the percentage.

Assignment 1 $\quad \dfrac{17}{20} \times 100\% = \dfrac{17 \times 100\%}{20 \times 1} = 85\%$

Assignment 2 $\quad \dfrac{8}{10} \times 100\% = \dfrac{8 \times 100\%}{10 \times 1} = 80\%$

Assignment 3 $\quad \dfrac{21}{25} \times 100\% = \dfrac{21 \times 100\%}{25 \times 1} = 84\%$

Note that we are not changing the value of the fraction, just the look of it:

17/20 = 85/100 = 85% the percent sign meaning out of 100.

We can generalise this process.

When converting to a percentage, form a fraction and multiply by 100%.

Example

Consider the fictional property Gunadoo, a farm grazing cattle and sheep. In 1991 stock numbers indicated that times were good and food plentiful. By 1994 drought had taken its toll and Gunadoo had reduced its heard numbers. As things picked up after the drought Gunadoo again built up stock numbers. Below are the stock numbers for Gunadoo.

	1991	**1994**	**1997**
Cattle	500	100	300
Sheep	2 000	500	900

In which year did the farm have the greatest percentage of cattle compared to the entire stock?

We must firstly form a fraction and then multiply by 100% for each year.

$$\begin{aligned}
\text{Percentage of cattle in 1991} &= \frac{\text{Number of cattle}}{\text{Total number of stock}} \times 100\% \\
&= \frac{500}{2\ 000} \times 100\% \\
&= \frac{500 \times 100\%}{2\ 500} \\
&= 20\%
\end{aligned}$$

$$\text{Percentage of cattle in 1994} = \frac{\text{Number of cattle}}{\text{Total number of stock}} \times 100\%$$
$$= \frac{100}{600} \times 100\%$$
$$= \frac{100 \times 100\%}{600}$$
$$\approx 17\%$$

$$\text{Percentage of cattle in 1997} = \frac{\text{Number of cattle}}{\text{Total number of stock}} \times 100\%$$
$$= \frac{300}{1\ 200} \times 100\%$$
$$= \frac{300 \times 100\%}{1\ 200}$$
$$= 25\%$$

We can see from these figures that in 1997 Gunnadoo had the highest percentage of cattle.

Example

You pour out 200 mL from a bottle containing 1 000 mL. What percentage of the liquid did you pour out?

$$\text{Percentage poured} = \frac{\text{amount poured out}}{\text{total amount}}$$
$$= \frac{200\text{ mL}}{1\ 000\text{ mL}} \times 100\%$$
$$= \frac{2\cancel{00}\text{ mL} \times 1\cancel{00}\%}{1\ \cancel{000}\text{ mL}}$$
$$= 20\%$$

To do this on your calculator you would press the following keys.

2 0 0 ÷ 1 0 0 0 × 1 0 0 =

The display should of course read 20 and you will know to add the % sign.

As you can see it is often easier and quicker to do the calculation by hand using the technique of cancelling zeros.

Example

A team won 7 matches out of 8. What percentage did they win?

$$\text{Percentage won} = \frac{7}{8} \times 100\% = 87.5\%$$

(Check this on your calculator)

Example

Suppose I had painted 75 cm of a 3 m post. What percentage of the post has been painted.

As with previous examples, we cannot compare these two numbers while they are in differing units.

$$
\begin{aligned}
\text{Percentage painted} &= \frac{75 \text{ cm}}{3 \text{ m}} \times 100\% \\
&= \frac{75 \text{ cm}}{300 \text{ cm}} \times 100\% \\
&= \frac{75 \text{ cm} \times 1\cancel{00}\%}{3\cancel{00} \text{ cm}} \\
&= 25\%
\end{aligned}
$$

(Check this on your calculator)

Activity 1.12

1. Write as a percentage.
   (a) 8 out of 10
   (b) 250 mL out of 400 mL
   (c) 800 g out of 2 000 g
   (d) 25 cm out of 80 cm
   (e) $25 out of $60
   (f) 50 mL out of 2 L
   (g) 2×10^4 light years out of 3.5×10^3 light years

2. In a class of 50 students taking a maths test, 45 passed. What percentage of the class passed?

3. In her life a green turtle lays an average of 1 800 eggs. Of these, some 1 395 don't hatch, 374 hatchlings quickly die, and of the remaining 31, only 3 live long enough to breed. What percentage of the green turtle's eggs hatch and live long enough to breed?

4. A survey of 200 people asking what cereal they ate for breakfast found the following results:

Cereal	**Number of people**	**Percentage of people**
Corn Flakes	50	
Rice Bubbles	42	
Nutri Grain	39	
Rolled Oats	23	
Muesli	11	
Coco Pops	10	
Other Cereals	25	

Complete the table by calculating the percentage of people eating each type of cereal.

5. Consider the following figures for pedestrians killed in Queensland in 1996.

ALL AGES	OVER 70	ALCOHOL LINKED
Killed – 55	Killed – 14	Killed – 16
Taken to hospital – 405	Taken to hospital – 46	Taken to hospital – 65
Treated at the scene – 381	Treated at the scene – 20	Treated at the scene – 43
Minor Injuries – 153	Minor Injuries – 12	Minor Injuries – 11
Total – 994	Total – 92	Total – 135

HOW PEDESTRIANS ARE KILLED
Crossing carriageway at traffic lights – 6
Crossing carriageway at pedestrian crossing – 1
Crossing carriageway with no pedestrian control – 30
Stationary on road side – 6
Walking against the traffic – 2
Walking with the traffic – 7
Playing on the roads – 1
Other – 2

(a) What percentage of all people involved in pedestrian accidents are killed?

(b) What percentage of the people involved in alcohol related accidents are taken to hospital?

(c) As a pedestrian, what is the most common way to be killed? What percentage of the total deaths, die in this way?

(d) We learn as a child that we should walk on the right hand side of the road so we are facing the approaching traffic. Do these figures still support this view? What figures did you compare to come to this decision?

What happens if we have a decimal that we wish to convert to a percentage? Since we can convert any fraction to a decimal equivalent, converting a decimal to a percentage is exactly the same process as converting a fraction to a percentage.

Example

Suppose that $\frac{3}{4}$ of the children under 6 believe in Santa Claus

$$\frac{3}{4} \times 100\% = 75\%$$

We are saying that 75% of the children under 6 believe in Santa Claus.

Now we know that $\frac{3}{4} = 3 \div 4 = 0.75$

and that $0.75 \times 100\% = 75\%$

Recall that multiplying by 100 moves the decimal point 2 places to the right.

So, $\frac{3}{4} = 0.75 = 75\%$

So we can say that $\frac{3}{4}$ or 0.75 or 75% of the children under 6 believe in Santa Claus.

Example

Convert 0.93 to a percentage.

$$\begin{aligned} 0.93 &= 0.93 \times 100\% \\ &= 93\% \end{aligned}$$

Example

Convert 4.23 to a percentage.

$$\begin{aligned} 4.23 &= 4.23 \times 100\% \\ &= 423\% \end{aligned}$$

1.6.2 Converting from a percentage

Example

A well known department store is offering 15% off everything in the shop on one particular day. You are very interested in a new clock which is normally selling for \$35. What will the clock cost after the 15% **discount**?

Recall from a previous section that 15% means 15 per 100. As a fraction we can represent this as $\frac{15}{100}$

We could simplify this to be $\frac{3}{20}$

We could also represent $\frac{15}{100}$ as a decimal.

$$15\% = 15/100 = 0.15$$

You can work with a fraction or a decimal whichever you find most convenient.

Let's return to our question. We must firstly calculate how much we would save.

Discount offered is 15% of \$35

We can write this as $15\% \times \$35$ We write the 'of' as a multiplication sign.

$$\begin{aligned} &= \frac{15}{100} \times \$35 \\ &= \frac{3}{20} \times \$35 \\ &= \$5.25 \end{aligned}$$

We could use decimals in the same way.

$$\begin{aligned}&15\% \times \$35\\&= \frac{15}{100} \times \$35\\&= 0.15 \times \$35\\&= \$5.25\end{aligned}$$

Either way we have a discount of $5.25.

The price you will pay for the clock on ‘discount day’ is $35 – $5.25 = $29.75

Example

Convert 75% to a fraction and a decimal.

$$\begin{aligned}75\% &= \frac{75}{100} = \frac{3}{4}\\&= 0.75\end{aligned}$$

Example

Convert 345% to a fraction and a decimal.

$$\begin{aligned}345\% &= \frac{345}{100} = \frac{69}{20}\\&= 3.45\end{aligned}$$

Did you notice that we ended up with a number greater than 1. This is because the percentage was greater than 100%.

Converting from a percentage to a fraction or decimal is a very useful skill. There are some percentages that we use so often that it is helpful to remember their fractional equivalents. This will also help in estimating answers which you should always be doing, even if only in your head.

Some of the common percentages are:

$$10\% = \frac{1}{10}$$

$$25\% = \frac{1}{4}$$

$$50\% = \frac{1}{2}$$

$$75\% = \frac{3}{4}$$

Example

Convert 25½% to a fraction and a decimal

You know that $25\% = \frac{1}{4} = 0.25$ so you would expect this answer to be very close to this result.

Depending on the circumstances, you might do this example in two different ways.

Depending on the circumstances, you might do the question like this:

$$\begin{aligned} 25\tfrac{1}{2}\% &= \frac{25.5}{100} \\ &= 0.255 \end{aligned}$$

Recall that dividing by 100 moves the decimal point 2 places to the left.

This method is convenient if you are going on to do a calculation with your calculator.

Or, if calculating by hand you might do it like this:

$$\begin{aligned} 25\tfrac{1}{2}\% &= \frac{25.5}{100} \\ &= \frac{25.5 \times 10}{100 \times 10} \\ &= \frac{255}{1\,000} \\ &\frac{51}{200} \end{aligned}$$

To remove the decimal point, multiply top and bottom by 10.

Both of these answers are close to our estimate of $\frac{1}{4}$ or 0.25.

Let's now look at some more practical uses for percentages.

Example

Interest is paid by a bank at a rate of 4% p.a. (Note: p.a. means per annum which is really saying per year). If you invested \$2 000 for 2 years, how much money would you have?

$$\begin{aligned} \text{Interest received for one year} &= 4\% \text{ of } \$2\,000 \\ &= \frac{4}{100} \times \$2\,000 \\ &= 0.04 \times \$2\,000 \\ &= \$80 \end{aligned}$$

You could think here that 10% would give \$200 a year and 5% is half of this at \$100, so 4% must be a bit less than \$100.

Interest for two years = $\$80 \times 2 = \$160$

At the end of the two years you would have \$2 000 + \$160 = \$2 160

Example

A hospital keeps on hand a 5% glucose solution (that is, a solution that is only 5% glucose). If the glucose container holds 500 mL what portion of the container is glucose?

5% of the container is actually glucose.

That is, 5% of 500 mL

$$\begin{aligned} &= \frac{5}{100} \times 500 \text{ mL} \\ &= \frac{5 \times 500 \text{ mL}}{100} \\ &= 25 \text{ mL} \end{aligned}$$

So in a 500 mL container of 5% glucose solution, 25 mL would be glucose.

Example

In a recent survey of adult students returning to studying mathematics it was found that 49% of these students expressed some anxiety about this return to mathematics study. If the group consisted of 63 students how many were expressing some anxiety?

Let's think about our answer before continuing. 49% is about 50% which we know is 1/2. If half the students are expressing anxiety this is about 31 or 32 students.

Now to the actual calculation.

We are to find 49% of 63

$$\begin{aligned} &49\% \times 63 \\ &= 0.49 \times 63 \\ &= 30.87 \\ &\approx 31 \end{aligned}$$

We can't have a part of a student so we will round up.

So approximately 31 of the 63 students expressed some degree of anxiety.

Example

Let's look again at Gunnadoo (from page 24). When the drought began to have an effect and stock numbers were reduced, what percentage of the cattle and sheep were removed?

Whenever we are looking at a **percentage reduction** or a **percentage increase** we must put the amount of the increase or decrease over the **original amount**.

Let's look at cattle firstly on Gunnadoo.

Number of cattle removed = 400 (500 – 100) This is the amount of the reduction
Original number of cattle = 500

If 400 out of 500 of the cattle were removed the percentage reduction is going to be quite high.

The percentage decrease in cattle numbers $= \dfrac{\text{Amount of decrease}}{\text{Original number}}$

$$= \frac{400}{500} \times 100\%$$

$$= 80\%$$

The percentage decrease in sheep numbers $= \dfrac{\text{Amount of decrease}}{\text{Original number}}$

$$= \frac{1\ 500}{2\ 000} \times 100\%$$

$$= 75\%$$

On Gunnadoo the cattle were reduced by 80% while the sheep were reduced by 75%. The cattle have been reduced to a greater extent than the sheep.

Activity 1.13

1. Complete the following table.

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{2}{5}$		
	0.25	
		80%
$\frac{3}{8}$		
	0.78	
$\frac{1}{3}$		$33\frac{1}{3}\%$

2. Calculate
   (a) 12% of 250 mL
   (b) 90% of $75
   (c) 30% of 645 g
   (d) 250% of $16.40
   (e) 2% of 900 mL
   (f) 15.6% of 300 mL
   (g) 5.5% of 350 g
   (h) 70.5% of 400 mL

3. Safety experts say that 60% of children's traffic injuries could be prevented by the use of child-restraint seats. If 6 100 children are injured each year in traffic accidents, how many injuries could be prevented with the use of child-restraint seats?

4. The longest bone in the human body is the thigh bone or femur. It normally measures about 27.5% of a person's height. Calculate the approximate length of your femur.

5. When the rains came to Gunnadoo (see page 25) and things were looking better, restocking began. What percentage increase in cattle and then sheep numbers took place in 1997?

Solutions

Activity 1.1

1.
   - (a) F
   - (b) F
   - (c) T
   - (d) T
   - (e) F
   - (f) T
   - (g) T

2.
   - (a) $6 < 12$
   - (b) $0.6 > 0.12$
   - (c) $-6 > -12$
   - (d) $17 > 0$
   - (e) $41 = 41$

Activity 1.2

	Number	To nearest thousand	To nearest hundred	To nearest ten
(a)	2 575	3 000	2 600	2 580
(b)	324	0	300	320
(c)	105	0	100	110
(d)	26 897	27 000	26 900	26 900
(e)	5 502 471	5 502 000	5 502 500	5 502 470

Activity 1.3

(a) 576.21
(b) 75 201
(c) –0.01
(d) 67.3
(e) –6 400.00

Activity 1.4

1.
   - (a) 119
   - (b) 981.83
   - (c) 1.776 2
   - (d) 658

2. 25 children

3. 36 428 people

4. 4 983 kJ

5.
(a) 19/20
(b) 21/4

6.
(a) –2
(b) 72.4
(c) –66.544
(d) –478
(e) –0.115

7.
(a) –25 + 5 = –$20. She still owes $20
(b) –37 + 23 = –14. Diver is 14 metres below sea level
(c) –134 + 76 = –58 still 58 runs behind
(d) –134 + 475 = 341. Australia is now ahead by 341 runs
(e) –3 +6 + ⁻8 = –5. It is now 5 floors below its starting point

Activity 1.5

1.
(a) 433
(b) 25.075
(c) –245.9357
(d) 3.7
(e) 7 + 2 = 9

2. 8 000 – ⁻5 000 = $13 000 difference

3. 3 500 – 542 = 2 958 mL of water

4.

Name	Gross Pay	Tax	Superannuation	Union Fees	Take Home Pay
Adams J	$500	$105	$30	$2	363
Bull P	$1 200	$407	$74	$2	717
Filbee Y	$678	$169	$41	$2	466
Hand I	$893	$261	$54	$2	576
Ruse K	$560	$127	$34	$2	397
Totals	3 831	1 069			

5.
(a) $-2\frac{19}{84}$
(b) –0.027 218 or 77/2 829

Activity 1.6

(a) 7.4 or 7.28

(b) 15 or –15

(c) 22 000 or 20 000

Activity 1.7

1.
   (a) 405
   (b) 6.3
   (c) 0.39
   (d) 692 000
   (e) 1.596
   (f) $^{-}2\ 000\ 000$
   (g) 1 092

2. $562.40

3. 0.04 grams/100 mL of blood

4.
   (a) $19
   (b) 5 × 19 = $95

5.
   (a) 36 – 14 = 22 passengers
   (b) 22 + 23 = 45

6. $-5 \times 2 = {}^{-}10^{c}$C

7.
   (a) $\frac{2}{21}$
   (b) 0.024
   (c) 0.148
   (d) 2.1

Activity 1.8

1.
   (a) 27
   (b) 16
   (c) 625
   (d) 4
   (e) 0.6561
   (f) 0.000 000 7

2.
   (a) 2
   (b) 5
   (c) 2
   (d) 4

3. $7^3 = 343$

4. 0.49

Activity 1.9

1.
   (a) 9
   (b) 6
   (c) 5
   (d) 8
   (e) 10
   (f) 0.9

2.
   (a) ±2.049
   (b) ±0.223 6
   (c) ±5.291 5 (you must add 25.3 and 2.7 first and press the equal sign)

3. 2.898

Activity 1.10

1.
   (a) 301
   (b) 36
   (c) 408
   (d) 86

2. $1 500

3. 450 grams

4. $595

5. 15 times round

6.
   (a) 7
   (b) $^{-}$37 142.857
   (c) 103½

Activity 1.11

1.

(a) $7 \times 5 + 4$
$= 35 + 4$
$= 39$

(b) $10 - 6 \times 7$
$= 10 - 42$
$= -32$

(c) $6 + (3 - 9)$
$= 6 + -6$
$= 0$

(d) $-2 - 2 \times -3$
$= -2 - -6$
$= -2 + 6$
$= 4$

(e) $9 \div 3 \times 7 + 3$
$= 3 \times 7 + 3$
$= 21 + 3$
$= 24$

(f) $-3 \div -1 + \times -2$
$= -3 \div -1 + 81 \times -2$
$= 3 + -162$
$= -159$

(g) $27 - \sqrt{9} + 21 \div -3$
$= 27 - 3 + 21 \div -3$
$= 27 - 3 + -7$
$= 24 + -7$
$= 17$

(h) $4 \times (2 + 5) \div (3 + 1)$
$= 4 \times 7 \div 4$
$= 28 \div 4$
$= 7$

(i) $17 - 5^3 \div -3 + 9 \times -5$
$= (17 - 125) \div -3 + 9 \times -5$
$= -108 \div -3 + 9 \times -5$
$= 36 + -45$
$= -9$

(j) $(-5 \times -4)^2 - 6 \div (-3 + \sqrt{4})$
$= 400 - 6 \div (-3 + 2)$
$= 400 - 6 \div -1$
$= 400 - -6$
$= 400 + 6$
$= 406$

2.

(a) $765 \div 15 + 822$

Estimate	Calculate	Check
$800 \div 20 + 800$	$765 \div 15 + 822$	
$= 40 + 800$	$= 51 + 822$	
$= 840$	$= 873$	Answer looks reasonable

(b) $89 + 21 - 48 \times 23$

Estimate	Calculate	Check
$90 + 20 - 50 \times 20$	$89 + 21 - 48 \times 23$	
$= 90 + 20 - 1\,000$	$= 89 + 21 - 1\,104$	
$= 110 - 1\,000$	$= 110 - 1\,104$	
$\approx 100 - 1\,000$	$= -994$	
$= -900$		Answer looks reasonable

(c) $591 + 37^2 \times \sqrt{49}$

Estimate	Calculate	Check
$600 + \ \times 7$	$591 + 37^2 \times \sqrt{49}$	
$= 600 + 1\,600 \times 7$	$= 591 + 1\,369 \times 7$	
$\approx 600 + 2\,000 \times 7$	$= 591 + 9\,583$	
$= 600 + 14\,000$	$= 10\,174$	
$= 14\,600$		Answer looks reasonable

(d) $4\,763 + 395 \div 5 \times 16$

Estimate	Calculate	Check
$5\,000 + 400 \div 5 \times 20$	$4\,763 + 395 \div 5 \times 16$	
$= 5\,000 + 80 \times 20$	$= 4\,763 + 79 \times 16$	
$= 5\,000 + 1\,600$	$= 4\,763 + 1\,264$	
$= 6\,600$	$= 6\,027$	Answer looks reasonable

(e) $(62 - 24^2 + (7 + 3 \times 81) - \sqrt{169}) + 61 \times 453$

Estimate: $(60 - 20^2 + (7 + 3 \times 80) - 13) + 60 \times 500$
$\approx (60 - 400 + (7 + 240) - 10) + 30\ 000$
$= (60 - 400 + 247 - 10) + 30\ 000$
$\approx (60 - 400 + 200 - 10) + 30\ 000$
$= -150 + 30\ 000$
$\approx -200 + 30\ 000$
$= 29\ 800$

Calculate: $(62 - 24^2 + (7 + 3 \times 81) - \sqrt{169}) + 61 \times 453$
$= (62 - 24^2 + (7 + 243) - \sqrt{169}) + 61 \times 453$
$= (62 - 24^2 + 250 - \sqrt{169}) + 61 \times 453$
$= (62 - 576 + 250 - 13) + 61 \times 453$
$= -277 + 27\ 633$
$= 27\ 356$

Check: Answer looks reasonable

3.

(a) Ahmid earns $32 \times \$14 + 8 \times \$28$

Estimate: $30 \times 10 + 8 \times 30$
$= 300 + 240$
$= 540$

Calculate: $32 \times 14 + 8 \times 28$
$= 448 + 224$
$= 672$

Ahmid earns \$672 for the week.

(b) Mary earns $8 \times \$28 + 4 \times 8 \times \$14 + 4 \times 2 \times \$21$

Estimate: $8 \times 30 + 4 \times 8 \times 10 + 4 \times 2 \times 20$
$= 240 + 320 + 160$
$\approx 200 + 300 + 200$
$= 700$

Calculate: $8 \times 28 + 4 \times 8 \times 14 + 4 \times 2 \times 21$
$= 224 + 448 + 168$
$= 840$

Mary earns \$840 for the week.

4.

Coach's load = $43 \times (54 + 12)$ kilograms

Estimate: $40 \times (50 + 10)$
$= 40 \times 60$
$= 2\ 400$

Calculate: $43 \times (54 + 12)$
$= 43 \times 66$
$= 2\ 838$

Coach's usual load is 2 838 kilograms

5.

Number of people going Brisbane to Melbourne = 36 – 14 = 22
Number of people going Brisbane to Sydney = 14
Number of people going Sydney to Melbourne = 23
Total amount paid = $22 \times \$120 + 14 \times \$75 + 23 \times \$68$

Estimate: $20 \times 100 + 10 \times 80 + 20 \times 70$
$= 2\ 000 + 800 + 1\ 400$
$= 4\ 200$

Calculate: $22 \times 120 + 14 \times 75 + 23 \times 68$
$= 2\ 640 + 1\ 050 + 1\ 564$
$= 5\ 254$

Total amount paid to bus company is \$5 254

Activity 1.12

1.

(a) 8 out of 10 = $\frac{8}{10} \times 100\% = 80\%$

(b) 250 mL out of 400 mL = $\frac{250}{400} \times 100\% = 62.5\%$

(c) 800 g out of 2 000 g = $\frac{800}{2\,000} \times 100\% = 40\%$

(d) 25 cm out of 80 cm = $\frac{25}{80} \times 100\% = 31.25\%$

(e) \$25 out of \$60 = $\frac{25}{60} \times 100\% \approx 41.7\%$

(f) 50 mL out of 2 L = 50 mL out of 2 000 mL = $\frac{50}{2\,000} \times 100\% = 2.5\%$

(g) 2×10^4 light years out of 3.5×10^3 light years

$$\begin{aligned} &= \frac{2 \times 10^4}{3.5 \times 10^3} \times 100\% \\ &= (0.5714... \times 10^{4-3}) \times 100\% \\ &\approx 571.43\% \end{aligned}$$

2. The percentage passing = $\frac{45}{50} \times 100\% = 90\%$

   Therefore 90% of the students pass.

3. Percentage living to breed = $\frac{3}{1\,800} \times 100\% \approx 0.17\%$

   The number of turtles living long enough to breed is only 0.17% of the eggs laid. Sometimes you will see this written as 0.17 of one percent, emphasising that this percentage is less than one percent.

4.

Cereal	**Number of people**	**Percentage of people**
Corn Flakes	50	$\frac{50}{200} \times 100\% = 25\%$
Rice Bubbles	42	$\frac{42}{200} \times 100\% = 21\%$
Nutri Grain	39	$\frac{39}{200} \times 100\% = 19.5\%$
Rolled Oats	23	$\frac{23}{200} \times 100\% = 11.5\%$
Muesli	11	$\frac{11}{200} \times 100\% = 5.5\%$
Coco Pops	10	$\frac{10}{200} \times 100\% = 5\%$
Other Cereals	25	$\frac{25}{200} \times 100\% = 12.5\%$
	200	100%

5.

(a) $\dfrac{\text{Number killed}}{\text{Total involved in accidents}} \times 100\% = \dfrac{55}{994} \times 100\% \approx 5.5\%$

Approximately 5.5% of pedestrians involved in accidents are killed.

(b) $\dfrac{\text{Taken to hospital}}{\text{Total alcohol linked}} \times 100\% = \dfrac{65}{135} \times 100\% \approx 48.1\%$

Approximately 48.1% of people involved in alcohol related accidents are taken to hospital.

(c) The most common way that pedestrians are killed is by crossing the road where there is no pedestrian control.

$$\frac{\text{Number killed crossing road}}{\text{Total number killed}} \times 100\% = \frac{30}{55} \times 100\% \approx 54.5\%$$

Approximately 54.5% of pedestrian deaths are caused by crossing the road where there is no pedestrian control.

(d) Of the 9 people killed walking near traffic, seven are killed walking with the traffic and only two are killed walking against the traffic as recommended. That is, about 78% are killed disobeying the childhood rules. This would suggest that this rule still holds true today.

Activity 1.13

1.

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{2}{5}$	0.4	40%
$\frac{1}{4}$	0.25	25%
$\frac{4}{5}$	0.8	80%
$\frac{3}{8}$	0.375	37.5%
$\frac{78}{100} = \frac{39}{50}$	0.78	78%
$\frac{1}{3}$	0.3	$33\frac{1}{3}\%$

2.

(a) 12% of 250 mL = $\frac{12}{100} \times 250$ mL = 30 mL

(b) 90% of \$75 = $\frac{90}{100} \times \$75 = \$67.50$

(c) 30% of 645 g = $\frac{30}{100} \times 645$ g = 193.5 g

(d) 250% of \$16.40 = $\frac{250}{100} \times \$16.40 = \$41.00$

(e) 2% of 900 mL = $\frac{2}{100} \times 900$ mL = 18 mL

(f) 15.6% of 300 mL = $\frac{15.6}{100} \times 300$ mL = 46.8 mL

(g) 5.5% of 350 g = $\frac{5.5}{100} \times 350$ g = 19.25 g

(h) 70.5% of 400 mL = $\frac{70.5}{100} \times 400$ mL = 282 mL

3. Injuries preventable = $\frac{60}{100} \times 6\,100 = 3\,660$

Therefore, 3 660 injuries could be prevented with the use of child-restraint seats.

4. For a person 165 cm tall the length of the femur will equal:

$\frac{27.5}{100} \times 165$ cm = 45.375 cm

5. Percentage increase in cattle:

$$= \frac{\text{Number of increase}}{\text{Original number}} \times 100\%$$
$$= \frac{200}{100} \times 100\%$$
$$= 200\%$$

Percentage increase in sheep:

$$= \frac{\text{Number of increase}}{\text{Original number}} \times 100\%$$
$$= \frac{400}{500} \times 100\%$$
$$= 80\%$$

In 1997 the cattle numbers were increased by 200% while the sheep numbers were increased by only 80%.

Module 2

Formulas

Aims

In Data Analysis we often generalise about and find relationships between data we collect. One of the best ways to show these relationships or generalisations is to use formulas. This module will revise some of the important rules when looking at formulas and look at how they can be applied in Data Analysis.

When you have successfully completed this module you should be able to:

- use mathematical formulas;
- rearrange and solve algebraic equations;
- develop and solve equations relating to practical situations.

2.1 Introduction

Consider the following sentences.

Gary once gave Gary's wife a gift which Gary's wife found so awful that Gary's wife threw this gift in the bin. No matter how many times Gary asked Gary's wife about the gift, Gary's wife never mentioned the gift again.

It is very difficult to read as it stands. In English we replace many of these repeated words with **pronouns** (words that replace nouns). We are very familiar and comfortable with these words in everyday life. Using pronouns, the above sentences become much easier to read.

Gary once gave **his** wife a gift which **she** found so awful that **she** threw **it** in the bin. No matter how many times Gary asked **his** wife about **it**, **she** never mentioned **it** again.

Whenever we see **she** in the above text, we know that this refers to Gary's wife, and whenever we see the word **it** we know it refers to the gift.

Let's look at another sentence.

Shirley is always 3 years older than Jeffrey.

We can say that if Jeffrey is 4 years old then Shirley will be three years older than Jeffrey, meaning that Shirley is 7 years old. Or when Jeffrey is nineteen years old, Shirley will be three years older than Jeffrey, meaning that Shirley is twenty-two years old.

These sentences become hard to understand as were the sentences above. So in mathematics we introduce **pronumerals** to make the sentences easier to understand. Pronumerals and **variables** are the same thing. We will use the word variable throughout this module.

If we let S represent Shirley's age in years and J represent Jeffrey's age in years we know that the relationship between Jeffrey and Shirley is $S = J + 3$.

We can now rewrite the above sentences using variables (pronumerals) just as we re-wrote the original sentences using pronouns.

If $J = 4$, then $S = J + 3$, so $S = 7$. Or when $J = 19$, $S = J + 3$, so $S = 22$.

As long as we know what is meant by J and S we will understand these new sentences.

In Data Analysis, we use a variety of variables. For example, some variables are related to what they stand for: r stands for the regression coefficient; n stands for the sample size. Sometimes we wish to differentiate between variables that are similar, Here we often use subscripts. For example h_A could stand for the heights of people in Asia and h_E could stand for the heights of people in Europe. Other times we will use letters from the Greek alphabet. An α (alpha) is a common Greek variable used in statistics, as is β (beta) and sometimes χ (chi) although this is often written as X.

2.2 Simplifying expressions

This process of using letters to represent numbers is called **algebra**. The word algebra comes from the name of a book *Al-jabr wa'l Muqabalah* written by an Arabic mathematician, Al-Khowarizmi, in the early ninth century. The title of the book means something like 'restoration and balancing' and this will become an important part of solving and rearranging equations in this module.

Let's look at some statements and their equivalent mathematical statements.

- The perimeter of a square is four times the length of one side.
  $P = 4 \times s$ where P represents the **perimeter**,
  $P = 4s$ and s represents the **length** of one side.

- The bank charges 8% interest on my loan.
  $I = 8\% \times A$ where I represents the **interest charged** on the loan in dollars,
  $I = 8/100 \times A$ and A represents the **amount** of the loan in dollars.
  $I = 0.08 \times A$

- The adult weighed three times as much as the child.
  $A = 3 \times C$ where A represents the **weight** of the adult,
  $A = 3C$ and C represents the **weight** of the child.

- The grevillea was half the height of the palm tree.
  $G = 1/2P$ where G represents the **height** of the grevillea,
  and P represents the **height** of the palm tree.

- The house was 15 metres longer than it was wide.
  $L = W + 15$ where L represents the **length** of the house,
  and W represents the **width** of the house.

- To find the deviation from the mean, we subtract the mean from the value.
  $D = x - \bar{x}$ where x is the value, and $\bar{x}$ is the mean.

- Find the proportion of women who are less than 175 cm tall.
  $p < 175$ where p represents the proportion of women.

Remember that the letters we have used are called variables because they can vary to take on any value that we give them.

We can also call these formulas **equations**, because the expression on the left hand side (LHS) is **equal** to the expression on the right hand side (RHS).

Let's look at some different examples.

Example

Write an equation to represent the following situation. Simplify the equation.

I think of a number and add 5. The result is 23.

Just as we define the variables when we write a formula, we must define the variables when we write an equation.

In this case we could say let N be the number I thought of.

