UNIVERSITY OF SOUTHERN QUEENSLAND

Significant Figures

The significant figures of a number are those digits that show the precision of a measurement. The concept of significant figures is often used in connection with rounding, for example, rounding to n significant figures is a more general-purpose technique than rounding to n decimal places.

If we perform a calculation involving a number of measurements we have to consider the level of accuracy of each measurement and understand how this could affect the accuracy of the final answer.

For example, suppose we are asked to find the volume of a container about the size and shape of a shoe box. The measuring instrument is a metre stick, graduated in millimetres, and the measurements we obtain, using the stick as accurately as we can are:

$$8.2\,\text{cm} \times 15.9\,\text{cm} \times 30.1\,\text{cm}\,.$$

Does this mean that the measurements are exact? Not necessarily. What we can say with some certainty is that the dimensions are:

$$\begin{aligned} 15.9 &\pm 0.1\,\text{cm} \\ 8.2 &\pm 0.1\,\text{cm} \\ 30.1 &\pm 0.1\,\text{cm} \end{aligned}$$

Now we find the volume. Multiplying the three dimensions together gives us 3924.438 cubic centimetres. How accurate is this? Let us consider the two worst cases in our measurement, when each measurement is 1 mm too small and when each measurement is 1 mm too large.

Firstly, consider the situation when each of the measurements is 1 mm too small. Therefore, the dimensions should be: $16.0 \times 8.3 \times 30.2$. In this situation, the volume would then be $4010.56\,\text{cm}^3$.

Secondly, consider the situation when each of the measurements is 1 mm too large. Therefore, the dimensions should be: $15.8 \times 8.1 \times 30$. In this situation, the volume would then be $3839.4\,\text{cm}^3$.

Now we have volume of our box ranging from $3839.4\,\text{cm}^3$ to $4010.56\,\text{cm}^3$ or approximately, $3800\,\text{cm}^3$ to $4000\,\text{cm}^3$. In this example, no more than the first two digits of our initial calculation can have any significance and the best we can say is that the volume is approximately $3900\,\text{cm}^3$.

General rule to determine significant figures in a calculation

When multiplying (or dividing) several quantities, the number of significant figures in the final result is the same as the number of significant figures in the least accurate of the quantities being multiplied, where 'least accurate' means 'having the lowest number of significant figures'. Since in the example above the 'least accurate' measurement is 8.2, we were entitled to claim only two significant figures in our answer, as found above.

$$\begin{aligned} \text{Volume} &= \text{length} \times \text{width} \times \text{height} \\ &= 15.9 \times 8.2 \times 30.1\,\text{cm}^3 \\ &= 3924.438\,\text{cm}^3 \\ &\approx 3900\,\text{cm}^3 \quad \text{to 2 significant figures.} \end{aligned}$$

When the operation used involves addition or subtraction, the number of significant figures should equal the **smallest number** of decimal places of any term involved.

For example, if the sides of a quadrilateral are 12.4 m, 16.5 m, 8.05 m, and 17 m, then the perimeter is 53.95 m by addition. However, because there are no decimals in the fourth measurement, only the whole number digits are significant, so we can say the perimeter is approximately 54 m.

Examples:

Rounding to 2 significant figures:

- 12 350 becomes 12 000;
- 13 stays as 13;
- 0.001 23 becomes 0.0012;
- 0.1 becomes 0.10 (the trailing zero indicates that we are rounding to 2 significant figures); and
- 0.020 84 becomes 0.021.

Rounding to 3 significant figures:

- 12 350 becomes 12 300;
- 13 becomes 13.0;
- 0.001 23 stays 0.001 23;
- 0.1 becomes 0.100 (the trailing zero indicates that we are rounding to 3 significant figures); and
- 0.020 84 becomes 0.0208.

Resources

- Other QuickTips flyers;
- Online resources at Study Support, USQ Library;
- Make a consultation with a Mathematics Learning Advisor.