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Substitution & Rearranging Equations

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Substituting into an equation

- Use substitution to replace the variable with a numeric value and then calculate the answer.
- For example, substituting $x = 2.5$ into $4(x+3)$ gives

$$\begin{aligned} & 4(x+3) \\ = \; & 4(2.5+3) \\ = \; & 4 \times 5.5 \\ = \; & 22\,. \end{aligned}$$

Substituting into formulas: Exercise

Observation towers are often built in forests to allow bushfires to be spotted quickly. The distance (D) in kilometres that can be seen from a tower of height h metres is given by:

$$D = 8\sqrt{\frac{h}{5}}\,.$$

Find the distance that can be seen from towers with the following heights. Round your answers to the nearest kilometre.

1. 10 metres
2. 25 metres
3. 17.5 metres

Substituting into formulas: Answers

1.

$$\begin{aligned} D &= 8\sqrt{\frac{h}{5}} \\ &= 8\sqrt{\frac{10}{5}} \\ &= 8 \times \sqrt{2} \\ &\approx 11\,. \end{aligned}$$

From this tower you could see about 11 kilometres.

Substituting into formulas: Answers

2.

$$\begin{aligned} D &= 8\sqrt{\frac{h}{5}} \\ &= 8\sqrt{\frac{25}{5}} \\ &= 8 \times \sqrt{5} \\ &\approx 18\,. \end{aligned}$$

From this tower you could see about 18 kilometres.

Substituting into formulas: Answers

3.

$$\begin{aligned} D &= 8\sqrt{\frac{h}{5}} \\ &= 8\sqrt{\frac{17.5}{5}} \\ &= 8 \times \sqrt{3.5} \\ &\approx 15\,. \end{aligned}$$

From this tower you could see about 15 kilometres.

Rearranging formulae

- A **formula** is a rule connecting different variables. For example, the area of a rectangle is found by multiplying its length by its width, that is,

$$A = lw\,.$$

- The **subject of the formula** is the variable on its own, usually on the left hand side, e.g. A is the subject of the Area formula.
- Sometimes we know the value of the subject but not that of one of the other variables. In such cases, we need to rearrange the formula so that the unknown variable becomes the ‘subject of the formula’. This is why this is an important algebraic skill to master.

Rearranging Formulae (cont.)

There are a number of operations which can be used to rearrange formulae. These include:

- adding or subtracting terms from both sides
- multiplying or dividing throughout
- changing the sign throughout
- raising both sides to the same power (e.g. squaring or taking the square root)
- taking reciprocals of both sides.

Rearranging Equations

The key point to remember when solving equations is to **isolate the variable**. To do this:

1. Remove all constants.
2. Make the coefficient in front of the variable one.

Use the opposite operation to achieve cancelling. A reminder:

Operation	Opposite
$+$	$-$
$-$	$+$
$\times$	$\div$
$\div$	$\times$

It is essential to maintain the balance of the equation throughout. Therefore, **whatever you do to one side you must do to the other**.

Equations involving powers and roots: Example

Make n the subject of this formula: $T = (3n+1)^2$.
Solution:

$$\begin{aligned} T &= (3n+1)^2 \\ \pm\sqrt{T} &= 3n+1 && \text{take the square root of both sides, both the positive and negative root must be included,} \\ \pm\sqrt{T}-1 &= 3n \\ \frac{\pm\sqrt{T}-1}{3} &= n \\ n &= \frac{\pm\sqrt{T}-1}{3}. \end{aligned}$$

Exercise:

Solve:

1. $-3+x=7$
2. $2.4-x=3.6$
3. $2x+1=-8$
4. $\frac{3}{4}(x+3)=\frac{3}{8}$
5. $4\left(3x-\frac{4}{9}\right)=3$
6. $2x+1=3x-5$

Solutions

$$\begin{aligned} -3+x &= 7 \\ -3+x+3 &= 7+3 && \text{Isolate } x \text{ by adding } 3 \text{ to LHS. Do the same to RHS} \\ x &= 10\,. \end{aligned}$$

Check: LHS$= -3+10 = 7 =$ RHS.

$$\begin{aligned} 2.4-x &= 3.6 \\ 2.4-x-2.4 &= 3.6-2.4 && \text{Isolate } x \text{ by taking } 2.4 \text{ from both sides} \\ -x &= 1.2 && \text{Multiply both side by } -1 \text{ to make the co-efficient of } x \text{ a one} \\ x &= -1.2\,. \end{aligned}$$

Check: LHS $= 2.4-(-1.2) = 2.4+1.2 = 3.6 =$ RHS.

Answers (cont):

$$\begin{aligned} 2x+1 &= -8 \\ 2x+1-1 &= -8-1 && \text{Isolate } x \text{ by taking } 1 \text{ from by sides} \\ 2x &= -9 \\ \tfrac{2x}{2} &= \tfrac{-9}{2} && \text{Divide LHS by } 2 \text{ to make the co-efficient of } x \text{ a one, do the same for same for RHS} \\ x &= -4\tfrac{1}{2}\,. \end{aligned}$$

Check: LHS $= 2\times\left(-4\tfrac{1}{2}\right)+1 = -9+1 = -8 =$ RHS.

Answers (cont):

$$\begin{aligned} \tfrac{3}{4}(x+3) &= \tfrac{3}{8} \\ \tfrac{4}{3}\times\tfrac{3}{4}(x+3) &= \tfrac{4}{3}\times\tfrac{3}{8} && \text{Remove the factor outside LHS by multiplying by its reciprocal, do the same for the RHS} \\ x+3 &= \tfrac{1}{2} \\ x+3-3 &= \tfrac{1}{2}-3 && \text{take } 3 \text{ from both sides} \\ x &= -2\tfrac{1}{2}\,. \end{aligned}$$

Check: LHS $= \tfrac{3}{4}(-2\tfrac{1}{2}+3) = \tfrac{3}{4}\times\tfrac{1}{2} = \tfrac{3}{8} =$ RHS.

Answers (cont):

$$\begin{aligned} 4\left(3x-\frac{4}{9}\right) &= 3 \\ \frac{4}{4}\left(3x-\frac{4}{9}\right) &= \frac{3}{4} && \text{divide both sides by } 4. \\ 3x-\frac{4}{9} &= \frac{3}{4} \\ 3x-\frac{4}{9}+\frac{4}{9} &= \frac{3}{4}+\frac{4}{9} && \text{add } \frac{4}{9} \text{ to both sides,} \\ 3x &= \frac{43}{36} \\ \frac{3x}{3} &= \frac{43}{36}\times\frac{1}{3} && \text{divide both sides by } 3, \\ x &= \frac{43}{108}\,. \end{aligned}$$

Check: LHS $= 4\left(3\times\frac{43}{108}-\frac{4}{9}\right) = 4\left(\frac{129}{108}-\frac{4}{9}\right) = 4\left(\frac{81}{108}\right) = 4\times\frac{3}{4} = 3 =$ RHS.

Answers (cont.):

$$\begin{aligned} 2x+1 &= 3x-5 \\ 2x+1-2x &= 3x-5-2x \\ 1 &= x-5 \\ 1+5 &= x-5+5 \\ 6 &= x\,. \end{aligned}$$

Check:
LHS $= 2\times 6+1 = 12+1 = 13\,,$
RHS $= 3\times 6-5 = 18-5 = 13 =$ LHS.

Further help
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