Then, $N + 5 = 23$

Example

If Toby weighs three times as much as Harold, write an equation to represent the situation.

$T = 3 \times H$ where T represents Toby's weight,
$T = 3H$ and H represents Harold's weight.

Here are some questions for you to do.

Activity 2.1

1. Write an equation to represent each of the following situations.
   (a) A number plus 5 gives 35.
   (b) A number minus 9 is equal to 2 times the same number.
   (c) 6.5 times a number equals –3.4
   (d) Five times a number gives 22.
2. Write an equation to represent each of the following situations.
   (a) Ben is five times older than John.
   (b) The length of the rectangle was 7 metres more than the width.
   (c) Katie is five years younger than Jack.
   (d) Barry earns one third the amount of money that Harry earns.
   (e) The sample proportion is between 0.25 and 0.35
   (f) At least 4 of the students are overweight.

In the previous Activity you were required to write some equations. Just to remind you of the difference between expressions and equations. An **expression** might involve variables, numbers and symbols (+, –, ×, ÷, √, ...) but **no equals sign**. An **equation** on the other hand has an **equals sign** and indicates that two expressions are equal.

Example

Write an expression to represent the following situation and then simplify the expression.

Two times a number plus five times the same number.

We must define the variable. Let the unknown number be x

Then, $2 \times x + 5 \times x$ becomes the required expression.

It is not necessary to include the multiplication sign, so we could rewrite this expression as:

$$2x + 5x$$

We call $2x$ and $5x$ **like terms** because they contain the **same power of the same variable**. Similarly, $3x^2$ and $9x^2$ are like terms because they contain the same power of the same variable. The number in front of the variable, the **coefficient**, does not influence whether terms are like or not.

When we have like terms, we can apply the distributive law to simplify the expressions.

That is,
$$\begin{aligned} & 2x + 5x \\ &= 2 \times x + 5 \times x \\ &= (2 + 5)x \qquad \text{applying the distributive law.} \\ &= 7x \end{aligned}$$

We could rewrite our expression as $7x$

Let's look a little more closely at like terms.

Examples

$6a$ and $2a$ are like terms.

$5x^3$ and $-7x^3$ are also like terms because they have the same power of x, that is, x^3. The coefficients, 5 and –7 do not influence our decision on like terms.

$6x^4$ and $3y^4$ are **not like terms** because they contain different variables.

$2x^2$ and $8x^3$ are **not like terms** because they have different powers of the same variable.

Example

Sort the following expressions into groups of like terms. $7x^2$, $5x$, $3x^2$, $6x$

We would group $7x^2$ and $3x^2$ together as like terms and $5x$ and $6x$ are like terms.

Example

Simplify $3x + 2x$

Are $3x + 2x$ like terms? Yes, because they have the same power of the same variable.

Then $3x + 2x$
$= (3 + 2)x$
$= 5x$

Example

Write an expression to represent the following situation, then simplify the expression.

Three times a number squared plus nine times the same number squared.

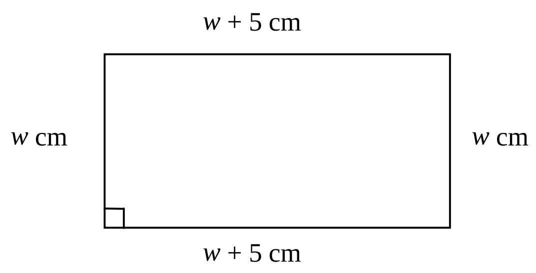
Let the number be x.

Then, $3 \times x^2 + 9 \times x^2$
$= 3x^2 + 9x^2$
$= (3 + 9)x^2$ applying the distributive law.
$= 12x^2$

Example

A rectangle has its length 5 cm longer than its width. If the perimeter is 62 cm, write an equation to represent the situation. Simplify the equation.

For questions like this a diagram is very helpful. Let the width of the rectangle be w cm, then the length must $w + 5$ cm, since the length is 5 cm more than the width.



Now, the perimeter as you will recall, is the distance around an object. In this case the distance around the rectangle is 62 cm. We represent this as the equation:

$$w + 5 + w + w + 5 + w = 62$$

To simplify the expression on the left hand side (LHS) we must look for like terms. Yes, there are like terms, because the w's are the same power of the same variable.

Now look at the coefficients. If there is no number in front of the variable, then the coefficient is understood to be 1. That is, if we have w it is understood that we have 1 lot of w, or $1w$.

Let's write our equation out with this new information.

$$1w + 5 + 1w + 1w + 5 + 1w = 62$$

On the LHS we can now group like terms.

$$\begin{aligned} 1w + 1w + 1w + 1w + 5 + 5 &= 62 & \\ (1 + 1 + 1 + 1)w + (5 + 5) &= 62 & \quad \text{Using the distributive law} \\ 4w + 10 &= 62 & \quad \text{Simplifying.} \end{aligned}$$

Remember:

- You can only add or subtract like terms.
- If a term is just x, x^2, etc., then the coefficient is one (that is, $1x$ and $1x^2$).
- Take care when regrouping terms with negatives.

Activity 2.2

1. Sort the following expressions $3x$, $5a$, $7a^2$, $6x^2$, $7x$, $9a$, $17x$, $5x^2$, $6a^2$, $8x$, $9x^2$, $12a$, $-4x$, $-11x^2$, $-3a$, $-2x$, $2a^2$ into groups of like terms.
2. Simplify the following expressions where possible.
   (a) $5x + 6x$
   (b) $6x^2 + 11x^2$
   (c) $x + 3x$
   (d) $7x - 3x$
   (e) $9x - 14x$
   (f) $-8x - 4x$
   (g) $-3.1x + {}^{-}2.4x$
   (h) $4x + 32x - 7x$
3. The length of a house is twice the width. If the perimeter of the house is 72 m write an equation to represent the situation. Simplify the equation.
4. A 75 metre long piece of rope is to be cut into two pieces. One piece is to be 15 metres longer than the other. Write an equation to represent the situation. Simplify the equation.

2.3 Relationships in words

This is probably the method of expressing a relationship that you are most familiar with at this stage.

For example,

- Chris earns twice as much as David.
- At the restaurant there are 5 more women than men.
- The bank charges 8% interest on my loan.
- The grevillea was half the height of the palm tree.
- The adult weighed three times as much as the child.
- The perimeter of a square is four times the length of one side.
- The house was 15 metres longer than it was wide.

While it is important that we are able to express in words what we are trying to find, it is more convenient to use a formula when we need to do some calculations using these relationships.

2.4 Relationships as formulas

We have already looked at expressing the relationship 'Shirley is 3 years older than Jeffrey' as a formula.

That is, $S = J + 3$ where S represents Shirley's age in years, and J represents Jeffrey's age in years.

It is important if we are going to express our relationship as a formula using letters, that we know exactly what these letters represent. In our case, we have let S equal Shirley's age in years and J equal Jeffrey's age in years. We could have chosen any letters we liked but these two clearly help us to identify the required relationship. It is also important to note that it is not just Shirley and Jeffrey that we are comparing, but their ages. We should also note that their ages are always measured in years.

In this case we call S and J **variables** because they can vary to take on any values for S and J that we give them.

Let's look at some of the word relationships from the previous section.

Example

Translate the following relationship into a formula.

Chris earns twice as much as David.

What are the variables in this case?

....

You could have let the variables be *C representing the amount of money that Chris earns* and *D representing the amount of money that David earns*. Note that it is the amount of money that each earns that is the variable, not the person.

Before you go on to write a formula, think about the question. You know that Chris is the person earning the most money. Remove from the question any unnecessary words.

Chris earns twice David
or, C earns 2 times D

Put the symbol $\times$ for times.

C earns $2 \times D$

Finally put in the equals sign to form the formula.

$C = 2 \times D$ where C represents the amount of money that Chris earns in dollars, and D represents the amount of money that David earns in dollars.

It is an acceptable shorthand to leave the multiplication sign out of expressions involving variables. We say the multiplication is **implied or understood**.

So we could write $C = 2D$ and we 'know' that we must multiply the 2 and the D.

Example

Translate the following relationship into a formula.

At the restaurant there are 5 more women than men.

We will firstly define the variables we will use.

Let W represent the number of women at the restaurant,
and M represent the number of men at the restaurant.

To write the formula you could use one of two methods.

The 'skeleton method' where you could set up the skeleton of the formula with lots of room.

$$W = M$$

There are more women than men to we must add the 5 to the number of men to get the number of women.

$$W = M + 5$$

The other method that you might use is to rearrange the question.

Five more women than men

or, Number of women is 5 more than number of men

Number of women is men plus 5

W is M plus 5

$W = M + 5$

Example

Translate the following relationship into a formula.

The bank charges 8% interest on my loan.

The formula using one of the above methods, could be written:

$I = 8\% \times A$

$I = \frac{8}{100} \times A$

$I = 0.08 \times A$

where I represents the interest charged on the loan in dollars, and A represents the amount of the loan in dollars.

We could also write this as $I = 0.08A$

Since it doesn't matter which letters are chosen for variables in a formula (provided that it is understood what the letters mean) this formula could also be written as:

$y = 0.08x$ where y represents the interest charged on the loan in dollars, and x represents the amount of the loan in dollars.

It is most important that the meaning of the variables be understood.

Formulas can involve two variables, just like the two examples above, but you might also see a relationship which involves only one variable and sometimes ones involving 3, 4 or more variables.

Example

The number of people in Toowoomba is approximately 90 000. We could write this as the formula

N = 90 000 where N is the approximate population of Toowoomba.

This is an example of a formula with one variable.

Example

The formula for finding the perimeter of a rectangle is

$P = 2(L + W)$	where	P represents the perimeter,
		L represents the length of the rectangle,
	and	W represents the width of the rectangle.

This is an example of a formula with three variables.

You might also notice that the formula for the perimeter of a rectangle contains two operations, namely addition and multiplication. The multiplication sign is understood to be after the 2, that is, $P = 2 \times (L + W)$

In general, a formula can contain any number of different operations (addition, subtraction, multiplication, division, powers, square roots.......) depending on the relationship being described.

Activity 2.3

1. Express the following relationships as formulas.

   (a) The distance (d) travelled be a car is equal to its speed (s) multiplied by the time (t) of the journey.

   $d =$

   (b) Profit (P) equals the revenue income (R) less the costs (C).

   $P =$

   (c) Electrical voltage (V) equals electrical current (I) times electrical resistance (R).

   $V =$

2. Express the following relationships as formulas.

   (a) The grevillea was half the height of the palm tree.
   (b) The adult weighed three times as much as the child.
   (c) The perimeter of a square is four times the length of one side.
   (d) The house was 15 metres longer than it was wide.

3. A shop stocks four times as many cola softdrink bottles as soda water. Write a formula to calculate the number of cola softdrink bottles given the number of soda water bottles.

4. If taxable income is greater than $33 500 in the Chook Islands, tax is calculated by subtracting $9 500 from the taxable income, then finding 39% of this difference, then adding $9 500. Write a formula to calculate the tax to be paid from the taxable income.

5. A small company pays their salesperson Mae Ihelpyu $200 per week plus $50 for each item that she sells in the week. Write a formula to calculate Mae's weekly income for any number of items sold.

2.5 Substituting into formulas

You will find that in subjects such as statistics, there are quite a lot of formulas. One of the main skills that are required of you, apart from determining which formula to use, is to be able to substitute values into a formula.

Recall our formula relating the ages of Shirley and Jeffrey.

$S = J + 3$ where S represents Shirley's age in years, and J represents Jeffrey's age in years.

By replacing the symbol for Jeffrey's age with a numerical value I am able to find Shirley's age.

When Jeffrey was 14 years old, how old was Shirley?

That is, when $J = 14$, what does S equal? We will **substitute** 14 for the J.

$$\begin{aligned} S &= 14 + 3 \\ &= 17 \end{aligned}$$

So, when Jeffrey was 14 years old, Shirley was 17 years old. (Shirley is still 3 years older than Jeffrey as she always will be.)

Find Shirley's age when Jeffrey is 19 and when he is 35.

You should have found that when Jeffrey is 19, Shirley is 22 and when he is 35, Shirley is 38.

Example

The formula for the area of a square is

$A = s^2$ where A represents the area of the square, and s represents the side length of the square.

When the length of the side of the square is 3 cm the area of the square will be

$$\begin{aligned} A &= s^2 \\ &= s \times s \\ &= 3\text{ cm} \times 3\text{ cm} \\ &= 9\text{ cm}^2 \end{aligned}$$

Remember to multiply the units. $\text{cm} \times \text{cm} = \text{cm}^2$

Find the area of a square with a side length of 9 cm.

$$\begin{aligned} A &= s^2 \\ &= s \times s \\ &= \boxed{}\text{ cm} \times \boxed{}\text{ cm} \\ &= \boxed{}\text{ cm}^2 \end{aligned}$$

Example

The formula for speed is

$s = d/t$ where s represents speed,
d represents the distance travelled,
and t represents the time taken.

A giant tortoise can cover 100 metres in 20 minutes, what is the speed of the tortoise?

We need to be very careful with units when calculating speeds.

$$s = \frac{d}{t}$$

$$s = \frac{100\text{ m}}{20\text{ minutes}}$$

$$s = 5\text{ m/minute}$$

If a human can walk at about 4 km/h how does this compare to the giant tortoise?

We cannot compare the two speeds at the moment because they are in different units.

Convert the tortoises speed to km/h. ....

Did you get 0.3 km/h? (5 m/minute = 0.005 km/minute = 0.005 × 60 km/hour)

We can now see that the human can walk much faster than the tortoise. In fact, using our knowledge from the previous module, the human can walk more than 13 times faster than the tortoise.

Activity 2.4

1. The formula for the area of a rectangle is:

   $A = lw$ Where l represents the length of the rectangle, and w represents the width of the rectangle.

   Find the area of the rectangle for the following lengths and widths.

   (a) l = 7 cm w = 5 cm
   (b) l = 34 cm w = 21 cm
   (c) l = 3 m w = 67 m
   (d) l = 4 cm w = 82 mm

2. In a square the perimeter is always 4 times the length of the side.

   (a) Write this as a formula.

   (b) Find the perimeter of squares with the following side lengths.

   (i) 4 cm
   (ii) 7 m
   (iii) 2.6 mm
   (iv) 3/4 m

(c) Suppose that you had a square chicken pen, of side length 3.2 metres. You wish to fence it with chicken wire costing $6.50 a metre. How much will it cost to fence the chicken pen?

3. The ratio of the circumference of a circle to its diameter is equal to π (Greek letter 'pi). Another way to look at this is to say that the circumference is equal to π times the diameter.

   (a) Write a formula that will allow us to find the circumference of the circle, given the diameter.

   (b) Find the length of the circumference, rounded to one decimal place, for circles with the following diameters:

   (i) 3 cm
   (ii) 72 mm
   (iii) 6.5 m
   (iv) 3/5 m

   (c) Suppose that you have built a circular rose garden with diameter 3.8 metres in your yard. You wish to purchase a log type edging that comes in rolls of three metres costing $18.95 for roll. How many rolls of edging should you purchase, and what will be the cost?

4. If I stand on top of a cliff S metres high and drop a stone, it will take t seconds to reach the ground. The height of the cliff can be found using the following formula.

   $S = 4.9\, t^2$

   Use this formula to find the height of the cliff when it takes the following times for the stone to reach the ground.

   (a) 3 seconds

   (b) 8 seconds

   (c) 1.4 seconds

5. Observation towers are often built in forests to allow bushfires to be spotted quickly. The distance (D) in kilometres that can be seen from a tower of height h metres is given by:

   $$D = 8\sqrt{\frac{h}{5}}$$

   Find the distance that can be seen from towers with the following heights. Round your answers to the nearest kilometre.

   (a) 10 metres

   (b) 25 metres

   (c) 17.5 metres

6. We can quickly find the approximate area of skin on our body by using the following formula.

   $A = \frac{3}{5} h^2$ where A represents the area of skin, and h represents the person's height.

   (a) This formula only gives approximate answers for average people. Explain why it wouldn't be accurate for all people?

   (b) Calculate the area of skin covering people with the following heights.

   (i) 165 cm
   (ii) 175.5 cm
   (iii) 1.68 m

   (c) If each of these people suffered burns to 24% of their bodies, calculate the area of skin that is burnt.

7. To check whether a person is the correct weight for their height, health workers now calculate the Body Mass Index. It is found by using the following formula.

   $\text{Body Mass Index} = \frac{w}{h^2}$ where w represents the weight in kilograms, and h represents the height in metres.

   For a person to be healthy their Body Mass Index should lie between 19 and 25 for a female and 20 to 26 for a male.

   Calculate the Body Mass Index for the following people.

   (a) Female, 165 cm, 65 kg

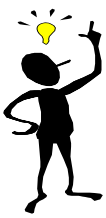
   (b) Male, 178 cm, 85 kg

   (c) Female, 150 cm, 75 kg

   (d) you

   Which of these people fall into the healthy category?

8. In statistics the formula for two sample t confidence interval is



   $$(\bar{x}_1 - \bar{x}_2) \pm t\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

   calculate the t confidence interval when:

   $\bar{x}_1 = 10$
   $\bar{x}_2 = 12$
   $t = 0.6$
   $s_1 = 2.7$
   $s_2 = 2.4$
   $n_1 = n_2 = 25$

2.6 Rearranging formulas

Occasionally we might be given the value of the subject of a formula and have to work out one of the other variables.

Let's return to Shirley and Jeffrey.

We know that Shirley is three years older than Jeffrey and that this relationship is represented by the following formula or equation.

$S = J + 3$ where S represents Shirley's age in years, and J represents Jeffrey's age in years.

By replacing the symbol for Jeffrey's age with a numerical value we are able to find Shirley's age.

Supposing, however, that we knew Shirley's age was 24 years and we had to work out Jeffrey's age.

That is, when $S = 24$, what does J equal? In this case we could **substitute** 24 for S.

$24 = J + 3$

Then **solve** the equation. Alternatively we could **re-arrange** the formula we started with and make J the subject instead of S, after all in this question we want to find Jeffrey's age!

Let's look at a simpler example to begin with.

Example

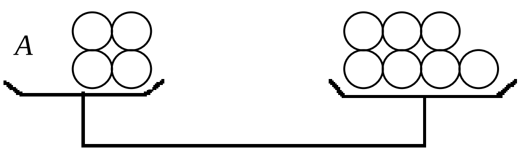
If Adam's age plus 4 equals 7, how old is Adam. (Pretend that you can't work it out!)

Representing this as an equation we get:

$A + 4 = 7$ where A represents Adam's age in years.

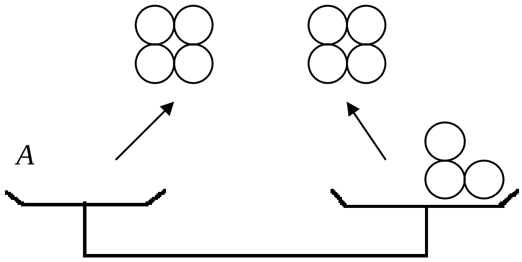
To find Adam's age we must make A the subject of the equation. That is, we must get the A on its own.

Now an equation can be thought of as a balance, if we add or take something from one side we must do the same to the other or we will upset the balance.



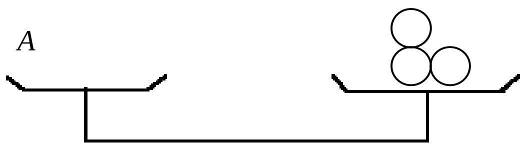
$A + 4 = 7$

If we want to leave the *A* on its own we must remove the 4 that is with it. If we take the 4 from one side we must do the same to the other.



$A + 4 - 4 = 7 - 4$

We are now left with:



$A = 3$

We can now say that Adam must have been 3 years old.

Let’s just check that this is true.

We started by saying that:

$A + 4 = 7$ where *A* represents Adam’s age in years.

and we **rearranged** (or **solved**) the equation:

$$\begin{aligned} A + 4 &= 7 \\ A + 4 - 4 &= 7 - 4 \\ A &= 3 \end{aligned}$$

When working with equations, try to keep the equals signs under each other.

to find that $A = 3$, or we found that Adam is 3 years old.

To check this answer we can substitute our answer into the original equation. We should look at the left and right hand sides of the equation separately.

The left hand side (LHS) of the equation becomes 3 + 4. We write this

$$\begin{aligned} \text{LHS} &= 3 + 4 \\ &= 7 \\ &= \text{RHS} \end{aligned}$$

Therefore our answer must be correct.

So, if $A + 4 = 7$
then $A = 3$

We have rearranged the equation to make *A* the subject.

Let's return to our original equation.

Example

We know that Shirley is three years older than Jeffrey and that Shirley is 24 years old. To find Jeffrey's age we can rearrange our original formula so that J is the subject and then substitute into this.

To make *J* the subject we must get the *J* on its own. To do this we must remove the 3, but we must keep the balance by removing 3 from the other side of the equation as well. Since the 3 is added we must undo the addition be subtracting 3 from both sides.

$S = J + 3$	
$S - 3 = J + 3 - 3$	Keep the equals signs under each other.
$S - 3 = J$	
$J = S - 3$	Writing the *J* on the left hand side of the equation.

Now if Shirley is 24 years old then:

$$J = S - 3$$
$$J = 24 - 3$$
$$J = 21$$

Therefore Jeffrey is 21 years old, three years younger than Shirley as we would expect.

Alternatively we could have substituted $S = 24$ into the original formula and then solved for T.

$$S = J + 3$$
$$24 = J + 3$$
$$24 - 3 = J + 3 - 3$$
$$21 = J$$

Again we show that if Shirley is 24 years old, then Jeffrey must be 21 years old.

So far we have only looked at rearranging equations that involved addition or subtraction. Let's look at some equations involving multiplication and division.

Example

The adult weighed three times as much as the child.

$A = 3 \times C$	where *A* represents the **weight** of the adult,
$A = 3C$	and *C* represents the **weight** of the child.

Now, what if we wanted to find the weight of the child given the weight of the adult? We will need to make *C* the subject of the equation. This time, the *C* is multiplied by the 3 so to remove the 3 we must do the opposite of multiplication, that is, division.

We must divide each side by 3.

That is,

$$A = 3C$$

$$\frac{A}{3} = \frac{3C}{3}$$ Cancel the 3's on the top and bottom.

$$\frac{A}{3} = C$$

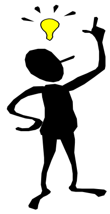
$$C = \frac{A}{3}$$ Writing the subject on the LHS.

To find the age of the child we must divide the adult's age by 3.

Example

Sometimes the formula that we are working with might have a number of variables to move. Consider the following equation that you would use in Data Analysis

$$z = \frac{x - \mu}{\sigma}$$



(The symbols μ and σ are Greek letters and represent the mean and standard deviation of a group of numbers. You will learn more about this in Data Analysis.)

Supposing that we know $z = 2$, $\mu = 3$ and $\sigma = 0.8$. To calculate the value of x we can either make x the subject and then substitute in the values **or** substitute the values into the formula and then solve it. We shall look at both methods.

Firstly we shall rearrange the formula to make x the subject.

In this example we have a division ($\div\ \sigma$) and a subtraction ($-\ \mu$) on the RHS.

Which do we attend to first?

Let's think about what you would do if you knew a value for x.

If you were to **substitute** this value for x into the equation, you would firstly have to subtract μ and then divide by σ to get a value for z.

When rearranging equations we must do the opposite of these steps in the reverse order. We can think of this in terms of the following everyday occurrence.

Think of getting dressed. The first item/s of clothing you put on will be your underclothes (unless of course you are superman). This is followed by your outer clothing for the day, and finally a coat if necessary.

To undress at a later stage, instead of adding clothes you are taking them off. You also need to take off your clothes in the opposite order to that in which you put them on. That is, you remove the coat, then the outer clothes and finally your underclothes.

Recall the opposite of each operation.

Operation	**Opposite**
+	–
–	+
×	÷
÷	×

So for our example above $z = \dfrac{x-\mu}{\sigma}$ to rearrange the equation we must multiply by σ and then add μ.

Let's look at this diagrammatically.

$$\boxed{x} \xrightarrow{\text{Subtract } \mu} \boxed{x-\mu} \xrightarrow{\text{Divide by } \sigma} \boxed{\dfrac{x-\mu}{\sigma}}$$

If you were to substitute a value for x into the equation, you would firstly have to subtract by μ and then divide by σ to get a value for y.

To rearrange the equation we must do the opposite of the above steps in the opposite order. Reading from right to left we have:

$$\boxed{x} \xleftarrow{\text{Add } \mu} \boxed{x-\mu} \xleftarrow{\text{Multiply by } \sigma} \boxed{\dfrac{x-\mu}{\sigma}}$$

$$\begin{aligned}
z &= \frac{x-\mu}{\sigma} \\
\sigma \times z &= \frac{x-\mu}{\sigma} \times \sigma && \text{(multiplying both sides by } \sigma\text{)} \\
\sigma \times z &= x-\mu \\
\sigma \times z + \mu &= x-\mu+\mu && \text{(adding } \mu \text{ to both sides)} \\
\sigma z + \mu &= x \\
x &= \sigma z + \mu && \text{(putting the subject on the left hand side)}
\end{aligned}$$

So in order to calculate our answer we then substitute $z = 2$, $\mu = 3$ and $\sigma = 0.8$ into the rearranged formula shown above.

$$\begin{aligned} x &= \sigma z + \mu \\ &= 0.8 \times 2 + 3 \\ &= 4.6 \end{aligned}$$

Alternatively we could have substituted the values into the original formula and solved (see below).

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ 2 &= \frac{x-3}{0.8} \\ 2 \times 0.8 &= \frac{x-3}{0.8} \times 0.8 && \text{(multiplying both sides by 0.8)} \\ 1.6 &= x - 3 \\ 1.6 + 3 &= x - 3 + 3 && \text{(adding 3 to both sides)} \\ 4.6 &= x \end{aligned}$$

Lets consider another example.

Example

In the following formula find the value of x if we know that $y = 2.3$.

$$y = 8 - 4x$$

We can make x the subject of the following equation and then substitute into this $y = 2.3$.

You should note that when looking at equations or expressions, the sign **in front** of a number is the one attached to the number.

In the above example there is no sign in front of the 8 so it understood that this is positive 8. The sign in front of the $4x$ is negative so this is $-4x$.

Firstly we must think about what steps we would take if we knew a value for x and were trying to find a value for y.

In words, we have to multiply by –4 and then add 8. So to rearrange we would subtract 8 and then divide by –4.

Diagrammatically,

$$\boxed{x} \xrightarrow{\text{Multiply by } -4} \boxed{-4 \times x} \xrightarrow{\text{Add 8}} \boxed{-4x + 8}$$

To rearrange the equation we must do the opposite of the above steps in the opposite order. Reading from right to left we have:

$$\boxed{x} \xleftarrow{\text{Divide by } -4} \boxed{-4 \times x} \xleftarrow{\text{Subtract } 8} \boxed{-4x + 8}$$

And algebraically,

$$y = 8 - 4x$$

$$y - 8 = 8 - 4x - 8 \quad \text{(subtracting 8 from both sides)}$$

$$y - 8 = -4x$$

$$\frac{y-8}{-4} = \frac{-4x}{-4} \quad \text{(dividing both sides by } -4\text{)}$$

$$\frac{y-8}{-4} = x$$

$$x = \frac{y-8}{-4} \quad \text{(writing the subject on the left hand side).}$$

Now if we substitute $y = 2.3$ into this formula we obtain:

$$x = \frac{2.3 - 8}{-4} = 1.425$$

Activity 2.5

1. Rearrange the following equations to make the variable in the bracket the subject.

   (a) $R = 4a$ (a)
   (b) $x + 2 = y$ (x)
   (c) $\frac{5}{b} = k$ (b)
   (d) $y = 5x + 2$ (x)
   (e) $7x - 8 = y$ (x)
   (f) $\frac{6}{t} + 7 = s$ (t)
   (g) $\frac{x+8}{3} = y$ (x)

2. The perimeter of a square is four times the length of one side.

   $P = 4 \times s$ where P represents the perimeter,
   $P = 4s$ and s represents the **length** of one side.

   Rewrite this formula to make s the subject.

3. Before the invention of mechanical clocks, candles were used to measure the passage of time. A formula for the height of such a candle related to time is given below.

   $h = 10 - 2t$ where h equals the height of the candle in centimetres, and t equals the time in hours that the candle has been burning.

   How long should it take before the height reaches 7.5cm? Use an equation to find this solution.

4. Economists have come up with the following formula relating the consumption of milk to the number of children in the family.

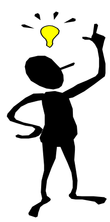
   $m = 3.5 + 2.5x$ where m is the number of litres of milk consumed per week, and x is the number of children in the family.

   How many children should there be in a family if 18.5 litres of milk are used. To do this question rewrite the formula to make x the subject and then substitute.

5. In statistics a formula to find the margin of error is

   $m = z \times \frac{\sigma}{\sqrt{n}}$ where z is the standard score, σ is the standard deviation and n is the sample size.

   In a situation we know that the standard deviation $\sigma = 0.8$ and $z = 2.1$. What sample size n must we use in order to have a margin of error of 0.28? Substitute these values into the formula and solve the corresponding equation.



s.

Solutions

Activity 2.1

1.

(a) $n + 5 = 35$ where n represents a number
(b) $n - 9 = 2n$
(c) $6.5n = -3.4$
(d) $5n = 22$

2.

(a) $B = 5J$ where B is Ben's age and J is John's age
(b) $L = W + 7$ where L is the length and W is the width
(c) $K = J - 5$ where K is Katie's age and J is Jack's age
(d) $B = \frac{1}{3}J$ where B is Barry's salary and J is Jack's salary
(e) $0.25 < p < 0.35$ where p is the sample proportion
(f) $s \geq 4$ where s is the number of students

Activity 2.2

1. $3x$; $7x$; $17x$; $8x$; $-4x$; $-2x$

$5a$; $9a$; $12a$; $-3a$

$7a^2$; $2a^2$; $6a^2$

$6x^2$; $5x^2$; $9x^2$; $-11x^2$

2.

(a) $11x$
(b) $17x^2$
(c) $4x$
(d) $4x$
(e) $-5x$
(f) $-12x$
(g) $-5.5x$
(h) $29x$

3.

$$\begin{aligned} 72 &= 2(L + 0.5L) \\ &= 2\,(1.5\,L) \\ &= 3L \end{aligned}$$

where L stands for the length of a house.

4.

$$\begin{aligned} 75 &= L + L + 15 \\ &= 2L + 15 \end{aligned}$$

where L stands for the shorter length of wire

Activity 2.3

1. (a) $d = st$
   (b) $P = R - C$
   (c) $V = IR$
2. (a) $G = 0.5P$ where G is the height of the gravillea and P is the height of the palm tree
   (b) $a = 3x$ where a is the height of the adult and x is the weight of the child
   (c) $p = 4l$ where p is the perimeter length and l is the length of one side
   (d) $x = 15 + y$ where x is the length of the house and y is the width
3. $c = 4s$ where c is the number of cola drinks and s is the number of soda drinks
4. $T = (I - 9\,500) \times 0.39 + 9\,500$ (if $I > 33\,500$) where T is the tax to be paid; I is the taxable income
5. $I = 200 + 50x$ where I is Mae's income and x is the number of items she sells in a week

Activity 2.4

1.
   (a) 35 cm^2
   (b) 714 cm^2
   (c) 201 cm^2
   (d) 3 280 mm^2 or 328 cm^2

2.
   (a) $P = 4s$
   (b)
      (i) 16 cm
      (ii) 28 m
      (iii) 10.4 mm
      (iv) 3 m

   (c) $c = 3.2 \times 4 \times 6.5$
   = $83.20

3.
   (a) $C = \pi d$
   (b)
      (i) 9.4 cm
      (ii) 226.2 mm
      (iii) 20.4 m
      (iv) 1.9 m

   (c) Length = 11.93 m. So I need to buy 4 rolls. Cost = $18.95 × 4 = $75.80

4.
   (a) 44.1 m
   (b) 313.6 m
   (c) 9.604 m

5.
   (a) 11 km
   (b) 18 km
   (c) 15 km

6.

