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# Inequalities

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## Inequalities

Inequalities are central to the definition of all limiting processes and are essential to many professions. When faced with a real world problem to translate to algebraic language, not all will result in simple expressions or equations.

Consider the following.

*The total cost of landscaping a garden includes the cost of plants (call it $P$) and the cost of the soil and mulch (call it $S$). The owner declares that the total cost of the job should not be greater than $\$2\,500$. Express this relationship in algebraic form.*

There is no way that this would be an equation. But rather this would be an *inequation* or an **inequality**. We would write it as $P + S \le 2\,500$, and say:

*The sum of the cost of plants and the cost of soil and mulch is less than or equal to two thousand and five hundred dollars.*

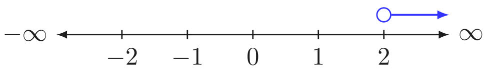


Recall from your previous experiences the meaning of the following signs:

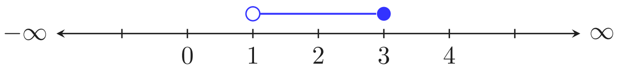
| | |
|---|---|
| $<$ | Less than |
| $\le$ | Less than or equal to |
| $>$ | Greater than |
| $\ge$ | Greater than or equal to |

## Graphing inequalities

- We can graph inequations to help visualise their properties and solution.
- The inequality $x > 2$ is read as '$x$ is greater than two'. We can represent this inequation on a real number line




- Another example is: $1 < x \leq 3$, is read as '$x$ is greater than 1 but less than or equal to 3'.






## Rearranging inequalities

- Simple versions of inequations can be solved or rearranged in ways similar to those used to solve equations.
- There are some differences, however,
  - When you switch sides in an inequality you must reverse the sign. For example, $2 < 3$ must become $3 > 2$, otherwise it is not true.
  - When you divide or multiply by a negative number you must reverse the inequality sign. For example, $2 < 3$, but when multiplied on both sides by $-1$ it must become $-2 > -3$, otherwise it is not true.



## Example

Find the value of $x$ when $2x - 1 < 4$.

$$\begin{aligned}
2x - 1 &< 4\,, \\
2x - 1 + 1 &< 4 + 1\,, && \text{To remove } -1 \text{ from the LHS add 1 to both sides.} \\
2x &< 5\,, \\
\frac{2x}{2} &< \frac{5}{2}\,, && \text{To isolate the } x \text{ divide both sides by 2.} \\
x &< \frac{5}{2}\,.
\end{aligned}$$

Note that our solution is not a single number as we would obtain from solving an equation. Rather, it is a range of numbers.



## Checking the Solution

To check our solution we could try numbers either side of $\frac{5}{2}$ in the original inequality.

Try a number less than $\frac{5}{2}$, say $x = 2$.

Substitute this into the original inequality:

$$\text{LHS} = 2 \times 2 - 1 = 3\,,$$

which indeed is less than 4 and the inequality is true.

Next try a number greater than $\frac{5}{2}$, say $x = 3$.

Substitute this into the original inequality:

$$\text{LHS} = 2 \times 3 - 1 = 5\,,$$

which is greater than 4. The inequality is not true for this value of $x$, which is what our solution predicted.

This procedure does not ensure that we are correct, but it does give an opportunity to pick up some errors.

## Another example

For example, solve the following inequation for $x$.

$$1-2x < x+2\,,$$

$$1-2x-x < x+2-x\,,$$

To group the $x$'s on the LHS subtract $x$ from both sides.

$$1-3x < 2\,,$$

$$1-3x-1 < 2-1\,,$$

To group the constants on the RHS subtract $1$ from both sides.

$$-3x < 1\,,$$

$$\frac{-3x}{-3} > \frac{1}{-3}\,,$$

To isolate $x$ on the LHS divide both sides by $-3$.

$$x > -\frac{1}{3}\,,$$

When you divide an inequation by a negative number you must reverse the inequality sign.



## Exercises

Solve the following inequations for $x$.

1. $$\frac{x}{3}+2<5$$
2. $$22 \le 5x-3 \le 32$$
3. $$\frac{3x}{5}-\frac{2x}{3} > -7$$
4. $$3x-2(2x-7) \le 2(3+x)-4$$
5. $$-3 \le \frac{2x-1}{3} < 3$$



## Answers Q1

$$\begin{aligned}
\frac{x}{3}+2 &< 5\,,\\
\frac{x}{3}+2-2 &< 5-2\,,\\
\frac{x}{3} &< 3\,,\\
\frac{x}{3}\times 3 &< 3\times 3\,,\\
x &< 9\,.
\end{aligned}$$



## Answers Q2

$$\begin{array}{ccccc}
22 & \le & 5x-3 & \le & 32\,,\\
22+3 & \le & 5x-3+3 & \le & 32+3\,,\\
25 & \le & 5x & \le & 35\,,\\
\frac{25}{5} & \le & \frac{5x}{5} & \le & \frac{35}{5}\,,\\
5 & \le & x & \le & 7\,.
\end{array}$$

## Answers Q3

$$\begin{aligned}
\frac{3x}{5} - \frac{2x}{3} &> -7\,,\\
15\times\frac{3x}{5} - 15\times\frac{2x}{3} &> 15\times -7\,,\\
3\times 3x - 5\times 2x &> -105\,,\\
9x - 10x &> -105\,,\\
-x &> -105\,,\\
-1\times -x &< -1\times -105\,,\\
x &< 105\,.
\end{aligned}$$



## Answers Q4

$$\begin{aligned}
3x - 2(2x-7) &\le 2(3+x) - 4\,,\\
3x - 4x + 14 &\le 6 + 2x - 4\,,\\
-x + 14 &\le 2 + 2x\,,\\
-x + x + 14 &\le 2 + 2x + x\,,\\
14 &\le 2 + 3x\,,\\
14 - 2 &\le 2 - 2 + 3x\,,\\
12 &\le 3x\,,\\
\frac{12}{3} &\le \frac{3x}{3}\,,\\
4 &\le x\,,\\
x &\ge 4\,.
\end{aligned}$$



## Answers Q5

$$\begin{array}{rcccl}
-3 & \le & \dfrac{2x-1}{3} & < & 3\,,\\
-3\times 3 & \le & \dfrac{2x-1}{3}\times 3 & < & 3\times 3\,,\\
-9 & \le & 2x-1 & < & 9\,,\\
-9+1 & \le & 2x-1+1 & < & 9+1\,,\\
-8 & \le & 2x & < & 10\,,\\
\dfrac{-8}{2} & \le & \dfrac{2x}{2} & < & \dfrac{10}{2}\,,\\
-4 & \le & x & < & 5\,.
\end{array}$$



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