Algebra: Solving Simultaneous Equations - Elimination Method

Overview

This presentation will cover the use of the elimination method to solve a system of two equations with two unknown variables.

Simultaneous equations

- A set of simultaneous equations is a set of equations for which common solutions are sought for a number of variables.
- You need as least the same number of equations as variables to be able to find a solution.
- This presentation will only focus on two equations with two unknown variables.
- There are a number of different ways that you can solve a set of simultaneous equations. All methods are equally valid. It is up to you to choose the method that is easiest for you to use.
- This presentation will cover the elimination method.

Elimination method

In this method we eliminate one variable and from one equation. The steps involved are:

1. Multiply one or both equations by constants so that one of the variables has the same coefficient.
2. Add or subtract one equation from the other so that the variable with the same coefficient is eliminated.
3. Solve this equation to find the value of the variable.
4. Substitute the value of this variable into one of the equations to find the value of the other variable.
5. Check your answer in both of the original equations.

Example

Let us follow the steps through in an example.

$$\begin{aligned} 2x + 5y &= 6 \qquad (1) \\ 3x + 2y &= -2 \qquad (2) \end{aligned}$$

Step 1: Multiply one or both equations by constants.

The coefficients of x are different in both equations.

If we multiplied Equation (1) by 3 and Equation (2) by 2, the coefficient of x in both equations would be 6.

Remember, we must multiply every term in the equation by it.

Step 1: continued

Multiply Equation (1) by 3:

$$\begin{aligned} 3 \times (2x + 5y) &= 3 \times 6\,, \\ 3 \times 2x + 3 \times 5y &= 18\,, \\ 6x + 15y &= 18\,. \qquad (3) \end{aligned}$$

Multiply Equation (2) by 2:

$$\begin{aligned} 2 \times (3x + 2y) &= 2 \times -2\,, \\ 2 \times 3x + 2 \times 2y &= -4\,, \\ 6x + 4y &= -4 \qquad (4) \end{aligned}$$

Step 2: Subtract the equations to eliminate a variable.

$$\begin{array}{rcrcr} 6x & + & 15y & = & 18 \\ 6x & + & 4y & = & -4 \\ \hline & + & 11y & = & 22\,. \end{array}$$

Thus the equation is

$$11y = 22\,.$$

Step 3: Solve the equation.

$$\begin{aligned} 11y &= 22\,, \\ y &= 2\,. \end{aligned}$$

Step 4: Substitute to find the other variable.

$$\begin{aligned} 2x + 5y &= 6\,, \\ 2x + 5 \times 2 &= 6\,, \\ 2x &= 6 - 10\,, \\ 2x &= -4\,, \\ x &= -2\,. \end{aligned}$$

From our calculations the answers are $x = -2$ and $y = 2$.

Step 5: Check the answer in both of the original equations.

Check: $x = -2$ and $y = 2$.

Equation (1):

$$2x + 5y = 6\,,$$

$$\begin{aligned} \text{LHS} &= 2 \times -2 + 5 \times 2 \\ &= 6 \\ &= \text{RHS}. \end{aligned}$$

Equation (2):

$$3x + 2y = -2\,,$$

$$\begin{aligned} \text{LHS} &= 3 \times -2 + 2 \times 2 \\ &= -2 \\ &= \text{RHS}. \end{aligned}$$

Both values substitute correctly into both equations so the answer must be correct.

Worded example:

A city bakery sold $1\,500$ bread rolls on Sunday, with sales receipts of $\$1\,460$. Plain rolls sold for 90 cents each while gourmet rolls sold for $\$1.45$ each. How many of each type of roll were sold?

Solution:

Firstly we need to develop the equations to solve simultaneously.

The two equations generated from these sentences are:

$$
\begin{aligned}
P + G &= 1\,500\,, && (5)\\
0.9P + 1.45G &= 1\,460\,. && (6)
\end{aligned}
$$

Where P is the number of plain rolls sold and G is the number of gourmet rolls sold.

Solution (continued)

Multiply Equation (5) by 0.9.

$$0.9P + 0.9G = 1\,350\,. \qquad (7)$$

Subtract Equation (7) from Equation (6).

$$
\begin{array}{rcrcl}
0.9P & + & 1.45G & = & 1\,460\\
0.9P & + & 0.9G & = & 1\,350\\
\hline
0P & + & 0.55G & = & 110
\end{array}
$$

Solution (continued)

Therefore the equation is

$$
\begin{aligned}
0.55G &= 110\,,\\
G &= 110 \div 0.55\,,\\
G &= 200\,.
\end{aligned}
$$

To find the value of P substitute $G = 200$ into equation (5).

$$
\begin{aligned}
P + G &= 1\,500\,,\\
P + 200 &= 1\,500\,,\\
P &= 1\,300\,.
\end{aligned}
$$

Thus the solution is $G = 200$ and $P = 1\,300$.