(a) not true for young babies since the ratio would be different

(b)

(i) 16 335 sq cm
(ii) 18 480.15 sq mm
(iii) 1.693 sq m

(c)

(i) 3 920.4 sq cm
(ii) 4 435.236 sq cm
(iii) 0.406 4 sq m

7.

(a) 23.9
(b) 26.8
(c) 33.3

8. $(10-12) \pm 0.6\sqrt{\frac{2.7^2}{25}+\frac{2.4^2}{25}}$

$= -2 \pm 0.6\sqrt{0.552}$

$= -2 \pm 0.433\,497\,404$

$= -1.566\,502\,596$ or

$-2.433\,417\,404$

If we round these figures we would then say that we are confident the result is between –2.43 and –1.57.

Activity 2.5

1.

(a) $a = \frac{R}{4}$

(b) $x = y - 2$

(c) $b = \frac{5}{k}$

(d) $x = \frac{y-2}{5}$

(e) $x = \frac{y+8}{7}$

(f) $\mathrm{t} = \frac{6}{s-7}$

(g) $x = 3y - 8$

2. $s = \frac{P}{4}$

3. If we substitute h = 7.5 into the formula and then solve we obtain

$$7.5 = 10 - 2t$$

$$-2.5 = -2t \quad \text{(after subtracting 10 from both sides)}$$

$$1.25 = t \quad \text{(after dividing both sides by –2)}$$

So it must take 1.25 hours for the candle to reach 7.5cm.

4. If we rearrange the original formula we obtain $x = \frac{m - 3.5}{2.5}$. Substituting m = 18.5 into this formula gives: $x = \frac{18.5 - 3.5}{2.5} = 6$. So there must be 6 children.

5. If we substitute $\sigma = 0.8$, z = 2.1 and m = 0.28 into the original formula and solve we obtain:

$$m = z \times \frac{\sigma}{\sqrt{n}}$$

$$0.28 = 2.1 \times \frac{0.8}{\sqrt{n}}$$

$$0.28\sqrt{n} = 2.1 \times 0.8 \quad \text{(after multiplying both sides by } \sqrt{n}\text{)}$$

$$\sqrt{n} = \frac{2.1 \times 0.8}{0.28} \quad \text{(after dividing both sides by 0.28)}$$

$$= 6$$

$$n = 36 \quad \text{(after squaring both sides)}$$

Module 3

Graphing

Aims

In Data Analysis you will need to be able to draw and interpret a number of different types of graphs. In this module we will revise some of the more important aspects of graphs.

When you have completed this module you should be able to:

- use the Cartesian Coordinate System to represent formulas;
- convert graphs and formulas into written descriptions;
- identify, draw and interpret linear graphs;
- predict the effects on linear graphs of changes to coefficients and constants in the equations;
- describe more complex graphs in words.

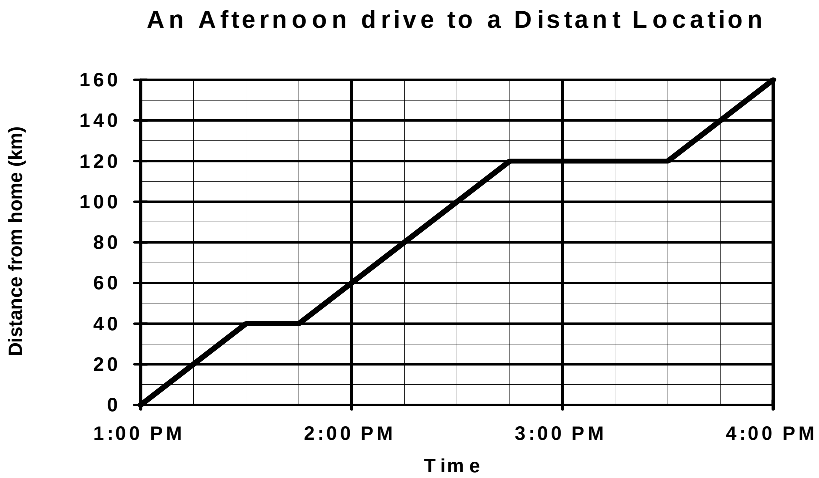
3.1 Describing a relationship from a graph

Graphs are a very convenient way to display information and it is up to the individual to interpret what the graph is 'saying'. A well presented graph will be clear and easy to follow. It will have many important features:

- a heading describing what the graph is about;
- clearly labelled horizontal and vertical axes;
- scales on the axes;
- lines or points plotted to indicate the relationship represented.

Example

Consider the following graph showing details of a Sunday drive.



When interpreting a graph we would firstly look at three items.

- The title is the first step. This should clearly tell us what the graph is about.
- Next look at the horizontal axis. In this case the axis represents time, from 1:00 pm to 4:00 pm.
- Finally look at the vertical axis. In this case the axis represents the distance from home in kilometres.

We should now have a clear picture on what the graph is about.

Next we look to the graph itself.

A series of questions might help you to clarify the meaning of the graph.

(a) At what time did the Sunday traveller leave home? ....
Did you say 1:00 pm? The distance from home at 1:00 pm was zero, so this is the time the traveller left home.

(b) At what time did the traveller reach their destination? ....
You should have said 4:00 pm.

(c) How far was the traveller from home when they reached their destination? ....
They were 160 km from home.

(d) How far did the traveller go in the first hour? ....
The first hour went from 1:00 pm to 2:00 pm and the distance from home at these times was 0 kilometres at 1:00 pm and 60 km at 2:00 pm. The distance travelled in the first hour was 60 kilometres.

(e) How far did the traveller go in the last hour? ....
Did you say 40 kilometres from 3:00 pm to 4:00 pm?

(f) What do you think happened between 1:30 and 1:45 pm? ....
What are your reasons for giving this answer? ....

....

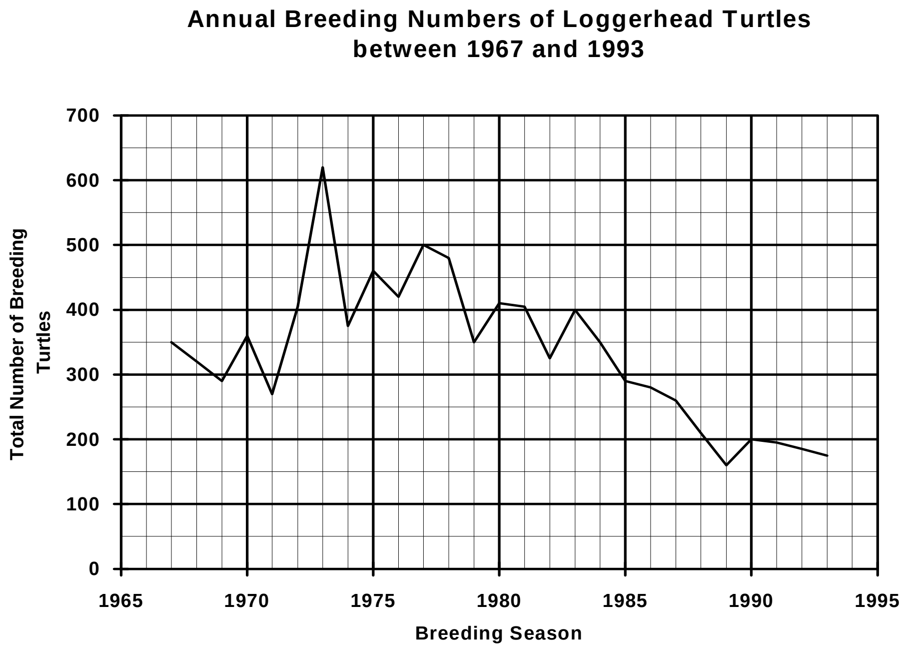
You should have indicated that the traveller was stopped over this time period (or the traveller was lost and travelling in circles) because there was no change in the distance from home, it remained at 40 km.

(g) Over what other time period was the traveller stopped? ....
The graph indicates that the traveller was stopped between 2:45 and 3:30 pm.

(h) What was the total time spent travelling? ....
This question requires you to do some calculations. The Sunday drive lasted for 3 hours but some of this time was spent not travelling. The total time spent travelling was 2 hours.

Activity 3.1

1. The following graph shows the annual breeding numbers of loggerhead turtles at Mon Repos on the Bundaberg coast over about 20 years.



   (a) In what year did the most turtles nest at Mon Repos?

   (b) In what year was there the least number of turtles?

   (c) Between which two years was there the greatest increase in turtle numbers?

   (d) Between which two years was there the greatest decrease in turtle numbers?

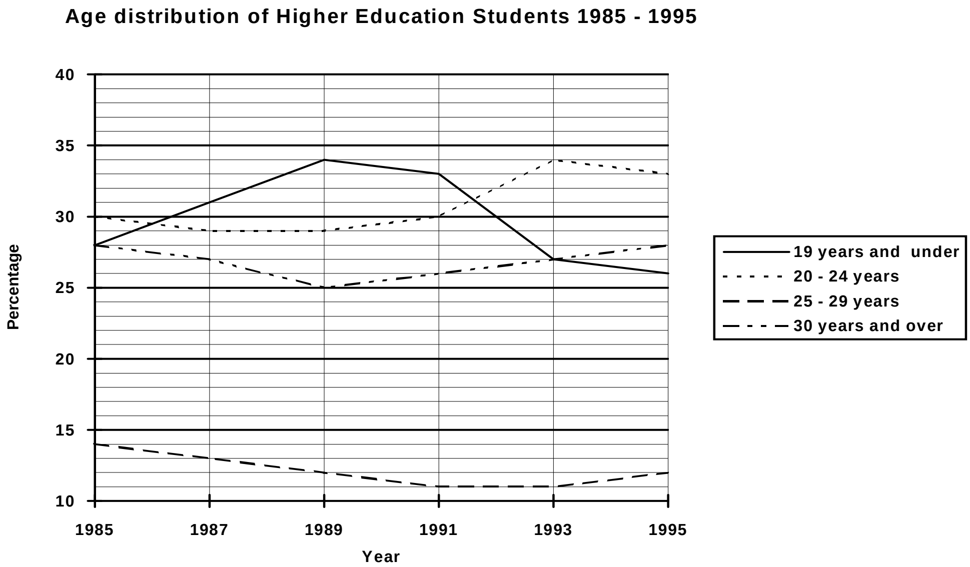
   (e) Scientists working in the Mon Repos area have predicted that this nesting sight will only last another 10 years if action is not taken. How do you think they might have come to this conclusion?

2. The following graph shows the age distribution of higher education students.

   (a) What percentage of students in higher education in 1991 were aged 20–24 years?

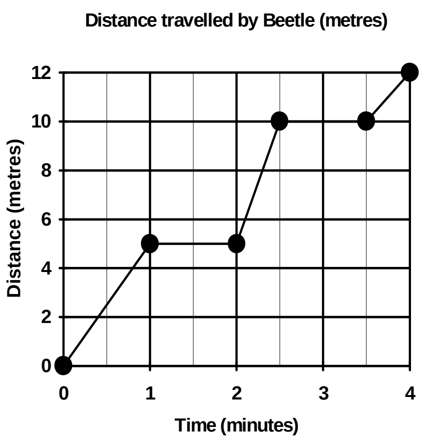
   (b) In which year did the 19 years and under group have the greatest representation in higher education?

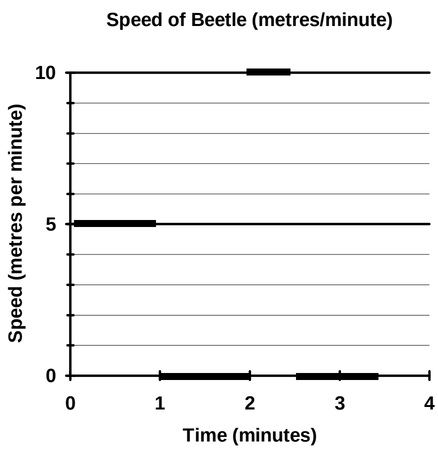
   (c) Which age group has the lowest representation in the higher education sector?



(Source: Selected Higher Education Student Statistics, DEET.)

3. Following are two graphs showing how a beetle travelled over a set distance. The first graph shows the distance the beetle walked over a given time. The second graph shows the speed at which the beetle was travelling at any particular time.





(a) Between which times is the beetle not moving? Explain how this is reflected in both graphs.

(b) From the first graph determine how far the beetle travelled between 0 and 1 minutes.

(c) From the second graph determine how fast the beetle travelled between 0 and 1 minutes. Can you see how this speed was calculated?

Recall from earlier in this module we gave the formula for speed as:

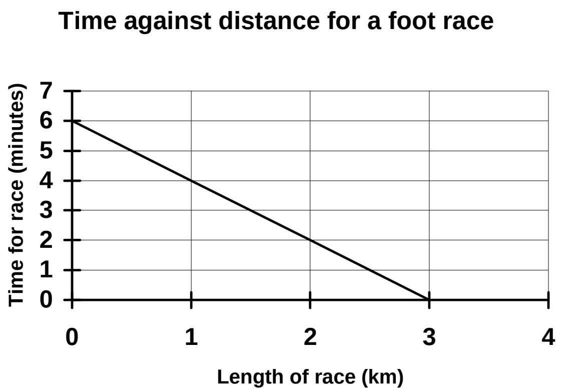
Speed = distance/time

From the first graph you found that the beetle covered 5 metres in the first minute.

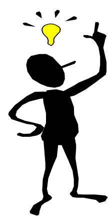
That is: $\text{speed} = \dfrac{\text{distance}}{\text{time}} = \dfrac{5\text{ m}}{1\text{ minute}} = 5\text{ m/minute}$, which is the speed you found from the second graph.

(d) From the first graph, calculate the speed at which the beetle travelled between 3.5 and 4 minutes. Mark this on the second graph.

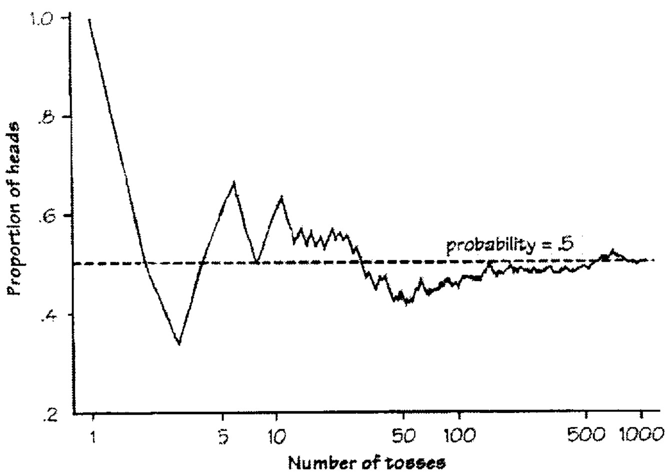
4. The following graph shows the time it takes to run a race plotted against the distance over which the race was run.



Look at this graph carefully until you understand what relationship the graph is describing. Write a sentence explaining what is wrong with this graph.



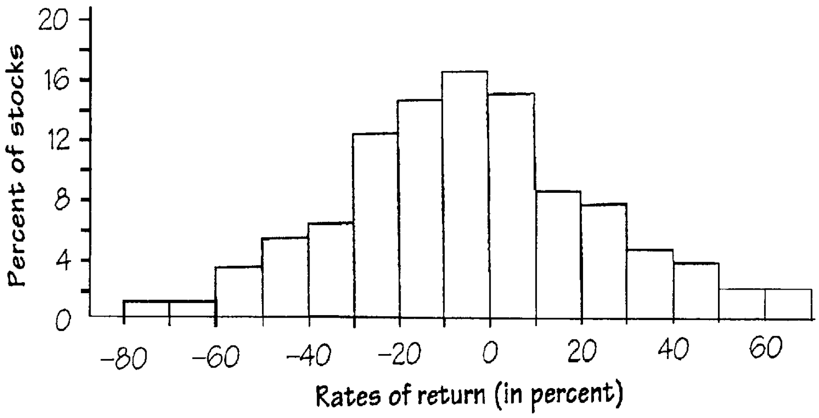
5. The following graph is from a Data Analysis text (Moore 1996 p. 243)



The behaviour of the proportion of tosses that give a head, from 1 to 1000 tosses of a coin.

What do you notice about the scale on the horizontal axis? You will find that it does not go up evenly, in fact it goes up in powers of 10 (the distance from 1 to 10 is the same as the distance from 10 to 100) This is called a log scale. It is used when there are large numbers on one or both of the scales. Write a few sentences describing the graph.

6. The graph below is from a Data Analysis text (Moore 1996, p. 292)



The distribution of returns for New York Stock Exchange common stocks since 1987

Write a few sentences describing the graph.

3.2 Relationships as graphs

You have probably used a map at some stage to find the location of a particular place. Map makers commonly set up a **grid** to allow you to do this. They use a letter to label a column and a number for a row (this will sometimes vary).

Look at the map of the University of Southern Queensland on the next page. Suppose you were looking for the Library and were given the grid reference J7. To find it you need to go across to column J and from there go up or down to row 7. The library will be found at this point. (It is actually labelled R on the map.)

Now find the new Office of Preparatory and Continuing Studies, the grid reference being K8. Go across to K and then up or down to 8. You should have found it labelled S on the map. If you are ever in Toowoomba call into S and say hello to us.

Activity 3.2

1. What is located at the following grid references.
   (a) I6
   (b) D5
   (c) N7
2. Find the grid reference for the following places.
   (a) Steele Rudd Courts.
   (b) W Block Sciences (nursing and psychology) and Shopping Complex.
   (c) Gymnasium

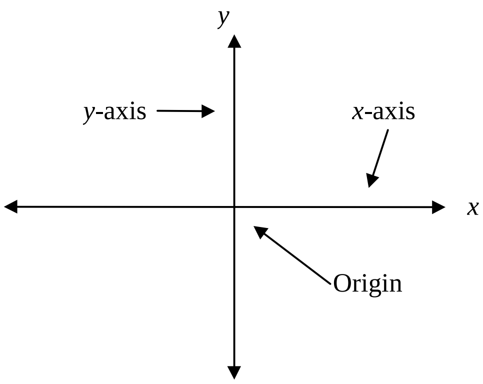
Obviously a grid system is a very useful tool because everyone agrees on the location of a particular point.

Mathematicians use a similar system to refer to a point in a plane (a flat surface extending infinitely in all directions) and to distinguish it from any other points in that plane.

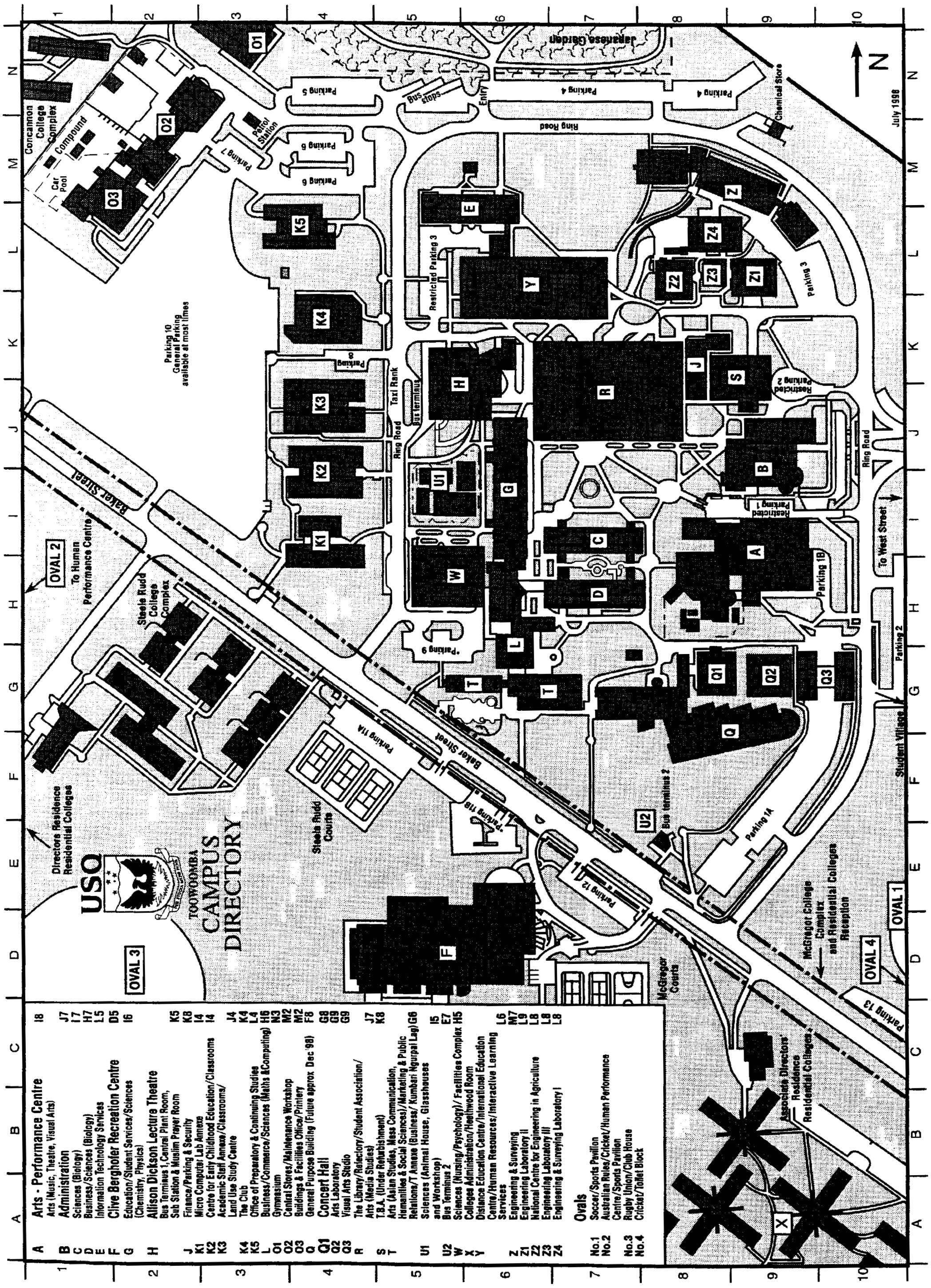
The system we use today was developed by a seventeenth century mathematician called René Descartes. Legend has it that the idea of coordinates came to him while he lay in bed and watched a fly crawling on the ceiling. Noting that each position of the fly could be expressed by two distances from the edges of the ceiling where the walls and ceiling met. He set up a **grid** using two number lines at right angles. The **horizontal** number line is usually called the ***x*-axis** and the **vertical** number line is usually called the ***y*-axis**. In fact these horizontal and vertical number lines can have any name as you will see as we move through this module. These two **axes** (the plural of axis and pronounced ax/ees) meet or **intersect** at a point called the **origin**. We call this grid system the **Cartesian Coordinate System** after its inventor.

3.2.1 The Cartesian plane

A Cartesian plane would look like this.

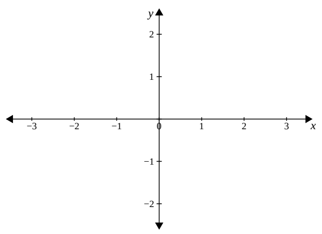


In this case I have labelled the axes x and y as you will often see, but I could also have labelled them J and S, or any other **variable** that I wish to represent.

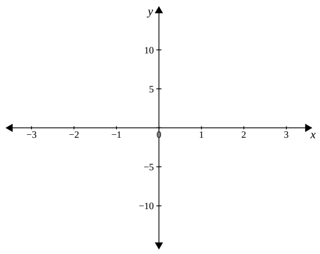


As each axis is a number line it must have a **scale** drawn on it as we have done before when constructing a number line. The numbers extend out infinitely in either direction from the origin. All the way along the axis **one centimetre must always represent the same number of units**. It is not necessary to have the same scale on both axes although this will often be the case.

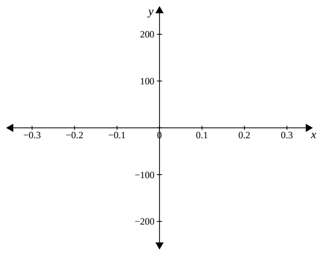
Look at the following Cartesian planes where different scales have been used. Note that only one scale has been used per axis. With a ruler, check that the scales are correct on each axis.



The same scale is used on each axis

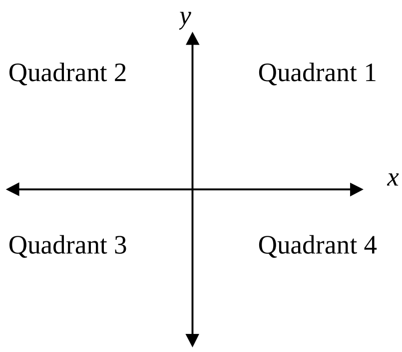


Different scales are used on each axis



Different scales are used on each axis

You will also have noticed that the two axes divide the plane into four regions. We call these regions **quadrants**. They are numbered as in the following diagram.



Any point can be represented on the Cartesian plane by using a pair of numbers that we call an **ordered pair**. To describe a point we give its horizontal or ***x*-coordinate** and its vertical or ***y*-coordinate**. This pair of numbers is always written in strict order, stating the *x*-coordinate first followed by the *y*-coordinate.

That is (*x*-coordinate , *y*-coordinate)

We abbreviate this to be (x , y)

Or, if we are not using *x* and *y* as our variables then in general we give the horizontal coordinate first and the vertical co-ordinate second.

That is (horizontal coordinate , vertical coordinate)

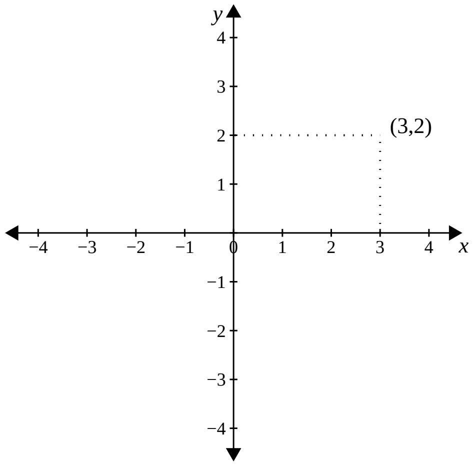
Let's now look at using this scheme to locate some points on the Cartesian plane.

To locate the point (3,2) firstly establish which is the *x*-coordinate and which is the

y-coordinate. As ordered pairs are always in the form (x,y) we know that the *x*-coordinate is 3 and the *y*-coordinate is 2.

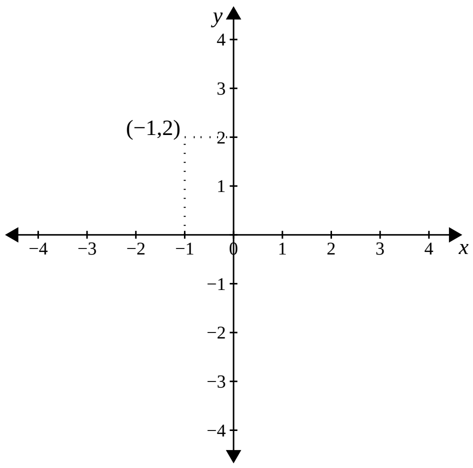
Having drawn a Cartesian plane, go across to 3 on the *x*-axis and up to 2 on the *y*-axis.

The dotted lines have been drawn to help you locate the point. These are only for guidance and would not normally be included on your diagram.

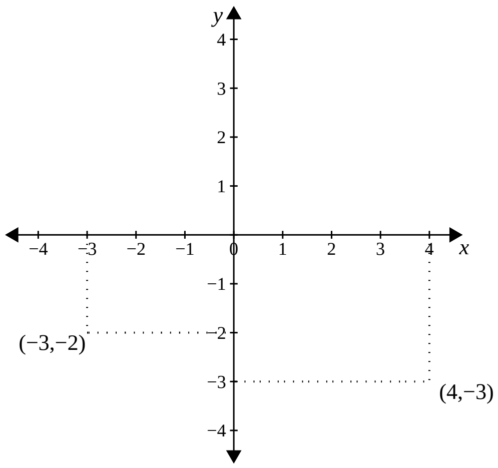


The point (3,2) lies in the first quadrant. We can locate points in other quadrants using the same technique.

Let's plot the point (–1,2). Again we must firstly determine that the *x*-coordinate is –1 and the *y*-coordinate is 2. This time we must move left from the origin to get to –1 and then up to 2 on the *y*-axis. You will notice that this point is in the second quadrant.



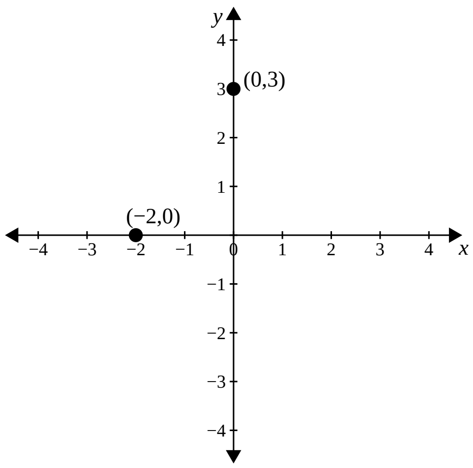
On the Cartesian plane below check the location of the points (–3,–2) and (4,–3)



You should have noticed that (–3,–2) is located in the third quadrant and (4,–3) in the fourth quadrant.

When you feel comfortable with locating points do not include the dotted lines.

What about points that lie on the axes rather than in one of the four quadrants. Take note of the location of the points (0,3) and (–2,0) below.



For the point (0,3) the *x*-coordinate is 0 which means we move neither left nor right from the origin, and then we move up to 3 on the *y*-axis. A point where the *x*-coordinate is 0 will always lie on the *y*-axis.

For the point (–2,0) the *y*-coordinate is 0. This means that after moving to –2 on the

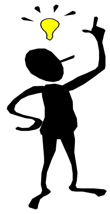
x-axis we move neither up nor down on the *y*-axis. A point where the *y*-coordinate is 0 will always lie on the *x*-axis.

Activity 3.3

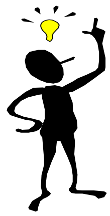
1. In Data Analysis you will be studying ‘regression’ which will require you to plot sets of ordered pairs that are related. The example below compares the household spending on tobacco products and alcoholic beverages in Great Britain.

   Plot the points on a Cartesian plane.

Household	Tobacco	Alcohol
1	2.6	4.7
2	2.9	4.5
3	3.2	5.9
4	3.3	4.8
5	3.5	5.6
6	3.7	6.2
7	3.8	6.3
8	4.1	6.5
9	4.5	6.2
10	4.6	4
11	3.6	5.3



2. Another graph that is used in Data Analysis is called a ‘residual plot’. Here we look at a model of what is predicted and we plot the difference between the observed and the predicted values. Below are the speeds of a British Ford Escort and the difference between the observed mileage and predicted mileage (e.g. at 10km/hr the mileage is 21 litres/100km but the predicted was 31.9 litres/100 km, so the difference is 10.9). Plot the information below.

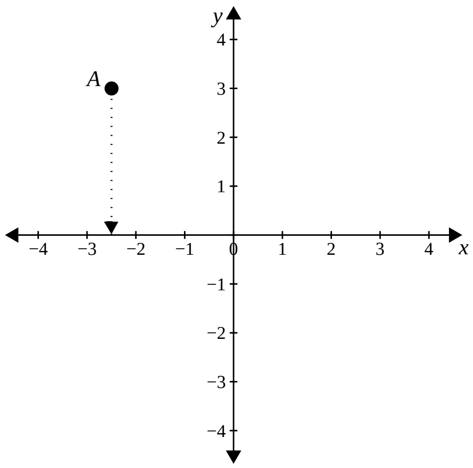


Speed (km/hr)	Observed – Predicted	Speed 80	Observed – predicted –2.94
10	10.9	90	–2.17
20	2.24	100	–1.32
30	–0.62	110	–0.42
40	–2.47	120	0.57
50	–3.33	130	1.64
60	–4.28	140	2.76
70	–3.73	150	3.97

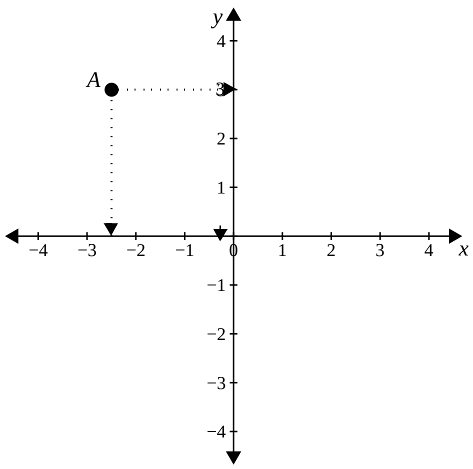
So far we have only looked at the case where you have been given an ordered pair and been required to locate it on the Cartesian plane. What if you were given the point located on the plane and needed to find its coordinates.