Solution (continued)

Check:

In Equation (5), LHS $= 200 + 1\,300 = 1\,500 =$ RHS.

In Equation (6), LHS $= 0.9 \times 1\,300 + 1.45 \times 200 = 1\,460 =$ RHS.

So at the end of the day the shop had sold 200 gourmet rolls and 1 300 plain rolls.

Note:

Finally, we have solved equations where multiples of the variables are only added to or subtracted from each other.

We call these **linear equations**.

Situations do arise where the variables are related in other ways (**non-linear equations**).

The elimination method can only be used for **linear equations**.

Exercise

Solve the following sets of simultaneous equations.

1.
$$\begin{aligned} 3x - y &= 12\,, \\ x + y &= 8\,. \end{aligned}$$

2.
$$\begin{aligned} 3x - 4y &= 5\,, \\ 5x - 12y &= 3\,. \end{aligned}$$

Solution: Question 1

$$3x - y = 12\,, \tag{8}$$
$$x + y = 8\,. \tag{9}$$

As y has the same coefficient (1) and opposite signs, this is the variable I will eliminate.

Adding Equation (8) and Equation (9) gives:

$$\begin{array}{rcrcl} 3x & - & y & = & 12 \\ x & + & y & = & 8 \\ \hline 4x & & & = & 20\,. \end{array}$$

Solution: Question 1 (continued)

Rearranging to give:

$$\begin{aligned} 4x &= 20 \\ x &= 5\,. \end{aligned}$$

Substituting this into Equation (9) gives

$$\begin{aligned} x + y &= 8 \\ 5 + y &= 8 \\ y &= 8 - 5 \\ &= 3\,. \end{aligned}$$

From our calculations the answers are $x = 5$ and $y = 3$.

Check the answer in both of the original equations.

Check: $x = 5$ and $y = 3$.

Equation (8):

$$3x - y = 12\,,$$

$$\begin{aligned} \text{LHS} &= 3 \times 5 - 3 \\ &= 12 \\ &= \text{RHS.} \end{aligned}$$

Equation (9):

$$x + y = 8\,,$$

$$\begin{aligned} \text{LHS} &= 5 + 3 \\ &= 8 \\ &= \text{RHS.} \end{aligned}$$

Both values substitute correctly into both equations so the answer must be correct.

Solution: Question 2

$$\begin{aligned} 3x - 4y &= 5\,, & (10) \\ 5x - 12y &= 3\,. & (11) \end{aligned}$$

Multiply Equation (10) by -3 gives:

$$-9x + 12y = -15\,. \tag{12}$$

Adding Equation (12) and Equation (11) gives:

$$\begin{array}{rcrcr} -9x & + & 12y & = & -15 \\ 5x & - & 12y & = & 3 \\ \hline -4x & & & = & -12 \end{array}$$

Solution: Question 2 (continued)

Rearranging to give:

$$\begin{aligned} -4x &= -12 \\ x &= 3\,. \end{aligned}$$

Substituting this into Equation (8) gives

$$\begin{aligned} 3x - 4y &= 5 \\ 3 \times 3 - 4y &= 5 \\ 9 - 4y &= 5 \\ -4y &= 5 - 9 \\ y &= 1\,. \end{aligned}$$

From our calculations the answers are $x = 3$ and $y = 1$.

Check the answer in both of the original equations.

Check: $x = 3$ and $y = 1$.

Equation (10):

$$3x - 4y = 5\,,$$

$$\begin{aligned} \text{LHS} &= 3 \times 3 - 4 \times 1 \\ &= 5 \\ &= \text{RHS.} \end{aligned}$$

Equation (11):

$$5x - 12y = 3\,,$$

$$\begin{aligned} \text{LHS} &= 5 \times 3 - 12 \times 1 \\ &= 3 \\ &= \text{RHS.} \end{aligned}$$

Both values substitute correctly into both equations so the answer must be correct.

Summary

This presentation covered the use of the elimination method to solve a system of two equations with two unknown variables.