Consider the point A marked on the plane below. We determine the coordinates by finding the *x*-coordinate first.

Imagine a vertical line from the point to the *x*-axis. (We will draw these dotted lines in for convenience only.) The point where this line cuts the *x*-axis gives the *x*-coordinate. We find that it cuts at –2.5



The next step is to imagine a horizontal line from the point to the *y*-axis to find the *y*-coordinate. We find that it cuts at $y = 3$

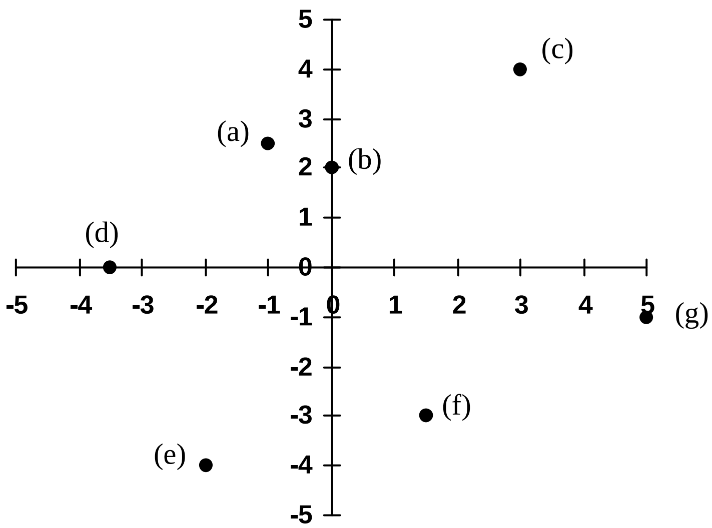


Therefore the coordinates of the point A are (–2.5,3)

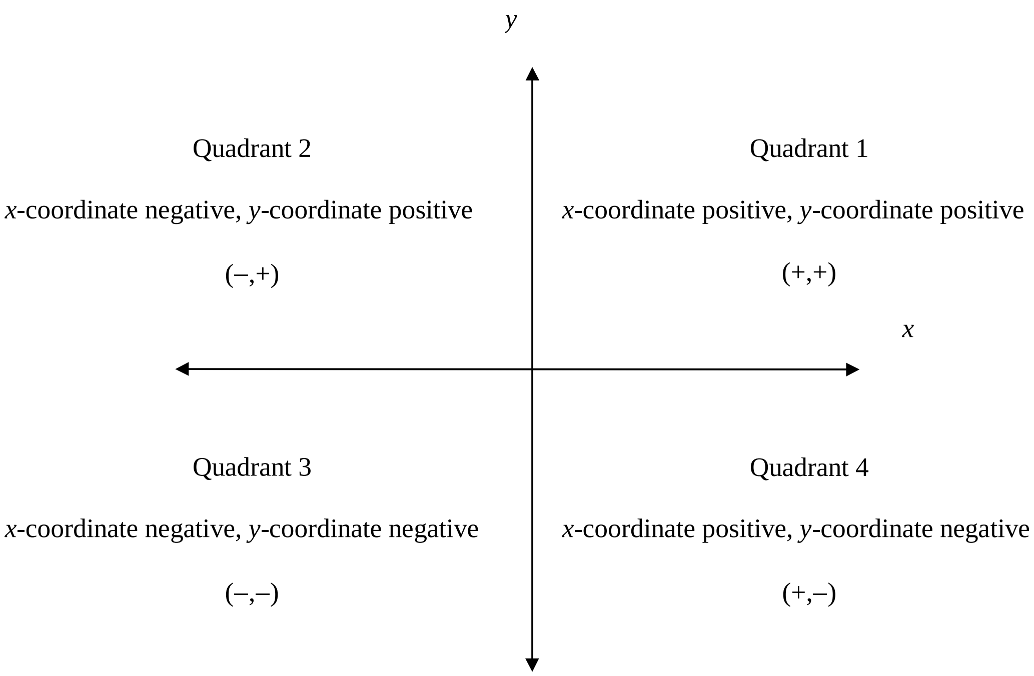
Practise, by finding the coordinates of the points below.

Activity 3.4

Find the coordinates of the points marked on the Cartesian plane below.



Let's summarise what we have learned about the points located in the various quadrants.



3.2.2 Drawing graphs

The general purpose of a graph is to give a visual representation of the relationship between two or more things. Graphs must contain clear, precise and meaningful information for the reader.

This section takes you step by step through the process of drawing a graph by plotting points. This is only one of many ways to develop a graph, but it is one that you can always fall back on, no matter what the shape of the graph. In this section we are only going to look at straight line graphs.

For this section you will need to have some 2 mm graph paper so that you can practise drawing graphs and reading off accurate answers. It is also helpful to have a well sharpened pencil to draw the graphs. You should draw the graphs on graph paper as we move through the steps.

Suppose there is a brother and sister Shirley and Jeffrey. Shirley is three years older than Jeffrey. This can be represented by the formula.

$S = J + 3$ where S represents Shirley's age in years,
and J represents Jeffrey's age in years.

Using the Cartesian coordinate system, if we could find a number of ordered pairs, we could locate these on the Cartesian plane and join them up to get a graph of this relationship.

When Jeffrey was 14, Shirley was 17
When Jeffrey was 19, Shirley was 22
When Jeffrey was 35, Shirley was 38

A convenient way to represent this information is to enter it in a **table of values**.

Jeffrey	14	19	35
Shirley	17	22	38

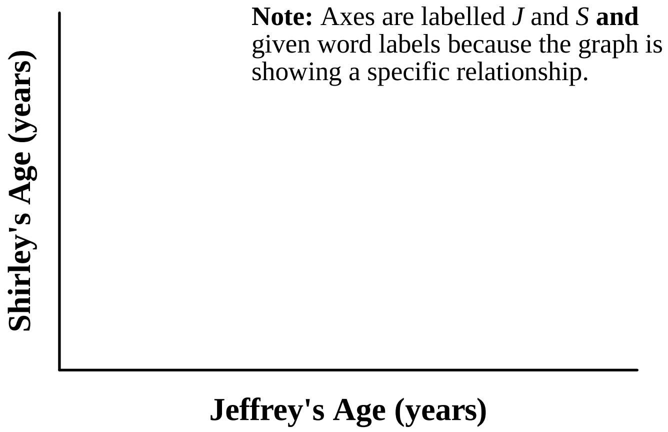
From our table of values we can write a series of ordered pairs that satisfy the formula.

(14,17) (19,22) (35,38)

In each case Jeffrey's age (horizontal axis) comes first and Shirley's age (vertical axis) is second.

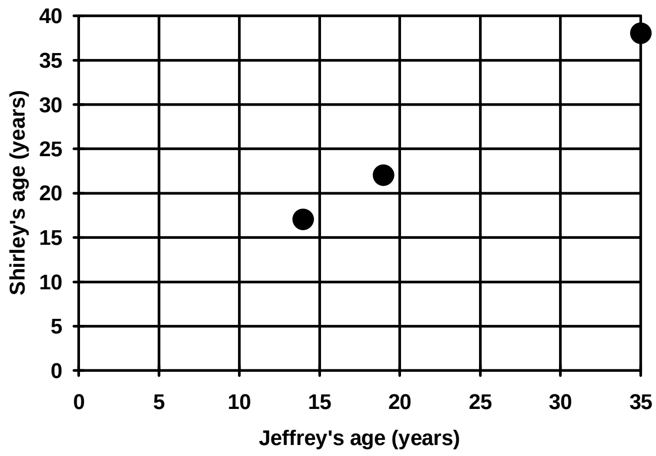
The series of ordered pairs can now be plotted on a Cartesian plane so that we can see the relationship between Jeffrey's age and Shirley's age.

Before we draw our axes let's think about the possible values for the ages. Since neither Jeffrey's age nor Shirley's age can be negative we are only interested in the first quadrant.



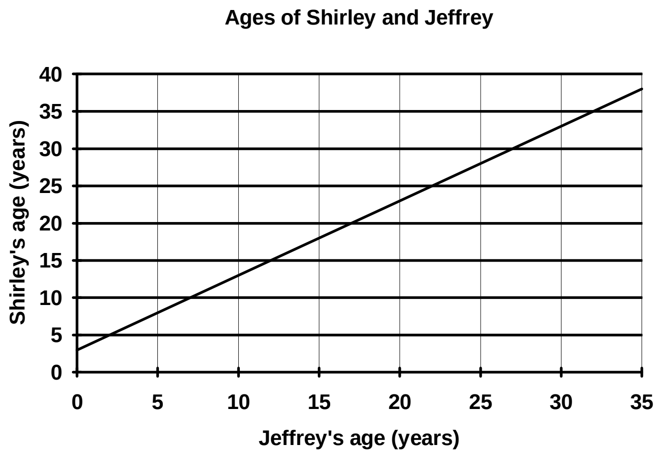
Now choose an appropriate scale to suit the ordered pairs. In our table of values, the maximum value for Shirley’s age is 38 and for Jeffrey, 35. Let’s then make the axis for Shirley’s age go up to 40 and for Jeffrey’s to 35.

Now plot the ordered pairs on the Cartesian plane drawn.



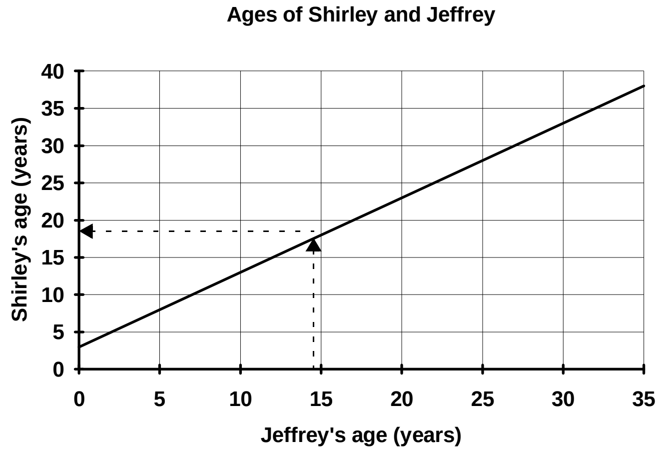
To illustrate the **trend in the graph** we draw a straight line through all of the plotted points. When drawing a trend line we need to think about whether all values on the *x*-axis are possible. In our case we are talking about ages in years. We can have 5 years, 10 years and so on, as marked on the graph, but we could also have 5 ½ years, or 6 years, 3 months and 4 days, so yes all values are possible on the *x*-axis. Since all values are possible we will draw a solid line when joining the plotted points.

Finally give the graph a title.



Notice that we do not stop at the points we have plotted when drawing in the trend line. The points we have plotted were only three of an infinite number of possibilities.

Once the graph is drawn it is easy to read off Shirley's age for any age of Jeffrey.



When Jeffrey was 15, Shirley was 18. This is not easy to find on the above graph, but having drawn yours on 2 mm graph paper you should be able to read off your answer fairly accurately.

Let's now summarise the steps we have taken to draw a graph.

- From the formula complete a table of values.
- Draw up a Cartesian plane and label the axes (don't forget the units if appropriate).
- Choose a suitable scale.
- Plot the ordered pairs from the table of values.
- Draw a trend line extending through and beyond the plotted points.
- Give your graph a title.

Let's now look at another example, following the same steps as last time.

Example

A small company pays their salesperson Mae Ihelpyu \$200 per week plus \$50 for each item that she sells in the week. Write a formula to calculate Mae's weekly income for any number of items sold.

The formula required was:

$I = 200 + 50n$ where I represents the total income in dollars, and n represents the number of items sold.

If you were to graph this formula you could find the total income for Mae selling any number of items in a given week. Or, given a set amount of money to be earned in one week, we could calculate the number of items to be sold.

Can you see that the amount of money Mae earns **depends** on the number of items she sells?

We call Mae's income the **dependent variable** because it depends on the number of items she sells, and we call the number of items sold the **independent variable** because this can be any number of items.

It is usual to put the independent variable first in the table of values.

Complete the following table of values.

Number of Items sold (*n*)	0	5	10
Total Income in \$ (*I*)			

When $n = 0$

$$\begin{aligned} I &= 200 + 50n \\ &= 200 + 50 \times 0 \\ &= 200 + 0 \\ &= 200 \end{aligned}$$

When $n = 5$

$$\begin{aligned} I &= 200 + 50n \\ &= 200 + 50 \times 5 \\ &= 200 + 250 \\ &= 450 \end{aligned}$$

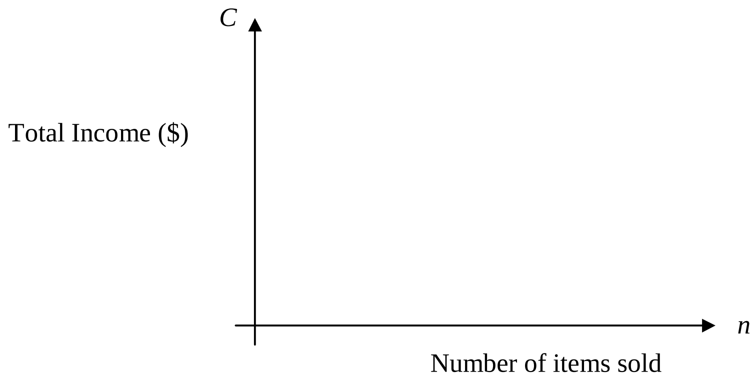
When $n = 10$

$$\begin{aligned} I &= 200 + 50n \\ &= 200 + 50 \times 10 \\ &= 200 + 500 \\ &= 700 \end{aligned}$$

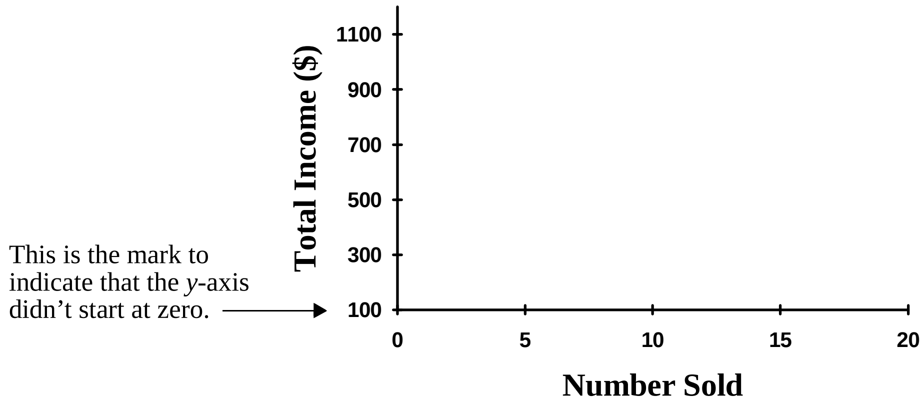
Completing the table of values we have:

Number of Items sold (*n*)	0	5	10
Total Income in \$ (*I*)	200	450	700

Now draw up the Cartesian plane and label the axes. Again we only require the first quadrant as the number of items sold can only be positive and the total cost must also be positive.



Now choose a suitable scale. The horizontal axis (number of items sold) will need to start at zero, the minimum number of items that could be sold. The vertical axis though, could start at \$100, a number just smaller than the least amount earned, which is \$200. If we do not start the scale on an axis at zero, we should indicate that we have not started at zero.



Now plot the ordered pairs from the table of values

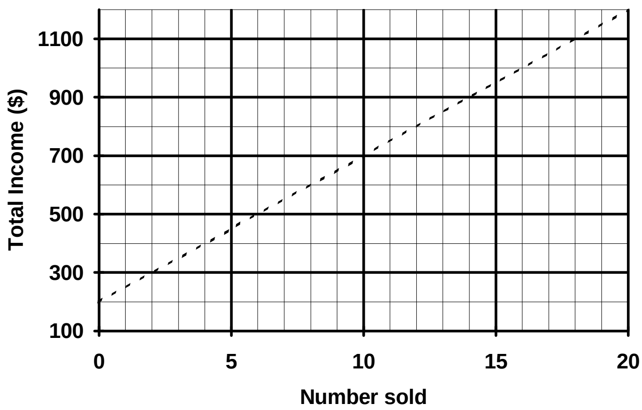
(0,200) (5,450) (10,700)

Again the **independent variable** comes first and is the one **plotted on the** ***x*****-axis**.

Before drawing a trend line through the points, we need to think about whether all values on the *x*-axis are possible. In this case we are talking about number of items sold. We can have 5 items sold, 10 items sold and so on, as marked on the graph, but in this case we could not have 5½ items sold (who wants to buy, say, 1/2 a TV?). So this time, all values are **not** possible on the *x*-axis. Since all values are not possible we will draw a dotted or broken line when joining the plotted points.

Finally give your graph a title.

Total Income for any given number of Items Sold



Let's now read off some values from your graph. These are not easy to find on the above graph, but having drawn yours on 2 mm graph paper you should be able to read off your answers fairly accurately.

What would Mae earn if she sold 7 items in one week? ....

How many items would Mae have to sell to earn \$600 in a week? ....

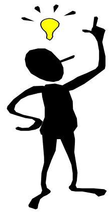
You should have found that Mae earned \$550 for selling 7 items, and to earn \$600 would need to sell 8 items.

You could **check** these answers by **substituting** them into the formula.

When $n = 7$
$I = 200 + 50n$
$= 200 + 50 \times 7$
$= 200 + 350$
$= 550$

When $n = 8$
$I = 200 + 50n$
$= 200 + 50 \times 8$
$= 200 + 400$
$= 600$

Example



Let's finally look at a graph from a statistical equation called the regression line. An equation is just another name for a formula. It is looking at the relationship between two variables as before. For example a university in the US conducted a study on the relationships between the average temperature (°F) during the month babies started to crawl and the baby's age. They found a relationship in the form of an equation:

$$y = -0.08x + 35.7$$

To sketch the graph of the formula, we follow the same steps as before.

The first thing to calculate is a table of values. You will need to decide what x values you will choose to put into your table. In this example there would be no negative values. The temperature range would be about 30° to 100° F.

x	30°	50°	90°
y			

When $x = 30$

$y = -0.08x + 35.7$
$= -0.08 \times 30 + 35.7$
$= 33.3$

When $x = 50$

$y = -0.08x + 35.7$
$= -0.08 \times 50 + 35.7$
$= 31.7$

When $x = 90$

$y = -0.08x + 35.7$
$= -0.08 \times 90 + 35.7$
$= 28.5$

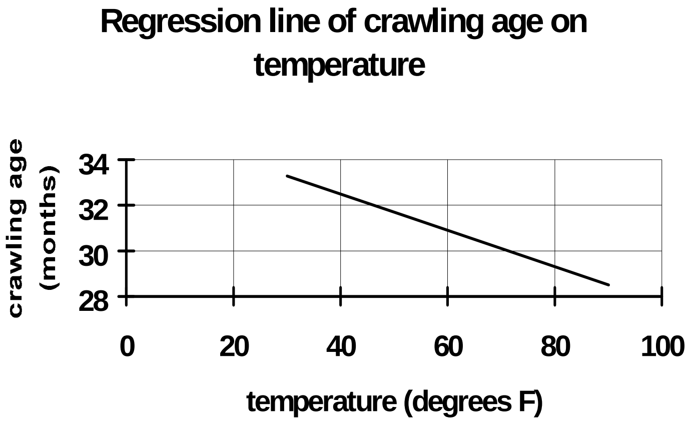
Now complete the table of values.

x	30°	50°	90°
y	33.3	31.7	28.5

The ordered pairs that we will plot are:

(30,33.3) (50,31.7) (90,28.5)

Plot the points, draw a line through and beyond the points. Finally label your graph.



Activity 3.5

To complete this Activity you will need graph paper.

1. Before the invention of mechanical clocks, candles were sometimes used to measure the passage of time. A formula for the height of such a candle related to time is given below.

   $h = 10 - 2t$

   where h equals the height of the candle in centimetres,
   and t equals the time in hours that the candle has been burning.

   (a) Use this formula to complete the following table.

t	0	1	2	3	4	5
h						

   (b) Graph this formula, letting the horizontal axis represent time.

   (c) What was the height of the candle before it was lit?

   (d) What does the 10 represent in the formula?

   (e) For how many hours will the candle burn?

2. Suppose that you were planning a holiday in the United States. You wanted to know what clothes you should take so you looked up the expected temperatures in the newspaper. Problem is the temperatures were in degrees Fahrenheit and you never knew or have long forgotten what this means. A **conversion graph** will be useful to convert between degrees Fahrenheit and degrees Celsius.

   (a) Draw a graph for the following information. You will need a full page graph with careful choice of scales to be able to read off accurate answers.

Degrees Celsius (°C)	0	10	20	40	80	100
Degrees Fahrenheit (°F)	32	50	68	104	176	212

(b) If you had found that the average temperature in the month you were planning to visit the USA was 59°F, what would this be in degrees Celsius?

(c) If it was 35°C in Australia, what would this be in degrees Fahrenheit?

3. The Leaky Boat Company hires out paddle boats for $7.50 an hour.

(a) Develop a formula relating the time you have the boat to the total cost of the hire. Don't forget to define your variables.

(b) Complete the following table of values.

Number of Hours	1	2	3	5	8
Total Cost ($)					

(c) Draw a graph showing the time you have the boat against the cost of the hire.

(d) If a family decides to hire the boat for 4 hours in the morning, what will be the cost of the boat?

(e) If another family comes to hire the boat for the afternoon but only has $24, for how many hours will they be able to hire the boat?

4. A car rental company charges an initial fee of $50 per day plus 15 cents per kilometre.

(a) Write a formula relating the kilometres travelled to the total cost for a days rental. Don't forget to define the variables.

(b) Draw up a table of values and construct a graph to represent this relationship.

(c) From your graph, determine the cost of renting the car for one day and driving 260 kilometres.

(d) If you had $110 to spend, how many kilometres could you travel in one day?

5. The amount of time that you can spend in the sun without burning is related to the sun protection factor (SPF) of the sunscreen lotion you use. The table below is for a person who can stay in the sun without any lotion for only 15 minutes without burning.

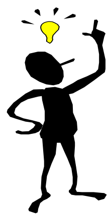
Sunscreen Factor	8	10	12	14
Time in minutes	120	150	180	210

(a) How does the increasing of the sunscreen factor by 2 change the time that can be spent in the sun?

(b) Draw a graph of this relationship.

(c) If this person wanted to stay in the sun for 1 hour without burning, what is the minimum sunscreen factor that would be required?

6. The regression equation: $y = -1.03 + 0.206x$, shows the relationship between increase in cigarette consumption and change in nicotine yields. x is the variable reduction in nicotine yield (%), and y is the variable increase in cigarette consumption (%).



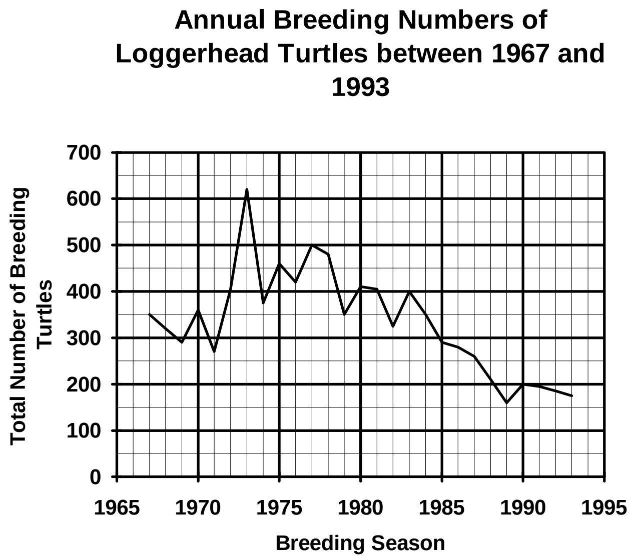
   (a) Draw a graph showing this relationship

   (b) A 50% reduction in nicotine yield might produce a rise in cigarette consumption of __________%

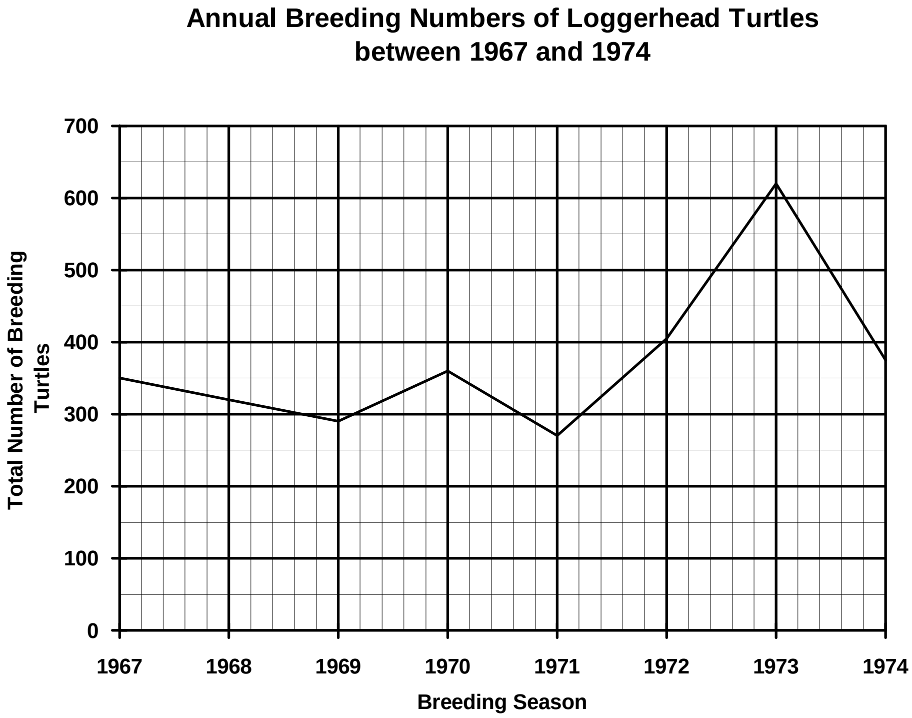
   (c) A person changed from cigarettes containing 2.1 mg of nicotine to ones that contained 1.3 mg. What increase in consumption would you expect?

3.3 Gradients of line graphs

Consider the following graph, showing the annual breeding numbers of loggerhead turtles at Mon Repos on the Bundaberg coast over about 20 years.



Part of this graph, from 1967 up to 1974 is enlarged and reproduced below.



What was the increase in breeding numbers between 1969 and 1970? ....

What was the increase in breeding numbers between 1972 and 1973? ....

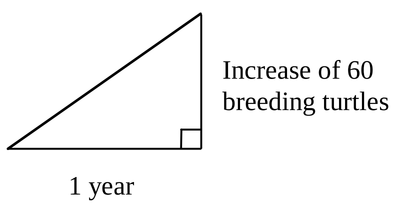
Between 1969 and 1970 turtle numbers increased by about 60, while between 1972 and 1973 numbers increased by about 220.

What do you notice about the steepness of the graph between 1969 and 1970 and between 1972 and 1973? ....

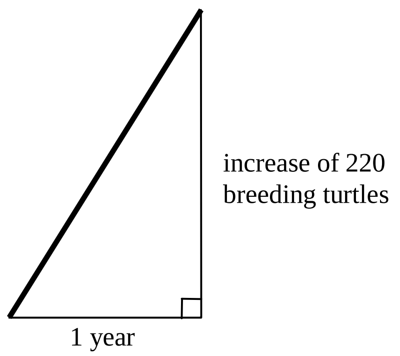
Did you say that the graph is steeper over the second period?

Let's look more closely at these two results. We will look at an even smaller part of the graph this time.

Over the 1 year period between 1969 and 1970 the number of turtles rose by 60.



Between 1972 and 1973, again a 1 year period, the number of turtles rose by 220.



Imagine walking up these two ‘hills’. It would be hard work climbing up the second much steeper ‘hill’. We say that the second of our diagrams has a much greater **slope** than the first diagram. Another word for the steepness or slope of a line is to talk of its **gradient**.

In fact we can put a value on the steepness or gradient of the line, by putting the value for the change in height over the change in horizontal distance.

That is, $\text{gradient} = \dfrac{\text{change in height}}{\text{change in horizontal distance}}$

You might also see this written as:

$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$

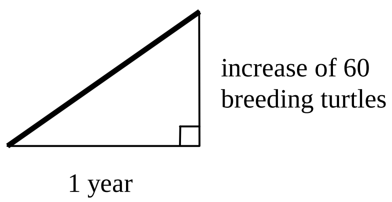
This formula means exactly the same thing as the formula above.

Let’s find the gradients of the two lines taken from our graph above.

Over the 1 year period between 1969 and 1970 the number of turtles rose by 60.

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

$$\text{gradient} = \frac{60}{1} = 60$$

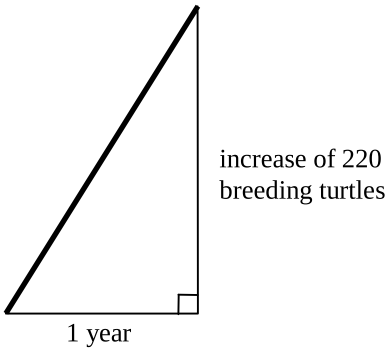


We can say that the **rate of change** in turtle numbers over this period was 60 turtles per year.

Between 1972 and 1973, again a 1 year period, the number of turtles rose by 220.

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

$$\text{gradient} = \frac{220}{1} = 220$$



Similarly we can say that the **rate of change** in turtle numbers over this period was 220 turtles per year.

So, the gradient of the graph between 1969 and 1970 is 60 while the gradient of the graph between 1972 and 1973 is 220. It can be seen from these figures that the gradient of the second part of the graph is much steeper than the gradient of the first part of the graph.

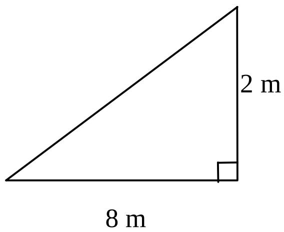
Let's now look at some other examples of gradients.

Example

Find the gradients of the following line segment. A line segment is just a part of a line.

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

$$\text{gradient} = \frac{2}{8} = \frac{1}{4}$$

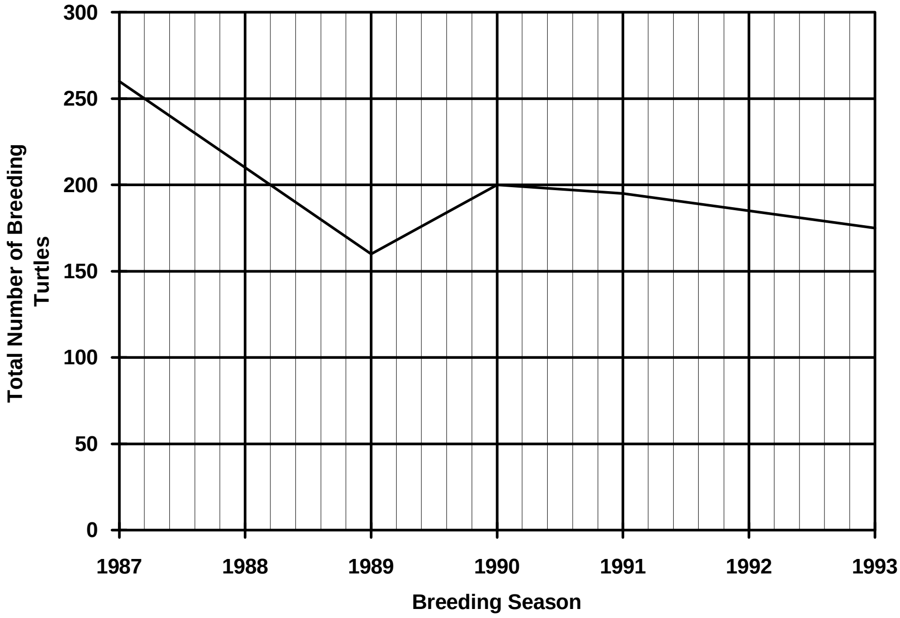


What the gradient is telling us is that for every 8 metres we move along in a horizontal distance, the vertical distance rises by 2 metres. We could also say that for every 4 metres in a horizontal direction we rise 1 metre. You may see engineers refer to this as a gradient of 1:4 using ratios as we have done in a previous module. If this was your block of land with this slope, the builder, the surveyor and the engineer would all be interested in the gradient of the block of land. If the land is too steep then slippage of the land could occur once the soil is disturbed. If the land is too steep there would need to be a deep cut in the land to make a level piece of ground on which to situate a concrete slab based house. This might mean that alternative methods of construction need to be considered.

In fact the building code prevents anyone building on land with a gradient greater than 1:4 due to the danger of land slippage.

Let's return to our turtles at Mon Repos. This time we are only going to look at that part of the graph from 1987 to 1993.

Annual Breeding Numbers of Loggerhead Turtles between 1987 and 1993



What was happening to the turtle numbers for the periods 1988 to 1989 and from 1991 to 1992?

....

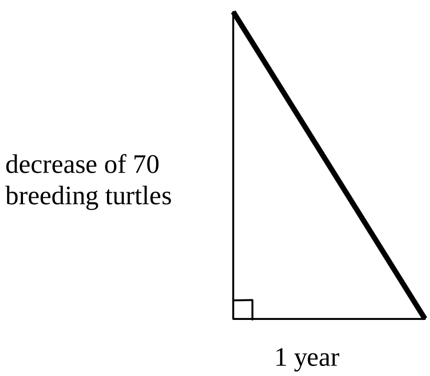
You should have said that the turtle numbers were decreasing.

Have a look at the two sections of graph for the above periods. How do they differ from the two periods that we looked at previously?

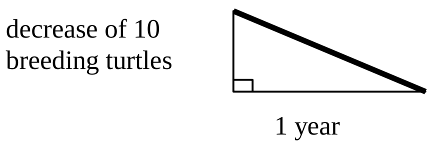
....

This time the two segments of graph are falling as we move along the graph from left to right.

1988–1989



1991–1992



To distinguish a graph that is rising as we move from left to right from a graph that is falling as we move from left to right we give the **falling** graph a **negative** gradient. We use the same formula as before but we must always check for rising or falling lines.

Let's find the gradients for the two periods of falling turtle numbers.

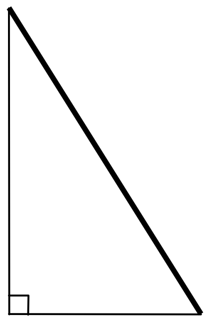
For the period 1988 to 1989:

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

$$\text{gradient} = \frac{-70}{1} = -70$$

1988–1989

decrease of 70
breeding turtles



1 year

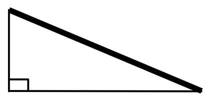
For the period 1991 to 1992:

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

$$\text{gradient} = \frac{-10}{1} = -10$$

1991–1992

decrease of 10
breeding turtles



1 year

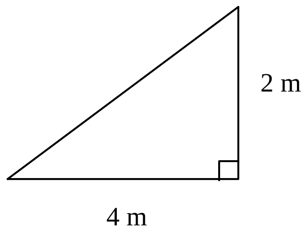
Notice again that the bigger the size of the number (ignoring the negative), the steeper the line.

Ski slopes are given ratings according to their gradients. A green run is a gentle slope suitable for beginners. A blue run is a steeper run suitable for the intermediate skier. The black run is steepest of all and is suitable only for the advanced skier.

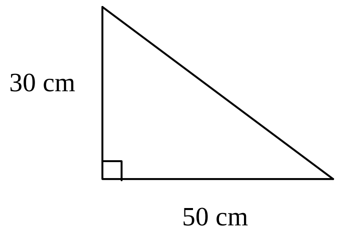
Activity 3.6

1. Find the gradient of the following line segments.

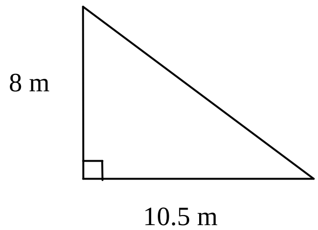
(a)



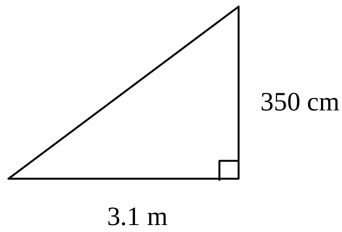
(b)



(c)



(d)



For the following questions, a **diagram** may help to clarify the situation.

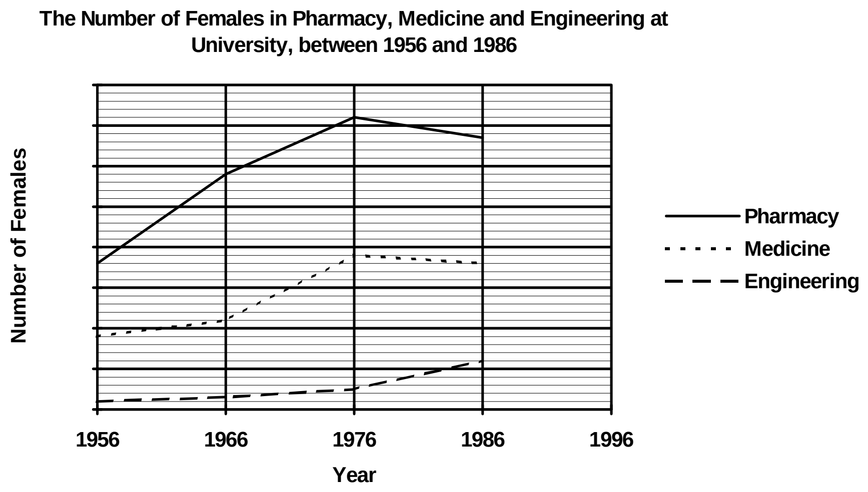
2. Interest rates rise from 3% to 5% in 2 years, what is the gradient? (Hint: think of the interest rate as the ‘rise’ and the years as the ‘run’.)

3. If you were skiing down a hill that fell 3 metres for every 2 metres of horizontal distance that you covered, what is the gradient of this hill?

4. Suppose you were riding the mine train down a tunnel to an underground mine. For every 10 metres of horizontal distance covered, you went 20 metres under the ground. What is the gradient of the mine train tunnel?

5. Suppose that you knew the gradient of a block of land you were looking at purchasing was 2. If you were to walk up the slope until you were 6 metres higher than your starting position, how many metres of horizontal distance would you have covered? A diagram might help you with your calculation.

6. The following graph has the scale on the vertical axis removed, but you can see that it represents the number of females in each of the given disciplines.

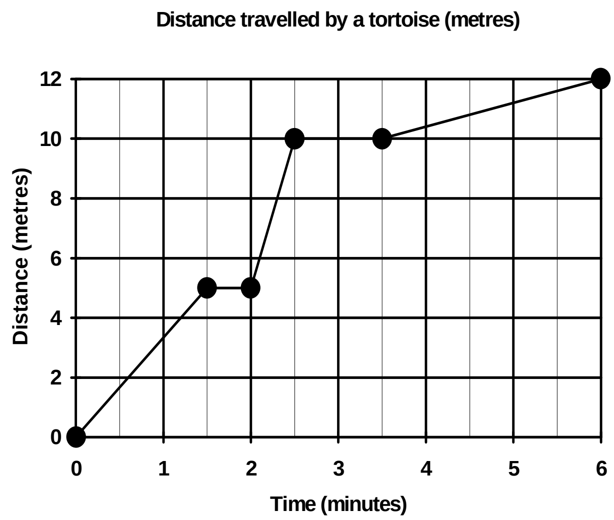


(a) Between 1956 and 1966, which course showed the greatest increase in numbers of females enrolled? How can you tell this from the graph?

(b) Over what periods and for what courses was there a decline in the number of females?

(c) Which course showed the greatest decline in numbers of females?

7. The following graph shows the distance covered by a tortoise over a given time.



(a) If we look at the gradient of this graph over any particular period, we would again put the change in height over the change in horizontal distance. Let's look at what this is telling us about this graph.

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

$$\text{gradient} = \frac{\text{distance}}{\text{time}}$$

Do you recognise the formula? What is the gradient telling us about this tortoise?

(b) Looking at the graph above, over which period of time is the tortoise travelling at the greatest speed? Calculate the gradient of the graph over this period.

(c) Over which period of time is the tortoise travelling at the least speed? Calculate the gradient of the graph over this period.

(d) Find the gradient of the graph from 2.5 to 3.5 minutes.

(e) Using your answer from part (d) complete the following sentence.

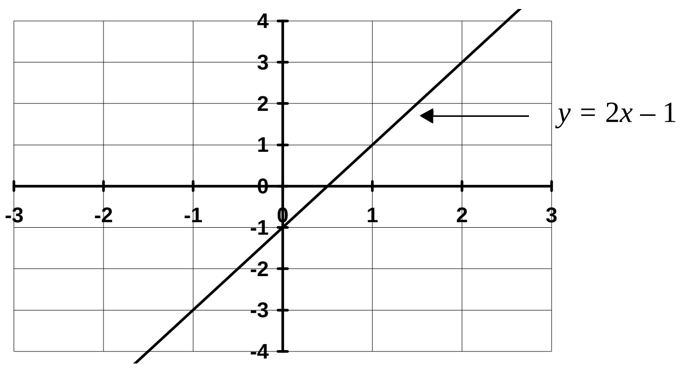
A horizontal line has a gradient of ....

3.3.1 Finding the gradient of a given line

In many situations the relationship between 2 variables can be represented by a formula or equation as we have seen already. It is often possible to tell a lot about the graph of this relationship without actually drawing the graph.

Example

Consider the following graph of the line $y = 2x - 1$.

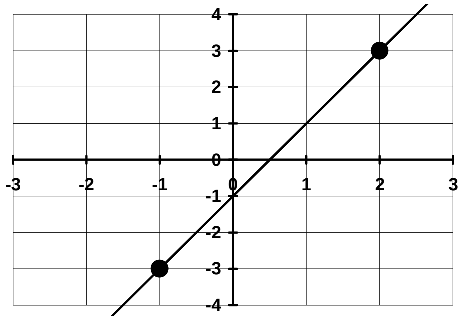


To find the gradient of this line we use the following procedure.

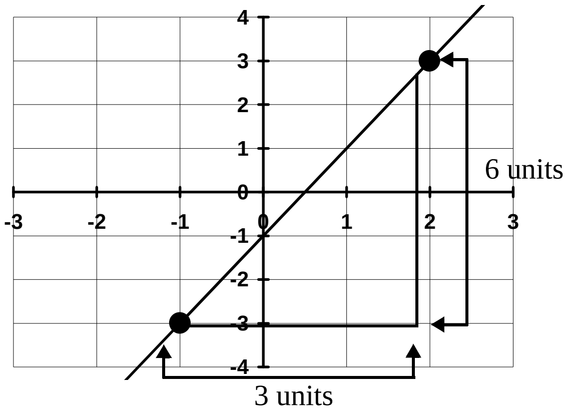
- Choose **any** two points on the line.
- Draw a triangle that shows the change in height over the change in horizontal distance between the two points.
- Calculate the gradient by putting the change in height over the change in horizontal distance. **Don't forget to check for a rising or falling line.**

Let's look at this for the above line.

Step 1. Choose **any** two points on the line.



Step 2. Draw a triangle that shows the change in height over the change in horizontal distance between the two points.



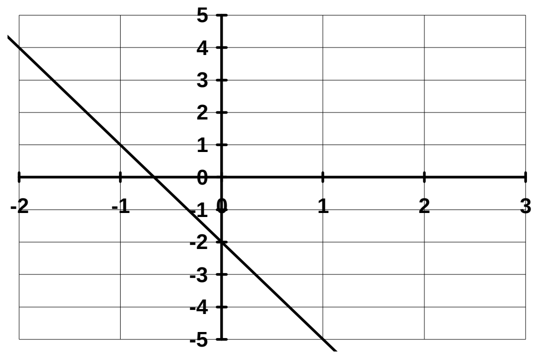
Step 3. Calculate the gradient by putting the change in height over the change in horizontal distance. Check to see if the line is rising or falling. This line is rising as we move from left to right and will thus have a positive gradient.

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

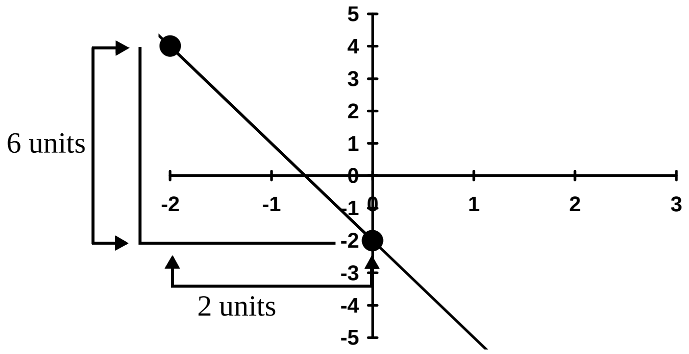
$$\text{gradient} = \frac{6}{3} = 2$$

Example

Find the gradient of the following line with equation $y = -3x - 2$



Two convenient points to choose this time might be (–2,4) and (0,–2).



Consider this time that the line is falling as we move from left to right so the gradient will be negative.

Therefore the gradient is:

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

$$\text{gradient} = \frac{-6}{2} = -3$$

3.3.2 Drawing a line, given the gradient

Using the skills we have just learnt, it is now possible to draw a line given its gradient and one point it passes through.

The steps that we will follow to do this are:

- plot the given point
- move horizontally and vertically according to the ‘instructions’ given by the gradient, and mark another point onto the Cartesian plane.
- Finally draw a line through and beyond these two plotted points.

Example

Let’s follow through these points and draw a line passing through the point (1,–2) with a gradient of 3.

Step 1 Plot the point (1,–2)

Step 2 Look at the gradient to determine the ‘instructions’ it is providing.

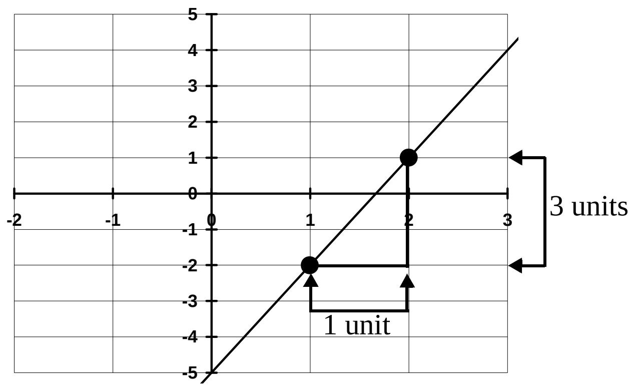
Now, $\text{gradient} = \dfrac{\text{change in height}}{\text{change in horizontal distance}}$

We know that the gradient for this question is 3, and we can express this as $\frac{3}{1}$, so we can write:

$$\text{gradient} = \frac{3}{1}$$

We take particular note that the gradient in this case is positive. This means that from our plotted point (1,–2), we will move 1 unit horizontally to the right and 3 units vertically upwards. This is the position of the second point.

Step 3 Now draw a line through and beyond the plotted points.



Example

This time we will look at an example where the gradient is negative.

Consider a line that passes through the point (–1,4) with a gradient of –3/2.

Step 1 Plot the point (–1,4)

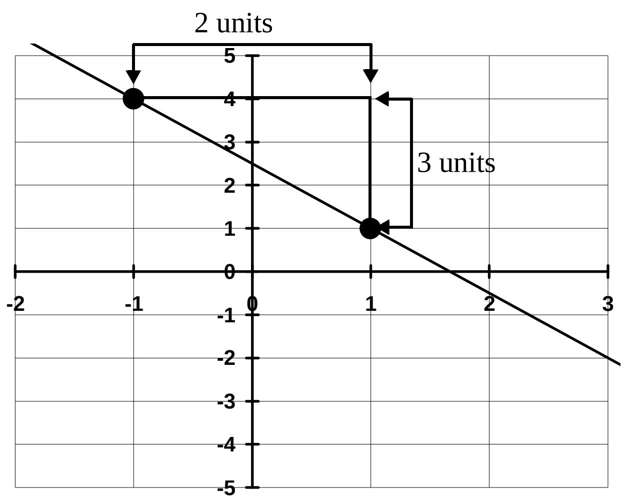
Step 2 Look at the gradient to determine the 'instructions' it is providing.

$$\text{Now,} \quad \text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

We know that the gradient for this question is –3/2, that is, the line falls 3 units for every 2 that it moves horizontally.

We take particular note that the gradient in this case is negative. This means that from our plotted point (–1,4), we will move 2 unit horizontally to the right and 3 units vertically downwards to show that it is a falling line. This is the position of the second point.

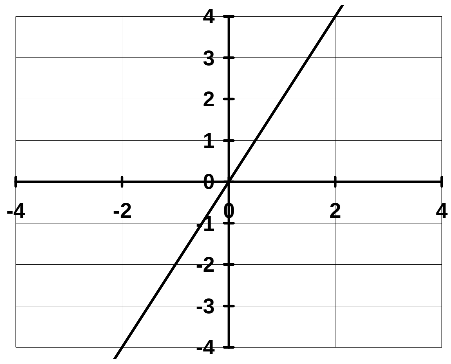
Step 3 Now draw a line through and beyond the plotted points.



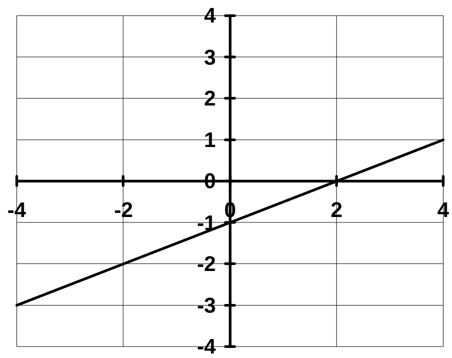
Activity 3.7

1. For each of the following lines, calculate the gradient. Don't forget to consider the scale on the axes and the rising or falling of the line.

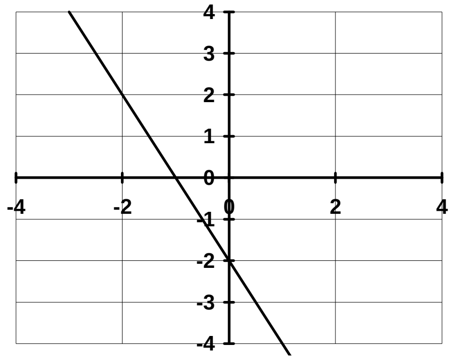
(a)



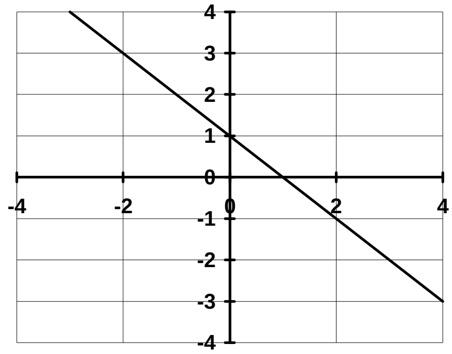
(b)



(c)



(d)



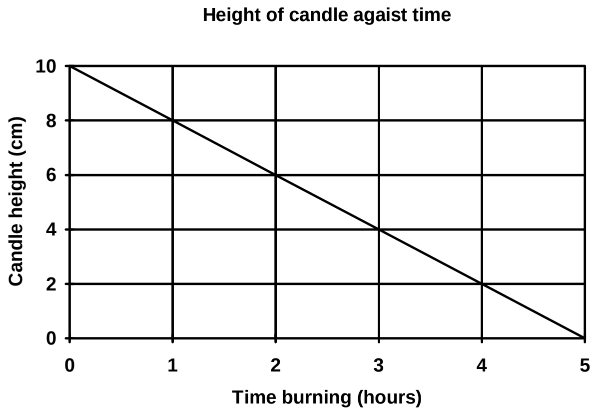
2. Draw a line with a gradient of 2 passing through (3,1)

3. Draw a line with a gradient of 2/3 passing through the point (-1,-2)

4. Before the invention of mechanical clocks, candles were sometimes used to measure the passage of time. A formula for the height of such a candle related to time is given below.

   $h = 10 - 2t$ where h equals the height of the candle in centimetres, and t equals the time in hours that the candle has been burning.

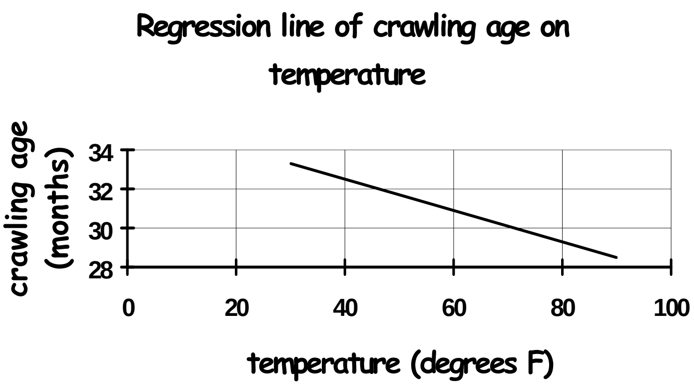
   The graph of this relationship looks like this.



(a) Find the gradient of this line.

(b) What is this graph 'telling' us about the rate at which the candle is burning?

5. The equation of the regression line for crawling babies was $y = -0.08x + 33.5$. The graph was:



(a) Find the gradient of this line.

(b) What is this graph 'telling' us about the relationship between temperature and crawling age?

We are now going to move on and look at a variety of equations and their graphical representation. You should be able to look at an equation and tell a lot about what the graph will look like before you draw it. This is a very valuable skill as it allows you to check that your calculating and plotting of values has been correct.

3.4 Linear equations

We call equations that produce straight line graphs, **linear equations**.

Here are some examples of linear equations that we have looked at in previous modules.

$C = 2D$	where C represents the amount of money that Chris earns in dollars, and D represents the amount of money that David earns in dollars.
$S = J + 3$	where S represents Shirley's age in years, and J represents Jeffrey's age in years.
$I = 200 + 50n$	where I represents the total income in dollars, and n represents the number of items sold.
$y = 3x - 2$	where x and y were not defined.
$h = 10 - 2t$	where h equals the height of the candle in centimetres, and t equals the time in hours that the candle has been burning.

Notice that in each case the variables are of power one and no two variables are multiplied together. This is the case for all linear equations.

Linear equations have both variables of power one and no variables multiplied together.

Activity 3.8

Which of the following are linear equations?

1. $y = 4x$
2. $y = 5x^2$
3. $3y + 2x = 6$
4. $xy = 4$
5. $3x = 7 - 2y$
6. $y = -2x$
7. $y = x^2 - 3x + 4$
8. $x = 3 - y$

You have learnt how to graph lines from equations in Activity 3.2.2. Try the following Activity.

Activity 3.9

1. Graph the following lines on the one set of axes on your graph paper.

   (a) $y = x$

   (b) $y = 2x$

   (c) $y = 0.5x$

   (d) $y = 3x$

2. Graph the following lines on another sex of axes on your graph paper.

   (a) $y = -x$

   (b) $y = -2x$

   (c) $y = -0.5x$

   (d) $y = -3x$

You should be able to see some patterns in these graphs.

Firstly, they all pass through the origin (0,0). We will discuss this point in a moment.

Look at the lines you have drawn in question 1 in the above activity. The coefficient of *x* in each case is positive and the slope or gradient of the lines is also positive (rising as we move from left to right). What do you notice above the value of the coefficient of *x* in the equation and the gradient of the line?

....

You should have said something about the greater the coefficient of *x* the steeper the line.

Let's now look at the equations and lines you drew in question 2 of the above activity.

This time the coefficients of *x* are negative and the gradients of the lines are negative (they fall as we move from left to right).

You should also note that $y = -3x$ is a much steeper line than $y = -1x$ (which may be written as $y = -x$).

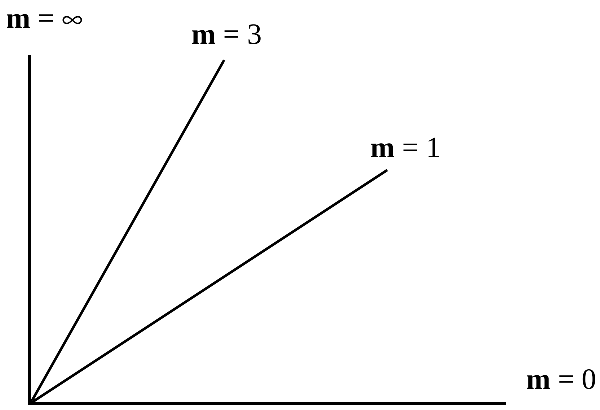
We can summarise this information grained from the last activity as follows:

The coefficient of *x* in a linear equation, is given the name gradient or slope (when the equation is in the form $y = ...$, for example $y = 5x$). We will use the letter m to represent the gradient.

Therefore in:

$y = 7x$	m = 7
$y = –3x$	m = –3
$y = 0.5x$	m = 0.5
$y = \frac{x}{5}$	m = $\frac{1}{5}$

Looking again at the solution to question 1 above, if we were to continue to increase the gradient the line will eventually be running vertical as is the y-axis. Such lines have infinite slope. Conversely, if we continue to decrease the slope, eventually the line will be horizontal and have a gradient of zero as we have discovered earlier in the module.



Activity 3.10

Graph the following lines on the one set of axes on your graph paper.

(a) $y = 2x$

(b) $y = 2x + 1$

(c) $y = 2x + 2$

(d) $y = 2x – 1$

(e) $y = 2x – 2$

Once again look for patterns in this set of graphs.

Notice that all the lines are parallel to each other – that is, if you extend them infinitely in either direction, they will never meet. Now look at the value of **m** in each equation. It is always 2 for these examples. This tells us that **lines with the same gradient are parallel**.

Can you see a relationship between the point where each line crosses the y-axis and the number on its own in each equation?

....

In fact this number on its own tells you the point at which the graph will cut the y-axis. We use the letter **c** to represent the ***y*-intercept** (the point where the line cuts the y-axis)

Therefore in:

$y = 7x + 3$	$c = 3$
$y = -3x + 7$	$c = 7$
$y = 0.5x - 1$	$c = -1$

In general one way to write a linear equation is in the form:

$$\boldsymbol{y = mx + c} \qquad \textbf{where} \quad \textbf{m = gradient or slope of the line}$$
$$\textbf{c = the } \boldsymbol{y}\textbf{-intercept}$$

Let's summarise what we have learnt so far about linear equations.

- If **m** is positive, the gradient is positive and the line rises as we move from left to right
- If m is negative, the gradient is negative and the line falls as we move from left to right.
- The greater the size of **m** the steeper the line.
- Parallel lines have the same gradient.
- The point where the line cuts the *y*-axis is called the *y*-intercept and is represented by the letter **c**.
- Lines parallel to the *y*-axis have infinite slope while lines parallel to the *x*-axis have zero slope.

This information now gives us a very powerful tool for **estimating** what a linear graph will look like given its equation.

Example

Find the slope and *y*-intercept of the following equation.

$$y = -3x + 2$$

Firstly, it is a linear equation. We should next check that it is in the form $y = \mathbf{m}x + c$

In this case it is in the correct form, therefore we can read off the values for **m** and **c**.

Gradient = -3

y-intercept = 2

Example

Find the slope and *y*-intercept of the following equation.

$$3x + y = 5$$

Firstly, it is a linear equation. We should next check that it is in the form y = m + c

In this case it is not in the correct form, therefore we must rearrange the equation using the techniques from module 7 before we can read off the values for **m** and **c**.

$3x + y = 5$	We want to make y the subject of the equation.
$3x + y - 3x = 5 - 3x$	
$y = 5 - 3x$	
$y = -3x + 5$	

It is now in the form $y = mx + c$ and we can read off the required values.

Gradient = –3

y-intercept = 5

Example

Find the slope and *y*-intercept of the following equation.

$$3x + 2y = 7$$

Firstly, it is a linear equation. We should next check that it is in the form $y = mx + c$

Again we must rearrange the equation before we can read off the values for m and c.

$3x + 2y = 7$	We want to make y the subject of the equation.
$3x + 2y - 3x = 7 - 3x$	
$2y = 7 - 3x$	
$2y = 7 - 3x$	Divide everything on both sides by 2.

$$y = \frac{7}{2} - \frac{3x}{2}$$

$$y = \frac{-3x}{2} + \frac{7}{2}$$

In Data Analysis an equation is often written in the form $y = a + bx$. Here the gradient is **b** and the *y*-intercept is **a**. (Can you see $y = a + bx$ is the same as $y = bx + a$?)

Example

Find the slope and *y*-intercept of the following equation.

$$5x - 3y = -7$$

Firstly, it is a linear equation. We should next check that it is in the form $y = a + bx$

Again we must rearrange the equation before we can read off the value for b and a

$5x - 3y = -7$	We want to make *y* the subject of the equation.
$5x - 3y - 5x = -7 - 5x$	
$-3y = -7 - 5x$	Divide everything on both sides by –3
$\frac{-3y}{-3} = \frac{-7}{-3} - \frac{5x}{-3}$	
$y = \frac{-7}{-3} - \frac{5x}{-3}$	Notice that top and bottom of these fractions are negative. Dividing two negative numbers gives a positive number.
$y = \frac{7}{3} + \frac{5x}{3}$	

It is now in the form $y = a + bx$ and we can read off the required values.

$$\text{Gradient} = \frac{5}{3}$$

$$y\text{-intercept} = \frac{7}{3}$$

Example

Just as we can find the gradient and *y*-intercept, given the equation of a line, it is also possible to form the equation, given the gradient and *y*-intercept.

Form an equation for a line with a gradient 3 and *y*-intercept $\frac{-4}{5}$ using the form $y = a + bx$.

We have $b = 3$ $a = \frac{-4}{5}$

So the equation becomes

$$y = \frac{-4}{5} + 3x$$

Activity 3.11

1. State the gradient and y-intercept of the graphs of the following linear equations.

   (a) $y = -2x + 3$

   (b) $y = 5 + 3x$

   (c) $y = 2 - 4x$

   (d) $y = x - 3$

   (e) $y = 5x$

   (f) $y + 4 = 6x$

   (g) $3x - y = 6$

   (h) $4y + 6 = -12x$

   (i) $2x + 2y - 7 = 0$

2. Write equations for lines with the following gradients and y-intercepts.

   (a) **m** = 5, **c** = 2

   (b) **m** = 3, **c** = –1

   (c) **b** = 1, **a** = 0

   (d) **b** = $\frac{1}{2}$, **a** = –5

   (e) **b** = $\frac{-3}{4}$ **a** = $\frac{7}{2}$

3. Write a few sentences comparing the graphs of the following pairs of equations.

   (a) $y = 3x + 1$

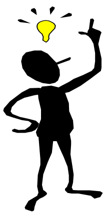
   $y = 3x + 4$

   (b) $y = 2 + 2x$

   $y = 2 + 3x$

   (c) $\hat{y} = 2 + 3x$

   $\hat{y} = 2 + -3x$



4. Check the above pairs to check your comparisons.

Solutions

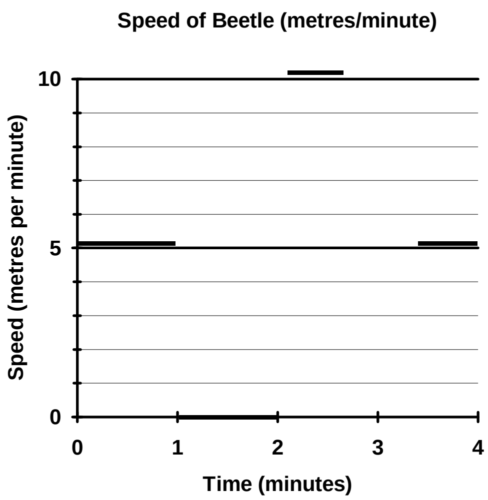
Activity 3.1

1.
   (a) 1973 had the greatest number, of breeding turtles.
   (b) The least number of turtles was in 1989.
   (c) Between 1972 and 1973 there was the greatest increase in turtle numbers.
   (d) Between 1973 and 1974 saw the greatest decrease in turtle numbers.
   (e) If you were to continue the graph following the same path from 1991 to 1993 this line would cut the horizontal axis at about the year 2005.

2.
   (a) 30% of students in higher education in 1991 were aged 20-24 years
   (b) 1989 saw the greatest representation of 19 years and under students.
   (c) Students aged 25 to 29 are the least represented in higher education sector for these years

3.
   (a) For the first graph, from 1 to 2 minutes and from 2.5 to 3.5 minutes, there is no change in distance so the beetle is not moving.
   For the second graph, from 1 to 2 minutes and from 2.5 to 3.5 minutes, the speed of the beetle is 0 so the beetle is not moving.
   (b) From 0 to 1 minute the beetle travelled 5 metres.
   (c) From the second graph the beetle is travelling at 5 metres per minute from 0 to 1 minutes.
   (d) Between, 3.5 and 4minutes the beetle travelled 2 metres.

   The speed will be $\frac{2\,\text{metres}}{0.5\,\text{metres}}$ = 4 metres/minute



4. This graph is showing time taken against distance travelled. The problem with the graph is that the longer the race the less time it takes. A 1 km race will take about 4 minutes while a 3 km race takes no time at all.

5. This is an explanation from the Data Analysis text.

When you toss a coin there are only 2 possible outcomes, heads or tails. Figure 4.6 shows the result of tossing a coin 1000 times. For each number of tosses from 1 to 1000, we have plotted the proportion of these tosses that gave a head. The first toss was a head, so the proportion of heads starts at 1. The second toss was a tail, reducing the proportion of heads to 0.5 after two tosses. The next three tosses gave a tail followed by two heads, so the proportion of heads after five tosses is 3/5, or 0.6.
The proportion of tosses that produces heads is quite variable at first, but settles down as we make more and more tosses. Eventually this proportion gets closer and closer to 0.5 and stays there. We say 0.5 is the probability of a head. The probability 0.5 appears as a horizontal line on the graph.

Activity 3.2

1.

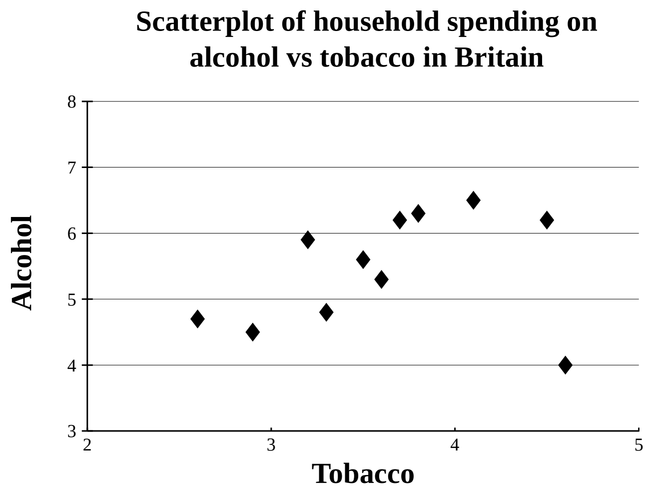
(a) I6 G block, Education, Student Services, Sciences.
(b) D5 F block, indoor Recreation Centre.
(c) N7 The Japanese Gardens.

2.

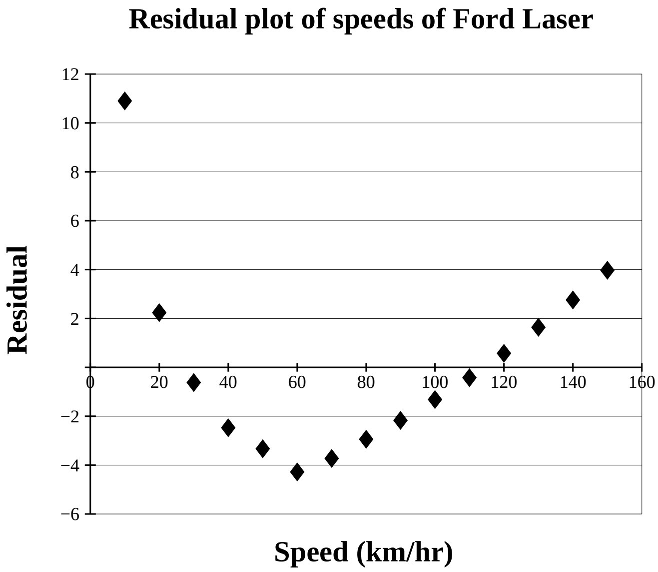
(a) Steele Rudd Courts. F4
(b) W Block Sciences (nursing, and psychology) and Shopping Complex.H5 or H6
(c) Gymnasium N3

Activity 3.3

1.



2.



Activity 3.4

(a) (–1,2.5)
(b) (0,2)
(c) (3,4)
(d) (–3.5,0)
(e) (–2,–4)
(f) (1.5,–3)
(g) (5,–1)

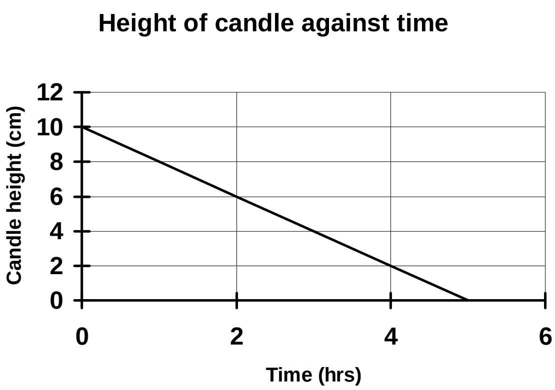
Activity 3.5

1.

(a)

t	0	1	2	3	4	5
h	10	8	6	4	2	0

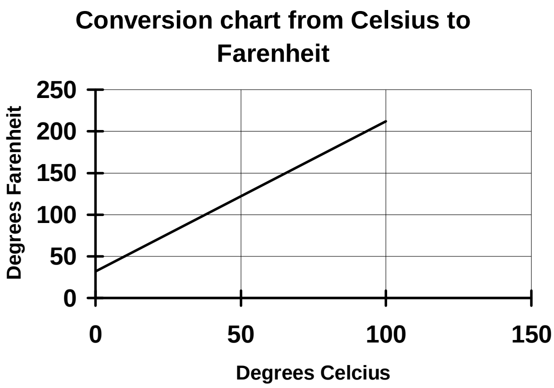
(b)



(c) At time 0 hours (before the candle was lit) the height of the candle was 10 cm.
(d) The 10 in the formula represents the height of the candle before it was lit.
(e) After 5 hrs the candle has a height of 0 cm so it will not burn any further. The candle burns for 5 hours.

2.

(a)



(b) From your graph you should be able to read that 59°F is the same as 15°C. Warm clothing will be needed for this trip.

(c) 35°C is equivalent to 95°F.

3.

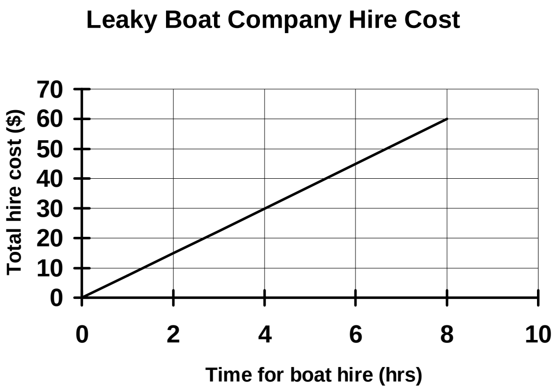
(a)

H = 7.50*t* where H represents the cost of hire in dollars, and *t* represents the time in hours that you have a boat.

(b)

Number of Hours	1	2	3	5	8
Total Cost ($)	7.50	15.00	22.50	37.50	60.00

(c)



(d) Cost for 4 hours hire will be $30.00

(e) For $24 you would be able to hire the boat for 3.2 hrs. This is 3 hrs and 12 mins, since 0.2 of an hour is 12 minutes.

4.

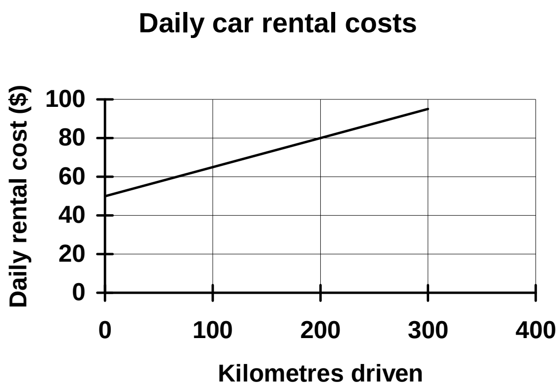
(a)

$C = 50 + 0.15k$ where C represents the total daily cost of hire in $, and k represents the number of kilometres travelled. Note that since the 50 is in dollars the 15 cents must also be written in dollars.

(b)

No. of km travelled	0	100	200	300
Total cost of hire	50	65	80	95

(c)



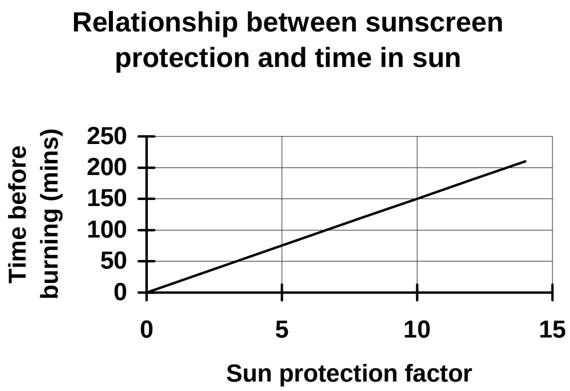
The cost for the day, having driven 260 kms, is $89.

(d) If you had $110 you could travel 400 kms.

5.

(a) For each increase of two in the sun protection factor, 30 extra minutes can be spent in the sun before burning.

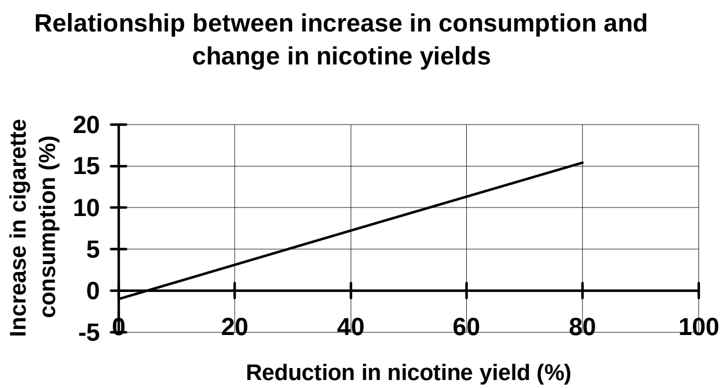
(b)



(c) Minimum sunscreen factor would be about 4.

6.

(a)



(b) 10% increase in cigarette consumption

(c) decrease = (2.1 – 1.3)/2.1 × 100% ≈ 38%. Therefore increase in consumption ≈ 7%

Activity 3.6

1.

(a) $\text{gradient} = \dfrac{\text{change in height}}{\text{change in horizontal distance}} = \dfrac{2}{4} = \dfrac{1}{2}$

$\text{gradient} = \dfrac{\text{change in height}}{\text{change in horizontal distance}} = \dfrac{-30}{50} = \dfrac{-3}{5}$

(b) $\text{gradient} = \dfrac{\text{change in height}}{\text{change in horizontal distance}} = \dfrac{-8}{10.5} = \dfrac{-80}{105} = \dfrac{-16}{21}$

(c) $\text{gradient} = \dfrac{\text{change in height}}{\text{change in horizontal distance}} = \dfrac{350}{310} = \dfrac{35}{31}$

Did you remember to change the measurements so that they are in the same units?

2. The rise is 2% and the run is 2 years so the gradient is 2 ÷ 2 = 1

3. The gradient of the hill would be –1.5

4. The gradient of the tunnel would be –20/10 = –2

5. A diagram should help with this situation.
To have a gradient of 2, the horizontal distance covered must have been 3 metres.

6.

(a) Between 1956 and 1966, Pharmacy showed the greatest increase in numbers. This can be seen became the Pharmacy line is steeper than the other two lines.
(b) Between 1976 and 1986 there was a decline in the number of women in Pharmacy and medicine.
(c) Pharmacy has shown the greatest decline in numbers of women because it is the steeper line than the Medicine line over the same period.

7.

(a) This formula is the formula for the speed that we have looked at previously. The gradient is telling us the speed of the tortoise in metres/minute.
(b) Between 2 and 2.5 minutes the tortoise is travelling at the greatest speed.

$$\frac{\text{change in height}}{\text{change in horizontal distance}} = \frac{5}{0.5} = 10$$

(c) The least speed (apart from when stopped) was over a period 3.5 to 6 minutes.

$$\frac{\text{change in height}}{\text{change in horizontal distance}} = \frac{2}{2.5} = 0.8$$

You can see from this result that the tortoise is indeed much slower over this period.

(d) $\dfrac{\text{change in height}}{\text{change in horizontal distance}} = \dfrac{0}{1} = 0$

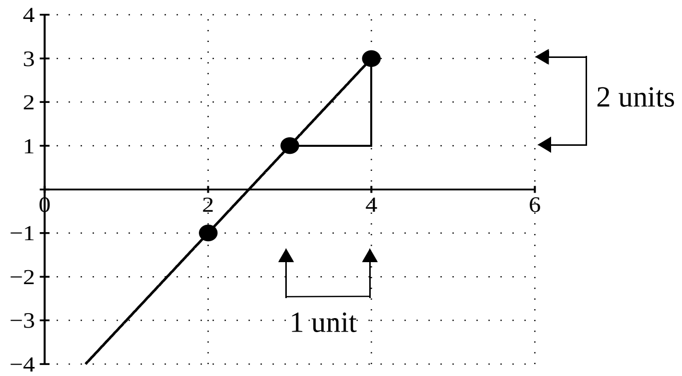
(e) A horizontal line has a gradient of zero.

Activity 3.7

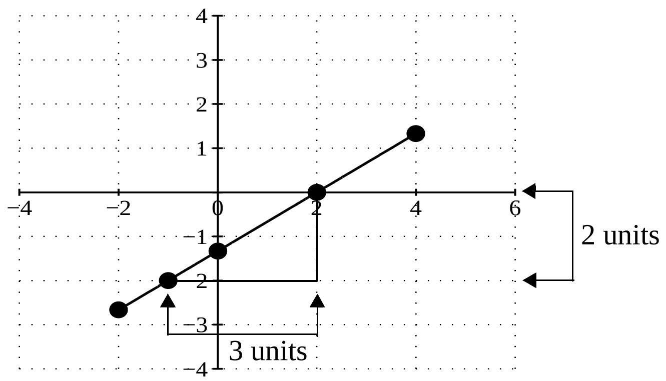
1. For these questions you might have chosen a variety of different points with which to work. With this in mind only the final gradient is given.

   (a) Gradient = 2
   (b) Gradient = 0.5
   (c) Gradient = –2
   (d) Gradient = –1

2. A gradient of 2 can be written $\frac{2}{1}$



3.



4.
   (a) The gradient of this line between any two points that you might have chosen is –2.
   (b) This gradient 'tells' us that the candle burns (decreases in height) at a rate of 2 cm for every hour.

5. $y = -0.08x + 33.5$

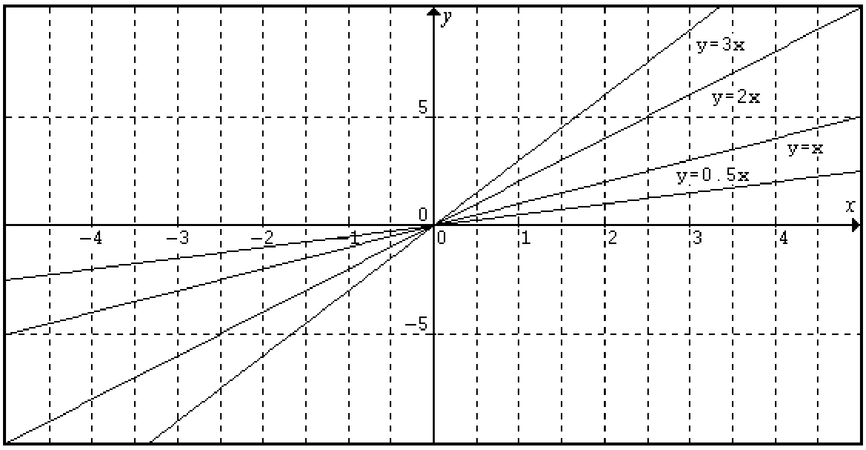
   (a) Gradient = –0.08
   (b) As the temperature increases, the crawling age in months decreases.

Activity 3.8

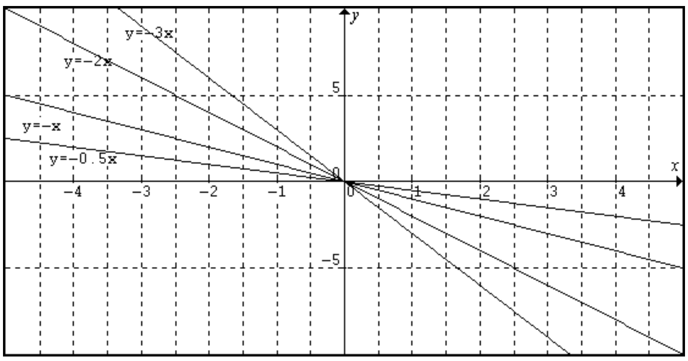
1. $y = 4x$	Linear
2. $y = 5x^2$	Not linear – power greater than one
3. $3y + 2x = 6$	Linear
4. $xy = 4$	Not linear – two variables multiplied together
5. $3x = 7 – 2y$	Linear
6. $y = –2x$	Linear
7. $y = x^2 – 3x + 4$	Not linear – power greater than one
8. $x = 3 – y$	Linear

Activity 3.9

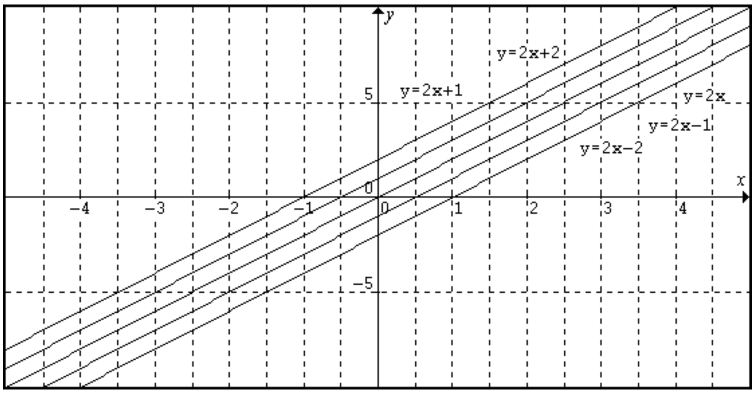
1.



2.



Activity 3.10



Activity 3.11

1.

(a) $y = -2x + 3$ gradient = –2 y-intercept = 3

(b) $y = 5 + 3x$ gradient = 3 y-intercept = 5

(c) $y = 2 - 4x$ gradient = –4 y-intercept = 2

(d) $y = x - 3$ gradient = 1 y-intercept = –3

(e) $y = 5x$ gradient = 5 y-intercept = 0

(f) $y + 4 = 6x$
$y = 6x - 4$ gradient = 6 y-intercept = –4

(g) $3x - y = 6$
$-y = 3x + 6$
$y = 3x - 6$ gradient = 3 y-intercept = –6

(h) $4y + 6 = -12x$
$4y = -12x - 6$
$y = -3x - \dfrac{3}{2}$ gradient = –3 y-intercept = $-\dfrac{3}{2}$

(i) $2x + 2y - 7 = 0$
$2y = -2x + 7$
$y = -x + \dfrac{7}{2}$ gradient = –1 y-intercept = $\dfrac{7}{2}$

2.

(a) **m** = 5 **c** = 2 $y = 5x + 2$

(b) **m** = 3, **c** = –1 $y = 3x - 1$

(c) b = 1, a = 0 $y = x + 0$ or $y = x$

(d) b = $\dfrac{1}{2}$ $a = -5$ $y = x - 5$ or $y = -5 + 0.5x$

(e) $b = \dfrac{-3}{4}$ $a = \dfrac{7}{2}$ $y = -\dfrac{3}{4}x + \dfrac{7}{2}$ or $y = 3.5 - 0.75x$

3.

(a) $y = 3x + 1$

$y = 3x + 4$

These two lines have the same gradient and are therefore parallel. The first will cut the *y*-axis at 1 and the second graph will cut the *y*-axis at 4.

(b) $y = 2x + 2$

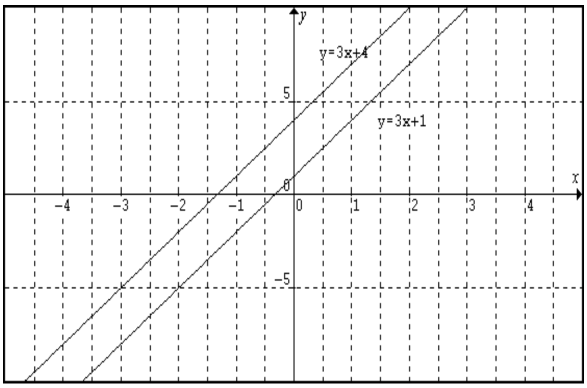
$y = 3x + 2$

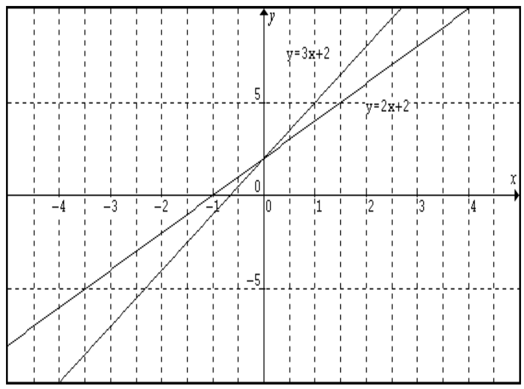
These two lines cut the *y*-axis at the same point, $y = 2$. The second graph will be slightly steeper than the first graph owing to the fact that it has a larger gradient.

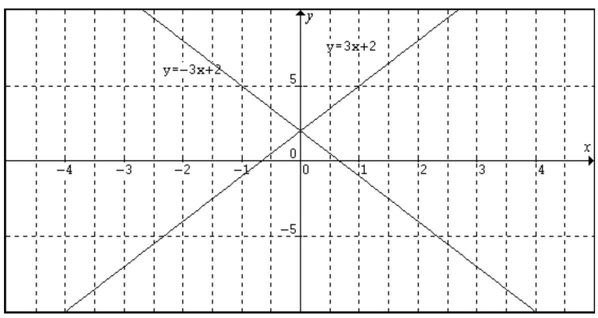
(c) $\hat{y} = 2 + 3x$

$\hat{y} = 2 + -3x$

These two lines cut the *y*-axis at the same place, $y = 2$. This time though, the first line has a positive gradient and the line rises as we move from left to right, while the second line has a negative gradient and falls as we move from left to right.







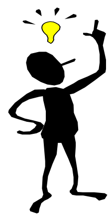
Module 4

Statistics

Aims

In this module we aim to help you in the process of collecting, organising, presenting and analysing data. More formally, on successful completion of this module you should be able to:

- categorise data into different types;
- define and give examples of population, sample, variables and randomness;
- summarise and interpret data in bar charts;
- summarise and interpret data in frequency distributions and histograms;
- use and construct stem-and-leaf plots to help interpret data;
- demonstrate an understanding of the centre and dispersion of data;
- calculate and interpret mean, median, mode, range and standard deviation of a sample
- summarise and interpret data with two variables using scatterplots and lines of best fit.

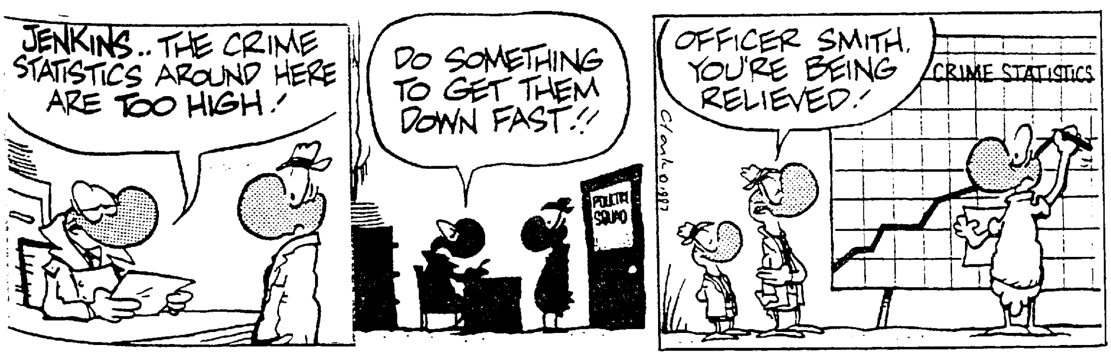


4.1 Introduction

It is often said that today we live in an information society. We just have to look at a newspaper or watch the TV to see that that is the case. Reporters often use expressions like:

> *"The statistics indicate....."*
> *"The data shows...."*
> *"Results of university tests show..."*

We are surrounded with advertising trying to persuade us that one product is better than another. In our society today if we are to be informed citizens and not be taken advantage of we need to know how to interpret such information for ourselves. Also in your future tertiary studies you might be required to collect large amounts of information which has to be summarised and presented in a report or essay. Statistics is the science of gaining information from numerical facts. These numerical facts are called data. These words were first coined in the eighteenth century by political scientists in Germany.



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4.2 Collecting data

We see data everywhere but where does it come from and what exactly is it? Data is collected for individuals.....how many people live in Australia, are these people male or female, how tall are most Australians. All these figures are data (data is the plural of datum from the Latin). However, data are not only collected about people we also might want to collect data from things or animals. **Individuals** are the objects described by a set of data. **Variables** are the characteristics of those individuals, for example, number of people, the gender or the height of the people. Note that a variable might thus take on different values for different individuals.

Data collected as variables can be of different types. For examples when we ask whether a person is male or female only a certain number of answers are possible (usually only male and female). The same goes for blood types, eye colour, country of birth. All these variables would be **categorical variables**. That is the values for the variable are just labels for the different categories. The categories should be non-overlapping and should cover all possibilities.

On the other hand the value of the variable could be found by counting or measuring a quantity. Such variables would be called **quantitative variables**. Some of these data would be of **continuous** (taking any value) such as height, temperature, money, while other data would be **discrete** (able to take only whole number values) such as number of people or number of trees in a paddock.

In summary

Variables are
- Categorical
- Quantitative
  - Discrete
  - Continuous

Example

Classify the following variables as either categorical, discrete or continuous

- Effect of new drug (successful or not successful)
- Rainfall in mm
- Number of words in a paragraph
- Australian State
- Blood Pressure in mmHg
- Number of faults at tennis

Variable	Type	Reason
Effect of new drug	categorical	only two non-overlapping alternatives present
Rainfall in mm	continuous	an unlimited number of smaller and smaller measurements are possible
Number of words in a paragraph	discrete	can take only values which are whole numbers; this is a result of counting
Australian State	categorical	only six non-overlapping alternatives present
Blood Pressure in mmHg	continuous	an unlimited number of smaller and smaller measurements are possible
Number of faults at tennis	discrete	can take only values which are whole numbers; this is a result of counting

Now that we know what data we might want to collect, how do we go about collecting it? Let’s look at the following example.

Suppose an entertainment park wishes to provide a family pass to their attraction. They would like their family pass to cater for families with the ‘average’ number of children. How do they go about finding the ‘average’ number of children in a family?

They must firstly find the number of children in every family with children in Australia. What a task!

All the children in Australia would be called a statistical **population**. That is, all possible values that could be collected: in our case the number of children in every family with children in Australia. In this and many other statistical investigations it is not practical to survey the entire population. In this case we might investigate a **sample** of the population. If this sample is to be later used to make conclusions about the entire population then it is important to make it representative of the entire population. It could include different types of families; country families, city families, or families from different nationalities, blended families and families with only one parent.

We call a sample that represents the entire population a **random sample**. This means that every member of the population has an equal chance of being selected for the sample. We have not gone out on purpose and selected every city family (for example) to survey and thus obtained a sample biased towards that group. In the remainder of this module we will be dealing only with samples collected randomly from a larger population.

Example

The following are some examples of populations, samples and variables that might have been of interest.

Population	Possible sample	Possible variable of interest
All words in a book	20 words from each page	length of words function of word (verb, noun) number of syllables in word
All trees in Australia	100 trees from each National Park	height of tree species of tree presence of flowers presence of possums
All Australian women who smoke	2000 women who smoke	age of women weight of women presence of cancer

Activity 4.1

1. Data from a study of local primary schools involving teachers, pupils and parents contained values of a range of different variables. Which of the variables were categorical, discrete or continuous?

   (a) Gender (male or female)

   (b) Age (years)

   (c) Income of parents (dollars and cents)

   (d) Siblings (number for each pupil)

   (e) Temperature of classrooms in winter (degrees Celsius)

   (f) Smoker (yes or no)

   (g) Class size (number of children)

2. You are interested in the relationship between English proficiency and 1st year university results of students whose second language is English. Answer the following questions.

   (a) Define the population

   (b) Define the sample(s) of the population you might decide to use.

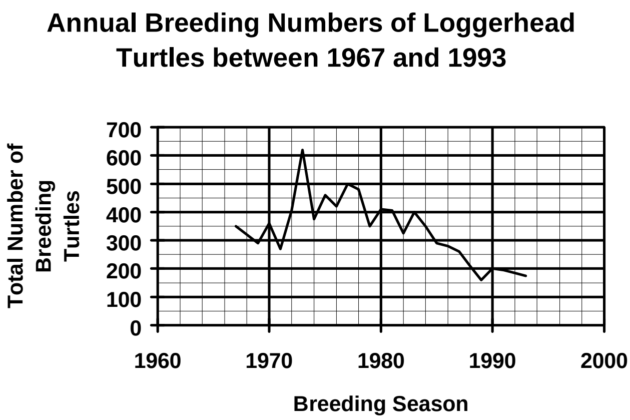
   (c) Define some variables which may be on some interest in this study.

4.3 Organising and displaying data from tables

Previously in these modules we have examined vast amounts of data. These data were collected from a range of different samples for a multitude of reasons, and often were summarised into tables. Tables are useful so that we can see exactly the details of the data but to gain the attention of the reader or audience and to show the main points in a set of data then tables are often turned into pictures. These pictures can take the form of graphs. The following are some types of graphs we have seen before.

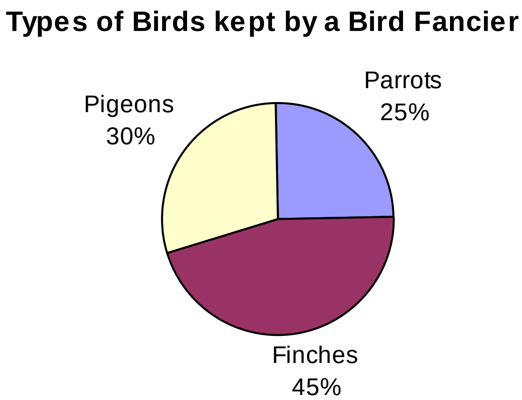
Line graphs

Line graphs are those where data have been plotted as a series of points that are joined by a line.



Pie charts

Pie charts are pictures where data are represented as a circle in which each sector represents each category under study. They are used primarily for categorical data



Bar charts

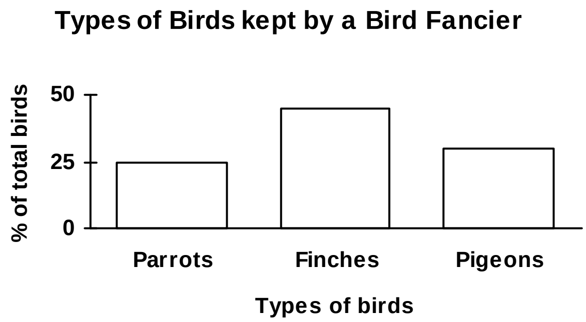
An alternative to the pie chart is a bar chart.

Let's examine the data from the pie chart above. These data were originally represented in a table.

Type of bird	% of Total birds
Parrots	25
Finches	45
Pigeons	30

In a bar chart the data are shown as a series of rectangles which represent each of the categories of the data. The height of the rectangle gives the value of the variable.

To draw this graph you must first consider what will be put on each axis. On the horizontal axis each category is given a certain length which will depend on the number of categories you have to fit across the page. The vertical axis shows the variable of interest this time the percentage of total birds. The graph should be high enough to cater for the maximum percentage shown in the table. The bar chart then looks like this.



From this graph we can quickly interpret that finches are the most common bird in the bird fancier's collection, with parrots being the least common.

Don't forget the following when drawing bar charts:

- label both horizontal and vertical axes;
- scale is correct and appropriate;
- make sure the rectangles are of equal width;
- make sure that the rectangles are the same distance apart;
- give the chart a title which details what the chart is about.

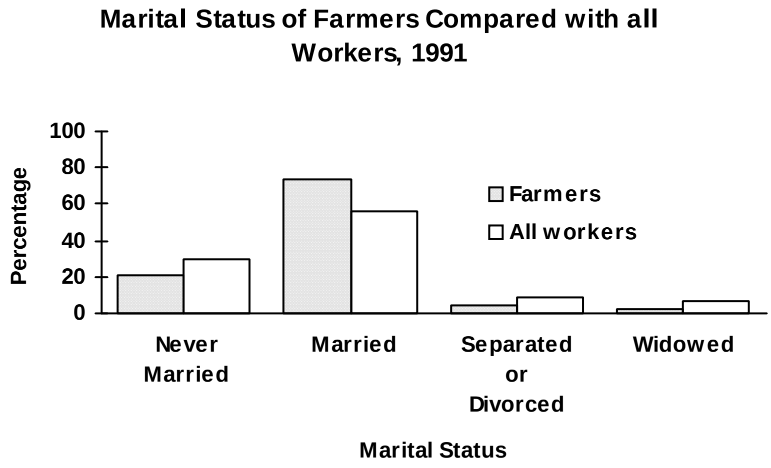
Often data contain information that has been broken into many different categories. This is where bar charts are more useful than pie charts. Consider the following set of data.

Marital status of farmers compared with all workers, 1991

	Never Married %	Married %	Separated or Divorced %	Widowed %
Farmers	20.8	73.2	4.0	2.0
All workers	29.6	55.7	8.3	6.4

(Source: Australian Bureau of Statistics 1991.

These data could be included in a **paired bar chart** as shown below. It is easy to see from this chart the differences between farmers and other workers.



Activity 4.2

1. A survey of 200 people asking what cereal they ate for breakfast found the following results:

Cereal	Number of people	Percentage of people
Corn Flakes	50	
Rice Bubbles	42	
Nutri Grain	39	
Rolled Oats	23	
Muesli	11	
Coco Pops	10	
Other Cereals	25	

   Draw a bar chart representing the cereal preferences of this group of people.

2. The following table shows the different items that have been found around the necks of 75 seals in the wild. This is a very serious problem in the wild and often leads to the death of the seal.

Item	Number	% of Total
Trawler nets	26	
Packaging bands	15	
Gillnet	8	
Rope	5	
Other	21	
Total	**75**	

   Complete the above table and present the data pictorially in a bar chart.

3. 500 females and 500 males in a city were asked which political party they would support if there was an election on the following day. The results are given below.

Political Party	Males	Females	Total
National	140	100	240
Liberal	65	135	200
Labour	260	180	440
Democrats	20	60	80
Other	15	25	40

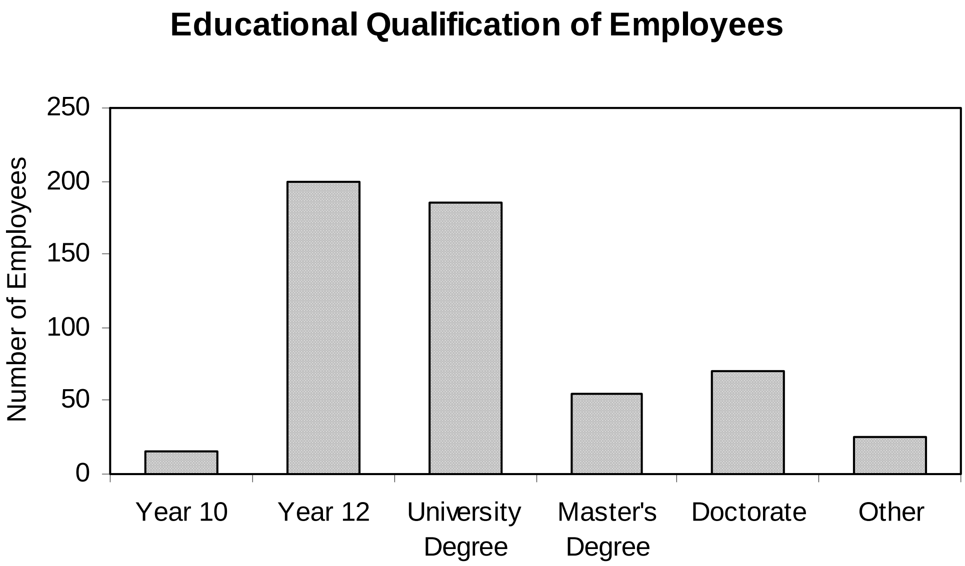
(a) Display these data as a paired bar chart from which you could compare male and female political preferences.

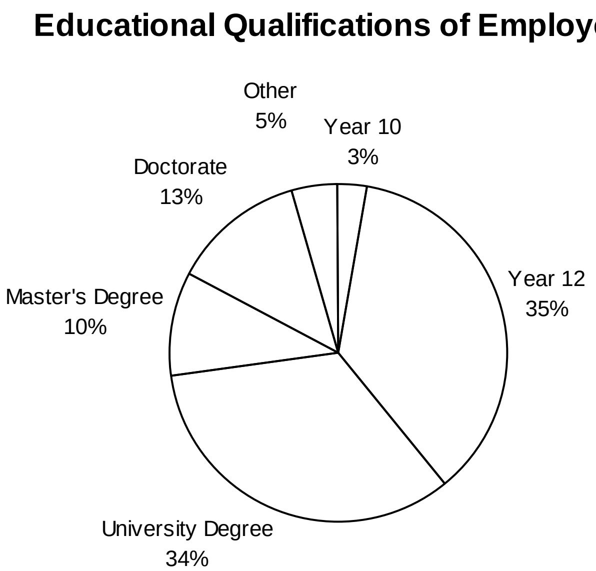
(b) Write a few sentences comparing the different preferences of males and females.

4. The following are the qualifications of a group of workers at a local company. The local newspaper collects these figures from the company census with the aim of writing an article on the educational background of a typical company in that town.

Qualification	Number of employees	% of total
Year 10 Secondary School	15	2.7
Year 12 Secondary School	200	36.4
University Degree	185	33.6
Master's Degree	55	10.0
Doctorate Degree	70	12.7
Other	25	4.5
Total	**550**	**100.0**

The newspaper is trying to decide how to best present these data pictorially. The reporter has suggested that they use one of the following graphs.

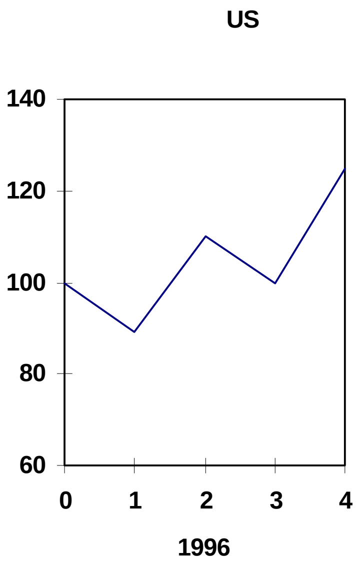


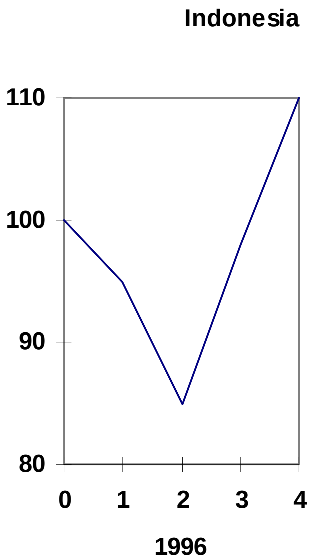


(a) Which graph would best describe the educational backgrounds for this set of workers. Explain the reason for your choice.

(b) Write a paragraph using one of the above graphs to describe the educational backgrounds of the workers. The paragraph should be suitable for use by the local newspaper.

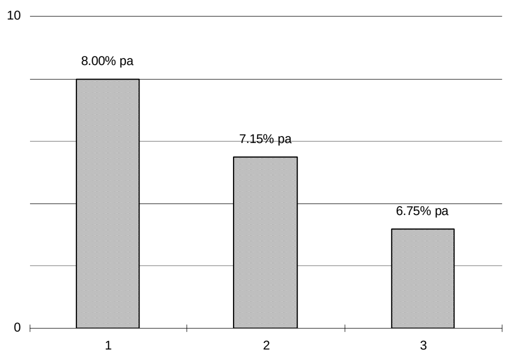
In this section of work we have looked at a three different types of graph. This is only the tip of the iceberg when it comes to graphs that you might see in the daily news. The way that different types of graphs can be presented is only limited by the our creativity. However, beware many graphs that we see daily are designed to deceive the unwary. It is up to us to be cautious in our interpretation of statistics. Here are a couple of examples of things to look for....





1,2,3,4 = first, second, third, fourth quarter changes

Graphs similar to these were seen in a financial newspaper. Note the different scales on the vertical axes making them difficult to compare. Also the titles and axes are lacking in details making them difficult to interpret.



Note: This graph represents rates for $20 000 – $40 000

This graph was seen in a advertisement for a popular credit union. Note that there is no detail on either axes and that the percentage written on the columns do not correspond with the scale on the left.

4.4 Organising and displaying raw data

So far we have looked at data that has already been collected and displayed in a table, but what if the data has just been collected and you are beginning with a large group of figures. For example:

You are a social worker who has just completed collecting data on the number of children in all the families within your jurisdiction. What you have as a result is a table which contains counts of the number of children in thirty families.

1	4	2	3	2	4	1	5	2	3
3	1	4	5	5	2	8	3	6	4
2	3	2	3	1	5	4	7	2	6

This type of table is very difficult to interpret, so we need a way to summarise these data into a form that is easier to read and interpret. The frequency distribution table is a way of doing this.

4.4.1 Frequency distribution table

The **frequency distribution table** is a common way of organising data so that it is very readable.

The first column of the table shows the variable being measured (in our case this is the number of children in the family). We often represent the variable as *x*. The second column is a tally column (if it is needed) and the third column is the **frequency** column. This column shows the number of times each score has been observed and is often represented by *f*. The table should always have a title. If we convert the table above into a frequency distribution table we get:

Frequency distribution table showing the number of children in each of 30 families surveyed

Number of children per family (x)	**Tally**	**Frequency (f)**
1		4
2		7
3		6
4		5
5		4
6		2
7		1
8		1
		$\Sigma f = 30$
This column shows all the possible values of the variable from smallest to largest	This column is the tally in groups of five. It is not really part of the formal table but is a way of keeping count.	This column is the frequency column and represents how often each value of the variable occurs.

Don't be put off by the use of Σf in the last column, it is just another way of saying the total or sum of the frequency column. It is the total frequency. Σ (sigma) is a Greek letter which means 'sum of'. It is always a good idea to find the sum of the frequency column as a check that you have not missed out any values when tallying. In our example the sum of the last column should be equal to 30 (because we started with 30 families).

It is much easier now to read off information from our survey. We can see that 7 families have 2 children but only 1 family had 8 children. We could also say that 8 of the 30 families have more than 4 children in their families (this means 5, 6, 7 or 8 children).

We could express some of this information as a percentage.

For example, what percentage of families have less than 4 children. Less than 4 children means 1, 2 or 3 children. From our table of values 17 families have less than 4 children. This represents $17/30 \times 100\% \approx 57\%$ of the families we have taken.

We will return to our survey later. For now try to construct some frequency distribution tables for the following data.

Activity 4.3

1. A survey of weights (to the nearest kilogram) of students in a maths tutorial produced the following data.

56	70	58	62	59	61	70	58
64	64	62	68	63	64	61	60
60	66	63	67	65	58	66	63
68	63	69	63	67	67	63	66

   (a) What was the lowest weight? What was the greatest weight?

   (b) Construct a frequency distribution table for the above data.

   (c) How many students weighed 70 kg?

   (d) How many students weighed less than 62 kg.

2. A golfer returned the following scores on 20 successive games on the same course.

85	81	81	83	84	78	87	79	82	86
83	80	82	81	79	84	85	81	79	80

   (a) Construct a frequency distribution table for the above data

   (b) On how many rounds did the golfer score 84?

   (c) On how many rounds did the golfer score less the 80?

3. When babies are born in hospital their blood group is determined as part of the routine testing performed on new-borns. Blood groups fall into one of four categories, A, B, AB or O.

   In a particular hospital the 24 babies born in one week had the following blood groups.

O	O	A	O	AB	B	O	A	B	O	O	A
A	O	B	O	A	A	O	O	A	O	A	A

   (a) Construct a frequency distribution table for the above data.

   (b) What was the most common blood group among these babies?

   (c) Did you know that approximately 49% of the Australian population has type O blood. What percentage of these babies had type O blood?

4. The following figures represent readings taken by a radar trap on a section of road with a speed limit of 80 km/h

81	80	70	76	90	82	73	78	81	80
81	83	73	75	78	75	79	76	84	102
93	81	100	85	73	81	76	77	76	73
74	83	71	71	85	96	86	79	81	83

   (a) Construct a frequency distribution table for the above data.

   (b) What number of the motorists were travelling above the speed limit?

5. A health clinic is having a free health check stand at the local shopping centre. The heights to the nearest centimetre of the first 50 clients are recorded below.

150	185	92	155	165	178	150	92	155	176
95	115	150	165	135	160	180	95	116	158
88	106	182	156	143	180	115	155	176	92
94	125	128	136	116	148	122	164	165	178
145	148	122	164	182	160	128	132	176	158

   (a) Construct a frequency distribution table for the above data.

   (b) Do you think that there would be a better way to present this data?

4.5 Analysing data

So far we have collected, organised and displayed data. However, this is often not enough. In many cases we need to abbreviate the data even further so that we have measures such as the 'average' or the most typical value. These types of measures are referring to centre of the data and as such are called measures of central tendency. It is also useful to obtain some measure of the spread in the data, that is whether they are grouped closely together in value or a long way apart. In the following sections we will look at 3 measures of the centre of a distribution of data and two measures of spread.

4.5.1 Where is the centre of these data?

The mean

The mean is the most commonly used measure of the centre of a group of data. You may have heard it referred to as the arithmetic average. In words we would say that the mean is equal to the sum of all the observations divided by the total number of observations.

$$\text{Mean} = \frac{\text{sum of all observations}}{\text{total number of all observations}}$$

This is sometimes abbreviated to the formula

$\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$ where n is the total sample size and $\sum_{i=1}^{n} x_i$ is the sum of all the observations.

Don't confuse mean with the English mean in the sense of 'nasty' although it might be considered mean to ask somebody all the things that mean can mean.

Example

If a nurse measured a patient's temperature (in °C) 4 times, what was the patient's mean temperature?

37.2 37.5 37.1 37.3

Add all the numbers given to get 149.1 then divide this total by 4 to get 37.275.

or

$$\begin{aligned}\text{Mean} &= \frac{\text{sum of all observations}}{\text{total number of observations}}\\ &= \frac{37.2 + 37.5 + 37.1 + 37.3}{4}\\ &= 37.275\\ &\approx 37.3 \text{ (rounded to 1 decimal place)}\end{aligned}$$

The mean temperature would be approximately 37.3 °C.

This method, however, becomes very tedious if we are working from a very large data set. To overcome this for large sets of data it is better to firstly arrange the data into a frequency distribution table and then calculate the mean from that table.

Recall this frequency distribution table from earlier. We can multiply the first two columns of this table together to produce a third column which we will call the *fx*.

Frequency distribution table showing the number of children in each of 30 families surveyed

Number of children per family (x)	**Frequency (f)**	$f \times x = fx$
1	4	$1 \times 4 = 4$
2	7	$2 \times 7 = 14$
3	6	$3 \times 6 = 18$
4	5	$4 \times 5 = 20$
5	4	$5 \times 4 = 20$
6	2	$6 \times 2 = 12$
7	1	$7 \times 1 = 7$
8	1	$8 \times 1 = 8$
	$\Sigma f = 30$	$\Sigma fx = 103$
	The total of this column is equal to the total number of observations	The total of this column is equivalent to the sum of all the observations.

This means that if we have a large number of observations that have been represented in a frequency distribution table then we can use this table to help us find the mean of the data.

$$\text{Mean} = \frac{\text{sum of all observations}}{\text{total number of all observations}}$$

This is sometimes abbreviated to the formula

$\bar{x} = \dfrac{\sum_{i=1}^{n} fx_i}{\sum_{i=1}^{n} f_i}$ where $\sum_{i=1}^{n} fx_i$ is the sum of all the observations and $\sum_{i=1}^{n} f_i$ is the total number of observations.

Example

Use the frequency distribution above to find the mean number of children per family.

$$\begin{aligned}\text{Mean} &= \frac{\text{sum of all observations}}{\text{total number of observations}}\\ &= \frac{\Sigma fx}{\Sigma f}\\ &= \frac{103}{30}\\ &\approx 3.433\end{aligned}$$

The mean number of children would be approximately 3.4.

Using the calculator to find the mean.

It is important that if you are going to use the calculator to find the mean that you put your calculator in the statistics mode. Check your manual to find out what this is for your calculator.

A small SD will appear on the display. Before you enter any data it is important to make sure that there is nothing already stored in the statistics memories of the calculators.

Data are then entered into the calculator usually by pressing the DATA key. If you input an incorrect value you can remove it from the data set by using DEL.

Always return your calculator to normal mode of operation after you have completed statistical calculations.

Example

If a nurse measured a patient's temperature (in °C) 4 times what was the patient's mean temperature?

37.2 37.5 37.1 37.3

The display should read 37.275.

Example

Use the frequency distribution table to find the mean number of children per family.

Frequency distribution table showing the number of children in each of 30 families surveyed

Number of children per family (x)	**Frequency (f)**	$f \times x = fx$
1	4	$1 \times 4 = 4$
2	7	$2 \times 7 = 14$
3	6	$3 \times 6 = 18$
4	5	$4 \times 5 = 20$
5	4	$5 \times 4 = 20$
6	2	$6 \times 2 = 12$
7	1	$7 \times 1 = 7$
8	1	$8 \times 1 = 8$
	$\Sigma f = 30$	$\Sigma fx = 103$

Note that when we enter these values into the calculator it is not necessary to enter each individually. The calculator is programmed so that it will allow you to enter multiples of the one value.

Activity 4.4

1. What is the mean rainfall for an 8 day period if the daily rainfall was 7.2, 13.5, 2.1, 25, 4.6, 0, 7.2,15.7 (rainfall is measured in mm).

2. Find the mean daily temperature if the temperature measured over a 21 day period are presented below.

Temperature in degrees Celsius	**Frequency of each temperature**
25	5
27	7
29	3
31	4
33	2

3. In a fishing competition there were 40 competitors. If the number of fish caught by each competitor was

8	7	7	4	7	3	7	7	6	7
3	2	7	9	7	3	8	2	6	7
5	10	8	7	8	9	7	7	8	10
7	6	5	7	3	7	7	5	4	10

What was the mean number of fish caught?

The mode

The mode is another measure of the centre of a set of data. It is the most common observation made in a set of observations derived from the French word for fashionable. For example if we have a set of examination scores 55 55 65 62 55 78 99 55, then 55 would be the mode of these scores as it is the most common score.

When sets of data are summarised into frequency distributions it is easy to read off the mode by looking for the observation with the highest frequency. Look again at the distribution of number of children per family. In that distribution the mode was 2. This means that 7 families had 2 children each and that this was the most common number of children to have. In this case there was only one mode but in other examples there may be more than one, or the mode may not exist as all scores had the same frequency.

Activity 4.5

Return to Activity 4.4 and calculate the mode for each of the distributions presented.

The median

The third measure of central tendency we shall discuss is the median. The median is the middle value in a set of observations, after they have been ranked in order (usually from smallest to largest). The median observation should therefore have the same number of observations on either side of it. If there are an odd number of observations the median is the middle observation but if there is an even number of observations the median will be the average of the two middle scores.

To find the median of a distribution.

- Arrange all observation in order of size, usually from smallest to largest.
- If the number of observations is odd, the median will be at the centre of the ordered list. You can find the middle score by adding one to the total number of observations and dividing by two. Then count up through the ordered set until you reach that observation.
- If the number of observations is even, the median will be midway between the two centre observations.

Example

The amount 5 people earned per hour is shown below.

\$12, \$18, \$14, \$11, \$15

Find the median amount earned.

11 12 14 15 18 Rearrange data into ascending order

As there are 5 observations the median position will be $\frac{5+1}{2} = 3$

If we count along the data set the 3rd value is \$14

The median is therefore \$14.

Example

Find the median of this maximum temperature data collected over 6 days during winter.

6.5 7.1 3.2 5.3 9.2 4.5 (all temperatures are in degrees Celsius)

3.2 4.5 5.3 6.5 7.1 9.2 Rearrange data into ascending order

As there are 6 observations the median position will be $\frac{6+1}{2} = 3.5$

This means that the median lies between the 3rd and 4th observation. To find the exact value of the median we take the midpoint between the 3rd and 4th value.

$$\frac{5.3 + 6.5}{2} = 5.9$$

The median is thus 5.9 degrees Celsius

Example

In a previous activity we looked at the distribution table which displayed daily temperature over a 21 day period. What would be the median of these data?

Temperature in degrees Celsius	Frequency of each temperature
25	5
27	7
29	3
31	4
33	2

The data are already arranged in descending order, so the first step is to find the position of the median. The median will be the $\frac{21+1}{2}$ term i.e. the 11 term.

To find the 11th term count down the 11 terms in the frequency distribution table. For example the first 5 terms are all 25, the next 7 are all 27 so that means that the 11th term is 27.

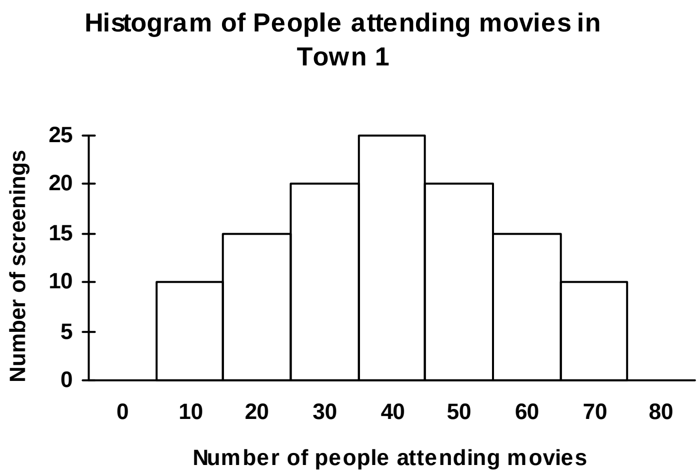
The median is 27 degrees Celsius.

Activity 4.6

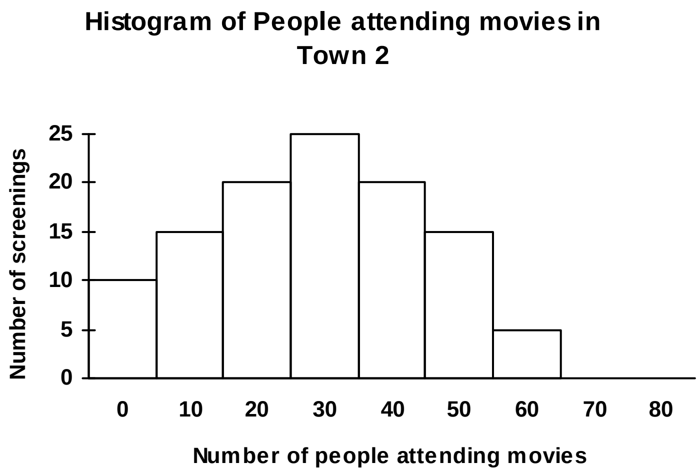
Find the median observation in the distributions detailed in Activity 4.4.

A comparison of mean, median and mode

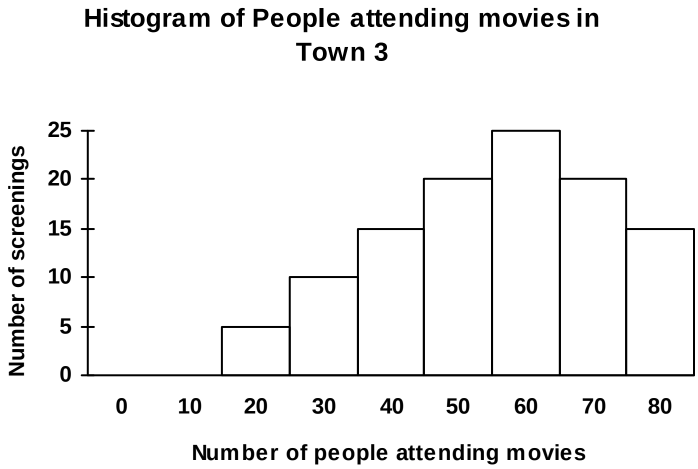
Now that we know how to calculate the mean, median and mode it is important to see when each of these measures is most useful. Let's look at the following three histograms of frequency distributions. The distributions represent the number of people attending three small cinemas around country Queensland.



Mean = 40
Median = 40
Mode = 40



Mean = 33
Median = 30
Mode = 30



Mean = 55.5
Median = 60
Mode = 60

Notice that although the histograms for these three distribution are about the same thing they have some distinct differences. Their general shapes are different. The first histogram is symmetrical (when it is cut in half both sides are the same). In the second histogram the rectangles are higher on the left hand side while on the third histogram the rectangles are higher on the right hand side. Also compare the values of the mean, median and mode for the three distributions. In the symmetrical histogram the mean, median and mode are the same, while in the unbalanced (or skewed) histograms the mean is different from the median and mode. This type of result will occur in many other distributions that are skewed to one side. In some cases the median and mode will also be different. It is often said that the mean is more liable to be influenced by extreme values in a distribution. However, it should not be concluded that because of this the mean is not useful. All three measures of central tendency measure slightly different aspects of a distribution and as such all are useful. In particular,

The mode is most useful when

- categorical variables are being considered
- qualities like sizes of products are being considered e.g. a manager of a supermarket will always be interested in the most frequently bought size rather than the mean size

The median is most useful when

- the distribution has values which are either very small or very large (outliers) e.g. reports about incomes, house prices or other very skewed distributions usually use the median rather than the mean

The mean is most useful because

- it uses all the numbers in the calculation and is sensitive to small changes
- many people intuitively understand the meaning of this measure
- many rigorous statistical theories have been developed around this measure

In conclusion all three measures of central tendency have their advantages and disadvantages. When selecting the measure to summarise data it is always necessary to first consider what the statistics will be used for.

Activity 4.7

1. Below is the number of people in the same family undergoing counselling after a bus crash.

   4 2 2 4 6 3 2 3

   As the psychologist in charge you have to present a report on this event. Calculate the mean, median and mode for these data and comment on the most appropriate measure to use.

2. Levels of a drug in the blood of ten patients admitted to the outpatients clinic of a major hospital were

   75 103 66 61 91 61 74 103 99 93 (micro units per mL).

   When reporting back to your superior about the group of admissions you would use either the mean, mean or mode to summarise these data. Calculate all three measures and give reasons for your decision to use the measure of your choice.

3. Eight houses all in the same street of a Toowoomba suburb had the following values

   $85 000 $115 000 $66 000 $77 000 $82 000 $89 000 $79 000 $91 000

   Which measure of central tendency would best summarise these data. Explain your answer showing calculations for mean, median and mode.

4.5.2 How much spread is in the data?

It is often the case that measures of centre alone do not give a good 'picture' of the data. Consider the following case where the yearly incomes from two different groups of people were recorded.

Group A: $35 500, $37 500, $44 000 and $45 000

Group B: $22 500, $40 000, $41 500 and $58 000.

The mean income for both groups is $40 500 and the median income for both groups is $40 750. These measures of centre are very similar, yet the groups are quite different. For this reason we need some measure of how the incomes are spread in each group.

The range

The range is a simple measure of the spread in data. It is merely the highest value in the data minus the lowest value. In the previous example, the range for Group A is $45 000 – $35 500 = $9 500 while the range for Group B is $58 000 – $22 500 = $35 500.

This tells us that the incomes are much more spread out in Group B.

The standard deviation

The standard deviation is a more common measure of spread, although a bit more difficult to calculate. Before we look at the standard deviation, it is useful to discuss the concept of deviation.

The deviation of a number in a sample is a measure of its distance from the mean of the sample. More specifically, the deviation for any number is the value of that number minus the mean. Mathematically this can be written: $x_i - \bar{x}$ where x_i is the number in question and $\bar{x}$ the mean of the sample.

Returning to our example above; the mean of the four numbers in Group A is. \$40 500. The deviation for the highest number is therefore \$45 000 – \$40 500 = \$4 500, while the deviation for the lowest number is \$35 500 – \$40 500 = –\$5 000. A negative deviation tells us that the number is below the mean.

The standard deviation is essentially the average of all deviations of numbers in a sample. It can be thought of as being the average ‘distance’ of each number in the sample from the mean. Mathematically the standard deviation or a sample can be expressed using the following formula:

$s = \sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{n-1}}$, where n is the number of numbers in the sample.

This formula tells us that in order to calculate the standard deviation we need to:

1. calculate the deviation $x_i - \bar{x}$ for each number;
2. square this deviation $(x_i - \bar{x})^2$ (this removes any of the negative numbers),
3. take the average of these square deviations $\dfrac{\Sigma(x_i - \bar{x})^2}{n-1}$; and,
4. finally take the square root of this number to undo the effect of squaring each deviation.

Note that when we took the average we divided by one less than the number of observations. This is done in statistics so that our sample standard deviation is a better estimate of the population standard deviation.

This standard calculation is best undertaken using a table. Let’s consider the numbers in Group A of our original example.

Number x_i	**Deviation** $x_i - \bar{x}$	**Square deviation** $(x_i - \bar{x})^2$
35 500	$35500 - 40500 = -5000$	$(-5000)^2 = 25000000$
37 500	$37500 - 40500 = -3000$	$(-3000)^2 = 9000000$
44 000	$44000 - 40500 = 3500$	$(3500)^2 = 12250000$
45 000	$45000 - 40500 = 4500$	$(4500)^2 = 20250000$
		$\sum(x_i - \bar{x})^2 = 66\,500\,000$

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{66\,500\,000}{4-1}} = \$4\,708.15$$

The standard deviation is not easy to compute and fortunately most scientific calculators have built in standard deviation functions. It is important that you have an understanding of the standard deviation and that you can calculate it on your calculator.

Activity 4.8

1. Use the same method as above to calculate the standard deviation for Group B in our original example. Then use your calculator to find the standard deviation for both groups.
2. The results in a test for 10 students are shown below. Calculate the mean and standard deviation for this test

16	20	17	26	17	21	20	22	18	23

4.6 Data with two variables

So far we have looked at data sets where only one variable has been measured, for example height of men or temperature of water. But what about the situation that often occurs where we measure two variables and want to know something about the relationship that occurs between them.

Consider the situation of four men:

Adam is tall and light. He is 180 cm tall and weighs 60 kg.
Bill is the same height as Adam (180 cm) but weighs 110 kg. He is the heaviest of the four men.
Charles is short and heavy. He is 140 cm tall and weighs 80 kg.
Don is the shortest and the lightest of the four men, he is only 120 cm tall and weighs 50 kg.

Two variables are presented in these data, height and weight. We could summarise this type of data by representing the two variables together on a graph. It may well be that there is some relationship between the height and weight of a man.

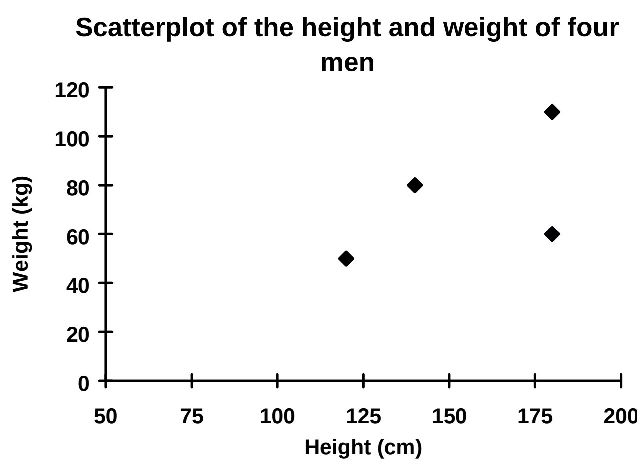
The best way to begin the investigation involving the relationship between these two variables is to draw a **scatterplot**. A scatterplot is a graph in which the values of one variable appear on the horizontal axis while those of the other variable appear on the vertical axis. The points drawn on the graph represent the values of each variable for that individual. From the data above we could construct a table that would help us plot the points on our scatterplot.

Name	Height (cm)	Weight (kg)
Adam	180	60
Bill	180	110
Charles	140	80
Don	120	50

To construct the scatterplot for these data you should:

- draw a Cartesian plane, using only the necessary quadrant (best to use graph paper).
- label the horizontal axis as height (cm) and the vertical axis as weight (kg)
- mark a scale on each axis that is appropriate for the range of each variable in this case height could go from 50 to 200 and weight from 0 to 120.
- plot a point on the graph for each pair of observations
- give the graph a title

The scatterplot from the above data could look like this:

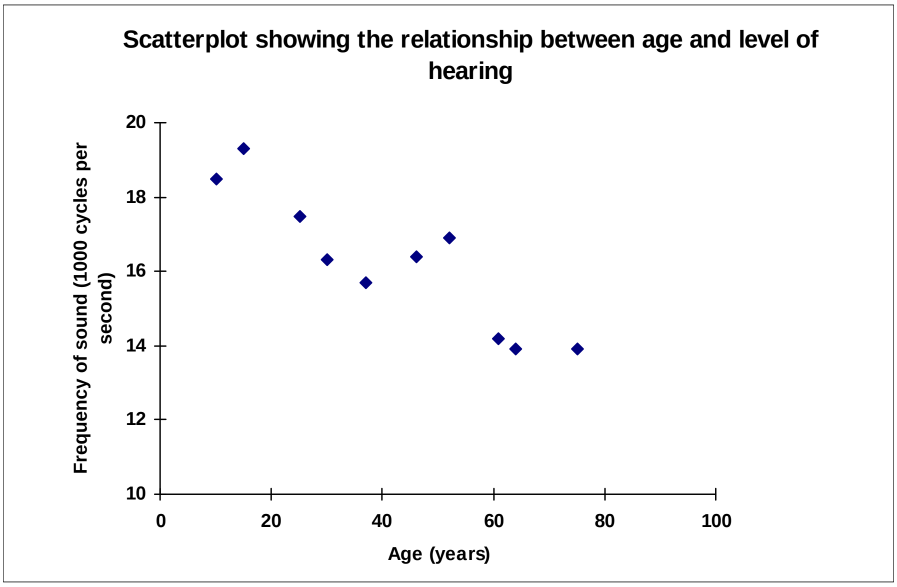


Let's consider another example.

Example

Ten people of different ages had their hearing checked to determine the relationship between age and hearing loss. Hearing was measured as the maximum frequency of the sound they could hear in 1000 cycles per second.

Person	Age (years)	Frequency (1000 cycles per second)
1	10	18.5
2	15	19.3
3	25	17.5
4	30	16.3
5	37	15.7
6	46	16.4
7	52	16.9
8	61	14.2
9	64	13.9
10	75	13.9



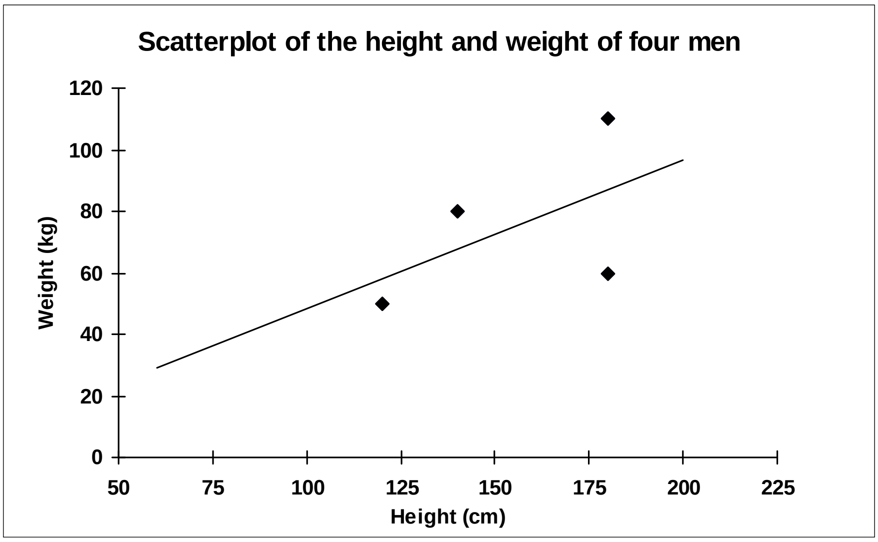
Important note: only the data for age and frequency are included in the plot. Do not include the person number. This is only included as a label for each observation and should not be a part of the scatterplot.

Before we go any further and draw some scatterplots, let's have a look at what these graphs tell us about the relationship between the two variables graphed.

In the first graph it looks like there is a relationship between the height and weight of a man. In particular as the height increases so does the weight. We can also see a relationship in the second graph. In this case as the age of the person increase their ability to hear decreases. Is it possible to make predictions about weight or hearing from these graphs?

One thing that will help us understand what is happening in the graph is to draw a line through the points. This line is called the **line of best fit** and will give us a good idea of the **trend** the data follows. Ideally the line should be drawn through the middle of the points, dividing them into two balanced groups (one group each side of the line). This should be done by 'sighting' a line - taking your ruler and drawing a line that appears balanced. This method is only an estimation of where the line could be and different people will produced slightly different lines. In Data Analysis the guess work is taken out of this method by a more mathematical approach to fitting a line called 'Regression Analysis'. However, for our purposes a line of best fit, fitted by eye will give us a good idea of the trend in the data.

Let's fit some lines to the two examples above.



When we examine the trend in this scatterplot it appears that there is a relationship between height and weight of men, that is, the taller a man is then the heavier he might be predicted to be. We say that this is a positive relationship between these two variables.

We could now use the line of best fit to make some predictions about the weight of men of heights that we did not measure. For example reading from the line of best fit when a man is 160 cm tall he might weigh about 75 kg. We could also predict that if a man was 200 cm tall (a basket ball player?) then from the line of best fit we would predict that he would weight 90 kg.

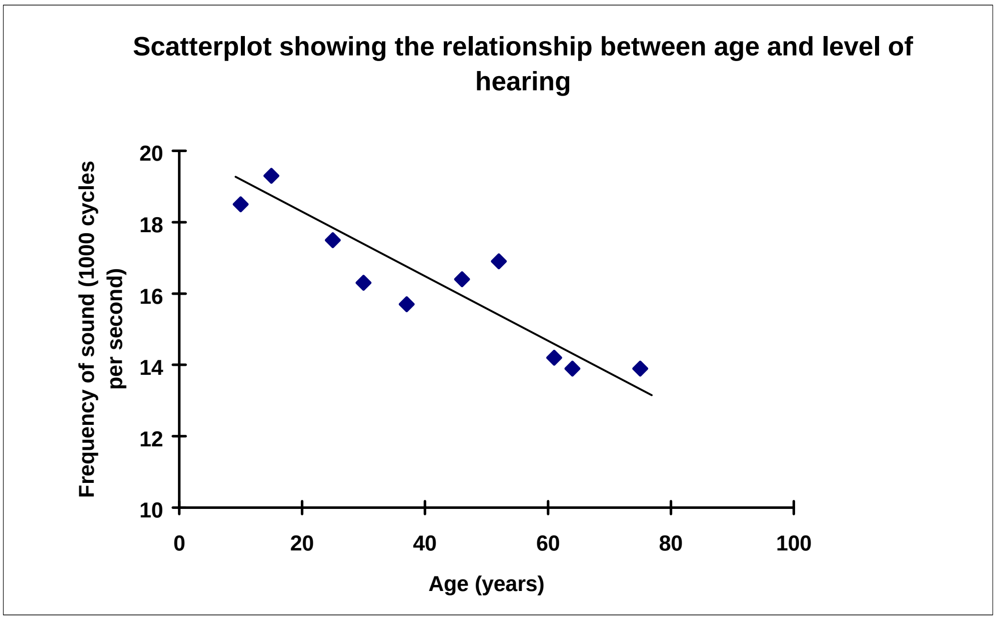
Similarly for the scatterplot of age versus level of hearing (on next page) we could draw a line of best fit. Notice that this line is sloping downwards indicating that as people age the frequency of sound that they can hear is reduced (the older you are the worse your hearing is). As we did before we could use this graph to predict some values of level of hearing for people we did not measure. For example using the line of best fit we could say that if you were 42 old you would be able to hear 17 000 cycles per second while if you were 90 years old you would be able to hear only 12 000 cycles per second.

The process of guessing values based on a known trend in some data is called:

interpolation if it is within the known range of values, and

extrapolation if it is outside the known range of values.

Watch out, however, for unrealistic predictions. For example could we predict the weight of a man who is 300 cm tall or the level of hearing of somebody 150 years old?



Activity 4.9

Draw scatterplots for the following data. Draw a line through the data points to illustrate the trend and then answer the questions.

1. The temperatures in a dam were measured at different depths producing the following data.

Depth (m)	Temperature (degrees)
0	24
1	20
2	18
3	15
4	10
5	9

(a) Is there a trend? If so describe it.

(b) What do you predict that the temperature would be at:

(i) 2.5 m

(ii) 6.0 m

2. Two students A and B sat for 6 tests in different subject areas. The teacher suspected these students of cheating so wanted to see if there was a relationship between the two results. The teacher examined the following data.

Subject	Person A's results (%)	Person B's results (%)
English	85	85
Mathematics	90	45
Art	40	95
History	55	80
Chemistry	80	50

(a) Is there a trend? If so describe it.

(b) Would this confirm the teacher's suspicions?

3. The percentage of the day that was sunshine was measured in winter and summer in a number of different towns to see if there was a relationship between percentage of sunshine in summer and winter. The following data resulted.

Town	Percentage sunshine	
	Summer (%)	Winter (%)
A	56	62
B	61	74
C	59	68
D	58	67
E	73	79

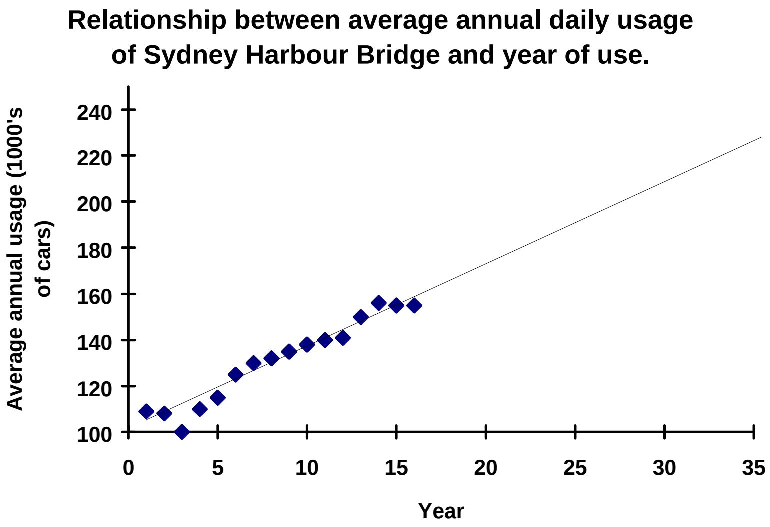
(a) Is there a trend? If so describe it.

(b) Predict the percentage summer sunshine in a town, if the percentage winter sunshine is

(i) 70

(ii) 78

4. In 1965 the NSW State Government began monitoring the annual average daily usage of the Sydney Harbour Bridge. The results of the survey are presented in the scatterplot below. The first year of the survey has been called Year 1 of the survey with the last year of the survey called Year 16.



(a) Describe in your own words the relationship between average annual daily usage and years.

(b) What year would be 35 years from the beginning of the survey?

(c) What would the average annual number of cars be predicted to be in this year?

(d) Will the prediction in part (c) be realistic? Explain your answer.

Solutions

Activity 4.1

1.
   (a) Gender is categorical
   (b) Age (years). If we consider whole years i.e. 1, 2 or 3 years then the variable could be discrete. If on the other hands we consider years as being able to take all parts of a year i.e. 1.27 years or 102.638 years then it will be continuous. In most instances age is considered to be a continuous variable.
   (c) Income is continuous variable
   (d) Number of siblings for each pupil is a discrete variable
   (e) Temperature is a continuous variable
   (f) Smoker (yes or no) is a categorical variable
   (g) Class size is discrete because it is considering numbers of students.

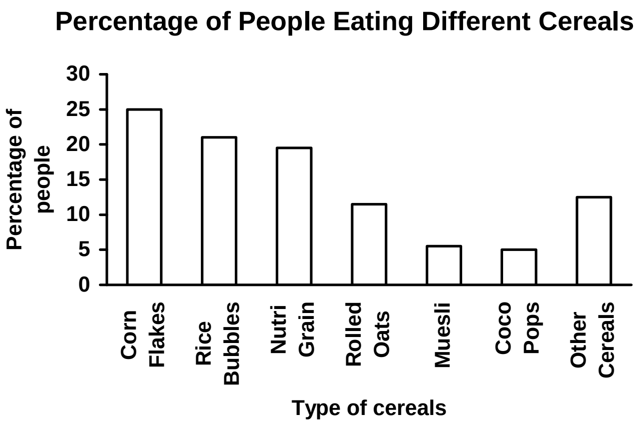
2.
   (a) Population could be all second language students
   (b) Sample is all 1st year university second language students
   (c) Variables of interest could be
   - number of words in vocabulary
   - number of complex sentences in written expression
   - number of questions correct in a test

   There are numerous other variables that could be considered. These will depend on the interests of the researcher.

Activity 4.2

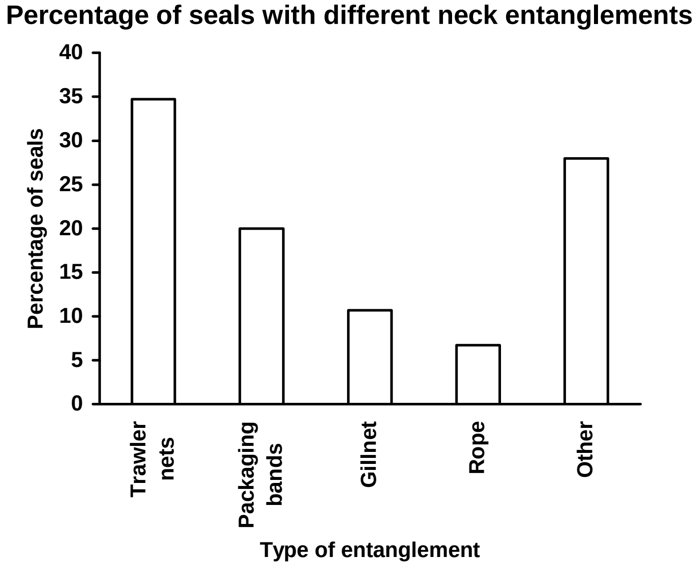
1.

Cereal	Number of people	Percentage of people
Corn Flakes	50	25
Rice Bubbles	42	21
Nutri Grain	39	19.5
Rolled Oats	23	11.5
Muesli	11	5.5
Coco Pops	10	5
Other Cereals	25	12.5



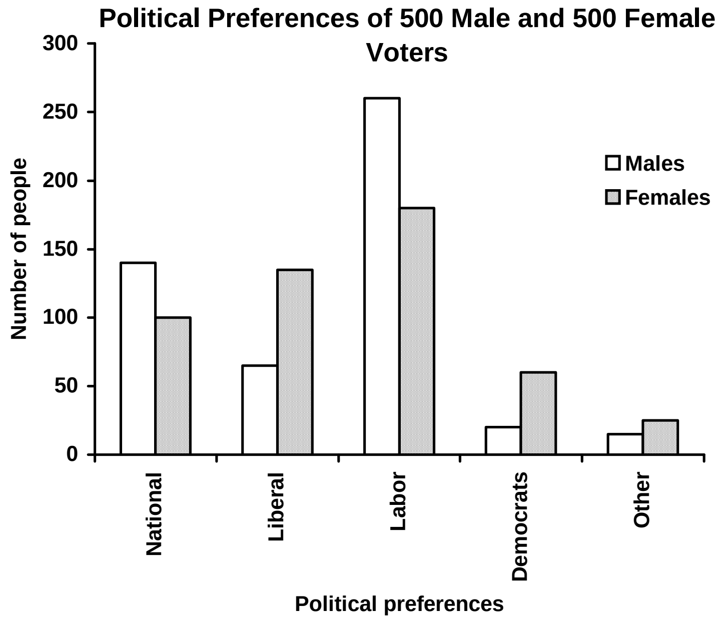
2.

Item	Number	% of total
Trawler nets	26	34.7
Packaging bands	15	20.0
Gillnet	8	10.7
Rope	5	6.7
Other	21	28.0
Total	75	100.0



3.

(a)



(b) The most popular party in this city was the Labor party. More males than females voted for this group. The National and the Liberal parties were the next most popular. The National party was the most popular with men while the Liberal party was the most popular with women. The remaining parties (Democrat and others) were the least popular, with more women voting for these in both cases.

4.

(a) Both graphs present the same data but in different ways. The bar chart presents the raw data. Many find these type of figures hard to compare but the bar chart which has rectangles next to each other makes this type of comparison easier. The pie chart does not present the raw data it uses percentages. The problem with this type of graph is that we do not know how many workers the graph is based on. For example if the chart was only based on 6 workers then it is not based on a very good sample. We cannot tell this from the graph as it is presented here. Note that some readers will prefer different types of graphs depending on their past experiences e.g. it could depend on whether they like to read from rectangles or circles?

(b) There are a range of answers you could provide to this question. To be acceptable your answer should cover the following points:
- educational backgrounds divided into 6 categories
- most workers had either year 12 or a university degree (385 out of 550) with year 12 being the most common.
- the least number of workers had year 10 (15 out of 550)
- postgraduate qualifications achieved by 125 out of 550 workers with doctorate being more common than master's degree
- the educational backgrounds of one group of workers could not be classified (25 out of 550)

Activity 4.3

1.

(a) Lowest weight 56 kg, greatest weight 70 kg.

(b)

Frequency distribution table of weights (kg) of students in a mathematics tutorial

Weight (kg)	**Frequency**
56	1
57	0
58	3
59	1
60	2
61	2
62	2
63	6
64	3
65	1
66	3
67	3
68	2
69	1
70	2
	$\Sigma f = 32$

(c) 2 students weighed 70 g.

(d) Totalling the frequencies of categories less than 62 kg we get, 1+0+3+1+2+2=9, 9 students weighed less than 62 kg.

2.

(a)

Frequency distribution table golf scores in 20 games	
Golf score	**Frequency**
78	1
79	3
80	2
81	4
82	2
83	2
84	2
85	2
86	1
87	1
	$\sum f = 20$

(b) 2 rounds

(c) Totalling the frequencies in groups less than 80 we get 1 + 3 = 4.
Golfer scored less than 80 on 4 rounds.

3.

(a)

Frequency distribution table of blood groups of 24 babies	
Blood groups	**Frequency**
A	9
B	3
AB	1
O	11
	$\sum f = 24$

(b) Most common blood group was O, with 11 people.

(c) Percentage of O group in this sample was $\frac{11}{24} \times 100\% = 45.8\%$

4.

(a)

Frequency distribution table of speed of cars caught by radar trap					
Speed (km/h)	**Frequency**	**Speed (km/h)**	**Frequency**	**Speed (km/h)**	**Frequency**
70	1	81	6	92	
71	2	82	1	93	1
72		83	3	94	
73	4	84	1	95	
74	1	85	2	96	1
75	2	86	1	97	
76	4	87		98	
77	1	88		99	
78	2	89		100	1
79	2	90	1	101	
80	2	91		102	1
					$\sum f = 40$

Note to save space this frequency distribution table has columns placed side by side.

(b) 19 motorists travelled above the speed limit of 80 km/h.

5.

(a)

Frequency distribution table of heights of 50 clients at a shopping centre							
Height (cm)	**f**	**Height (cm)**	**f**	**Height (cm)**	**f**	**Height (cm)**	**f**
88	1	111		135	1	159	
89		112		136	1	160	2
90		113		137		161	
91		114		138		162	
92	3	115	2	139		163	
93		116	2	140		164	2
94	1	117		141		165	3
95	2	118		142		166	
96		119		143	1	167	
97		120		144		168	
98		121		145	1	169	
99		122	2	146		170	
100		123		147		171	
101		124		148	2	172	
102		125	1	149		173	
103		126		150	3	174	
104		127		151		175	
105		128	2	152		176	3
106	1	129		153		177	
107		130		154		178	1
108		131		155	3	179	
109		132	1	156	1	180	2
110		133		157		181	
				158	2	182	2
						183	
						184	
						185	1
							$\sum f = 50$

Note to save space this frequency distribution table has columns placed side by side.

(b) Definitely, we could either use a computer to do this time consuming activity or present the data in a grouped distribution. See the next section of work.

Activity 4.4

1. Mean $= \dfrac{7.2 + 13.5 + 2.1 + 25 + 4.6 + 0 + 7.2 + 15.7}{8} = 9.4$ mm

2. To calculate the mean use this frequency distribution table and construct a further column for *fx*. Note you could also use the calculator to do all of this if you desired.

Temperature in degrees Celsius	**Frequency of each temperature**	***fx***
25	5	125
27	7	189
29	3	87
31	4	124
33	2	66
	$\sum f = 21$	$\sum fx = 591$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{591}{21} = 28.14$$

Mean temperature is 28.14 degrees Celsius.

3. Mean is the sum of all the observations divided by the total number of observations. In this case it is 257 divided by 40. The mean is thus approximately 6.425 fish.

Activity 4.5

1. Mode is 7.2 mm

2. Mode is 27 degrees Celsius

3. First it is best to construct a frequency distribution table.

Frequency distribution table of number of fish caught at a fishing competition	
1	0
2	2
3	4
4	2
5	3
6	3
7	16
8	5
9	2
10	3
Total	40

Mode is 7 fish.

Activity 4.6

1. Place the measurements in order 0 2.1 4.6 7.2 7.2 13.5 15.7, 25
   Median position will be (8+1)/2 = 4.5. The 5th measurement is 7.2.
   The median is the 7.2 mm.

2. These reading are already in order, so we can read straight from the frequency distribution table.
   Median position will be (21+1)/2 = 11. The 11th reading is 27.
   Median is 27 degrees Celsius.

3. Using the frequency distribution we constructed for the previous question we can again read straight from the frequency distribution table. Median position will be (40+1)/2 = 20.5. Median lies between the 20th and 21st number. The 20th number is 7 and the 21st number is also 7. Median is thus 7 fish.

Activity 4.7

1. First arrange these data in order. 2 2 2 3 3 4 4 6

   $$\text{Mean} = \frac{2+2+2+3+3+4+4+6}{8} = 3.25 \text{ mm}$$

   Mode is 2 people
   Median: median position will be (8+1)/2 = 4.5, the median will lie between the 4th and 5th terms. By counting along the row of numbers we can see that the median is 3.
   Median is 3 people.
   Mean, median and mode are all different. In this instance it may be that the median is the best measure of central tendency because of the effect of the 6.

2. First arrange these data in order. 61 61 66 74 75 91 93 99 103 103

   Mean will be 82.6 microunits per mL.
   Mode is 61 and 103 microunits per mL (note that in this instance there are two modes, it is bimodal)
   Median lies between 5th and 6th terms i.e. between 75 and 91. We can find the midway point between these two numbers by adding them and then dividing by 2, (75+91)/2 = 83.
   The median is 83 microunits per mL.

   In this instance either the mean or median would have been useful. Because the distribution has two modes at either end of the distribution the mode is not useful as a measure of central tendency.

3. First put these data into order

   66 000 77 000 79 000 82 000 85 000 89 000 91 000 115 000

   Mean is \$85 500.
   Mode: there is no distinct mode as there are no repeated values.
   Median: the position of the median will be the (8+1)/2 =4.5 term. That is, the median will be between the 4th and 5th terms. The values of these terms are 82 000 and 85 000.
   Midway between these terms is (82 000+85 000)/2 = 83 500.
   The median is \$83 500.

   In this instance because of the extreme value of \$115 000 the median would be the best measure of central tendency. Note that this is often the case with house and property prices.

Activity 4.8

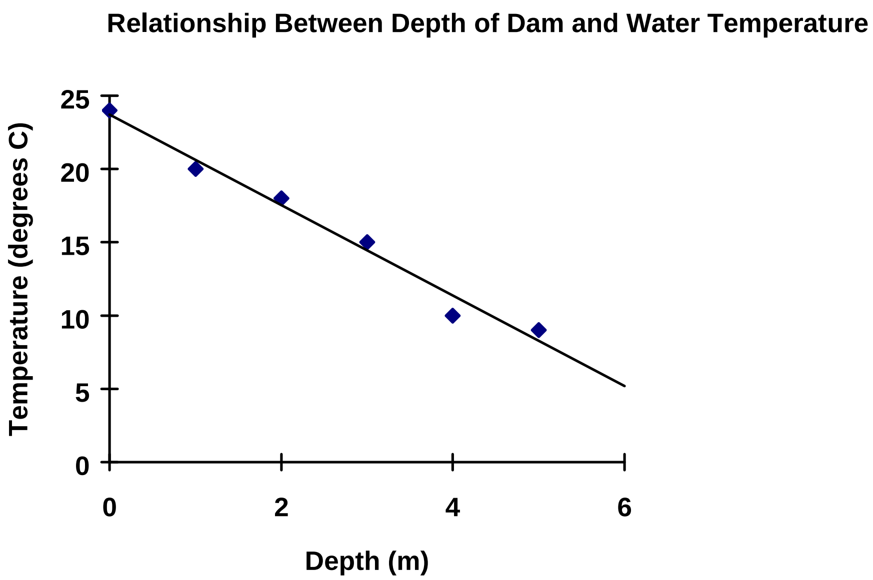
1. This calculation is best undertaken using a table as in our example.

Number x_i	**Deviation** $x_i - \overline{x}$	**Square deviation** $(x_i - \overline{x})^2$
22 500	$22500 - 40500 = -18000$	$(-18000)^2 = 324000000$
40 000	$40000 - 40500 = -500$	$(-500)^2 = 250000$
41 500	$41500 - 40500 = 1000$	$(1000)^2 = 1000000$
58 000	$58000 - 40500 = 17500$	$(17500)^2 = 306250000$
		$\sum(x_i - \overline{x})^2 = 631\,500\,000$

$$s = \sqrt{\frac{\sum(x_i - \overline{x})^2}{n-1}} = \sqrt{\frac{631\,500\,000}{4-1}} = \$14\,508.62$$

2. The mean score is 20 and the standard deviation 3.13.

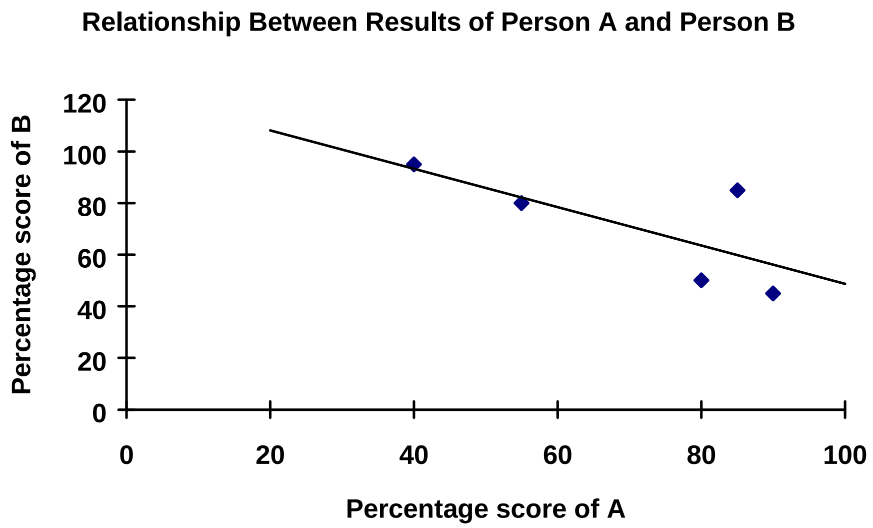
1. **Activity 4.9**



(a) Yes there is a trend. From the trend line we can see that as the depth of water in the dam increased the water temperature decreased.

(b) When depth is 2.5 m, temperature would be approximately 15 degrees C.
When depth is 6.0 m, temperature would be approximately 6 degrees C.

2.

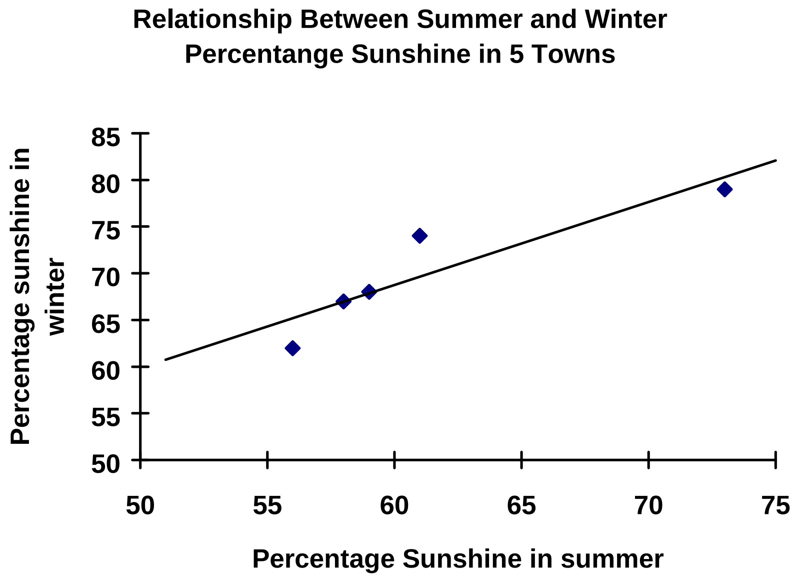


(a) Yes there is a trend. As the marks for person A decrease the marks for person B increase.

(b) There is little justification for the teacher's concern but they only have 5 instances on which to base the decision. They may like to consider some variables other than cheating.

3.

(a)



Yes there is a trend. Percentage sunshine in summer and winter are related. As the percentage sunshine in summer increases then the percentage sunshine in winter will also increase.

(b)

(i) If winter percentage is 70 then the summer percentage should be approximately 60.

(ii) If the winter percentage is 78 then the summer percentage should be approximately 70.

4.

(a) A number of answers would be possible, here is an example.

The relationship between years of survey and average annual usage indicates that as years increased the number of cars surveyed increased steadily. When the survey commenced cars totalled 110 000 while 16 years later they totalled 150 000.

(b) 2 000

(c) Using the trend line, when year 2 000 the number of cars would be 222 000 (approximately).

(d) One should be cautious when making predictions well into the future away from that last available data point. The early data suggests that the relationship increases steadily however, with a gap of 17 years between the last measurement and the prediction one doesn’t know what other variable could have now come into play. For example an unexpected surge in population could cause car number to rise greatly, or changes in public transport of availability of parking could cause car numbers to stagnate or drop.