Module A3

THE POWER OF NUMBERS

Table of Contents

Introduction

In module 2 we looked at calculating with different types of numbers. Generally we kept the numbers relatively small and our pen and paper or calculator were able to handle them relatively easily. There are many instances where you may be required to work with very large numbers or very small numbers.

In the health professions you may be working with viruses with masses as small as 0.000 000 000 000 000 003 kilograms.

In engineering you might look at the breaking stress of steel at 430 000 000 Pascals.

In business you might be interested to know that the profit in the 1995/6 financial year of one of our major banks was $1 119 000 000. That is over one billion dollars in profit!!!

This module will look at ways of handling calculations involving these very large and very small numbers.

On successful completion of this module you should be able to:

- demonstrate an understanding of power notation;
- evaluate and simplify expressions involving powers;
- demonstrate an understanding of scientific notation, and the metric system;
- solve problems using scientific notation; and
- solve problems using metric units, including conversions between units.

3.1 Power notation

Power notation is a very convenient way of writing very large and very small numbers. Before we move on to look at the uses for power notation we will look at what power notation means and how to work with numbers that are written in this form.

This method of writing numbers is so convenient, it is surprising that it has only been in general use for a little less than 400 years. The power notation that we use today was first written in a book by the French mathematician René Descartes, in 1637.

Recall from module 2 that a shorthand way of writing a series of like numbers being multiplied together was to use **power notation.**

Recall:
Module 2
Section 2.2.3

We could represent $2 \times 2 \times 2 \times 2 \times 2$ as 2^5.

That is, 5 numbers which are all 2 are being multiplied together.

Think of 1 000, we can write this as $10 \times 10 \times 10$ or 10^3 (Note that there were also three zeros in 1 000). This notation becomes really useful when our numbers get even larger.

Consider 100 000 000. We can write this in power notation as 10^8 (again note that there were 8 zeros in this number) which is much easier to read and understand.

Let's consider the parts of an expression in power notation.

For example, in 2^5 we call 2 the **base** (the number being multiplied) while 5 is called the **index** or **exponent** (how many times the base is being multiplied). This is shown below:

Base → 2^5 ← Index

Note here that the plural of index is indices (pronounced in/dis/ees).

Let's begin by considering cases where the index is positive.

Follow through the examples in the following table:

Product	**Think**	**Power notation**	**Say**
5	One number, 5	5^1 Normally written as 5	*5 to the power 1*
5×5	2 numbers each equal to 5	5^2	*5 to the power 2 or 5 squared*
$5 \times 5 \times 5$	3 numbers each equal to 5	5^3	*5 to the power 3 or 5 cubed*
$5 \times 5 \times 5 \times 5$	4 numbers each equal to 5	5^4	*5 to the power 4*
$5 \times 5 \times 5 \times 5 \times \ldots$ *m* (for many) times	*m* numbers each equal to 5	5^m	*5 to the power m*

Look carefully at the last row of the table above.

In general, if we multiply *m* (for many) numbers all the same, let's call them *a* (for anything) then we write

$a \times a \times a \times a \times$ .... *m* times and we can write this as a^m (read as *a to the power m)*

Here are some more examples.

7^3 means 3 numbers each being 7 are multiplied together.

$$\begin{aligned} 7^3 &= 7 \times 7 \times 7 \\ &= 49 \times 7 \\ &= 343 \end{aligned}$$

Now try this on your calculator.

Write down the keystrokes you have used if they are different.

[7] [y^x] [3] [=]

Recall:
Module 2
Section 2.2.3

$(-3)^2$ means 2 numbers each being –3 are multiplied together.

$(-3)^2 = -3 \times -3$ Recall that two negatives multiply to give a positive.

$= 9$

Now try this on your calculator.

Write down the keystrokes you have used if they are different.

[(] [(–)] [3] [)] [x^2] [=]

Activity 3.1

1. Write the following expressions in power notation and evaluate on your calculator.

(a) $2 \times 2 \times 2$

(b) 3×3

(c) $0.5 \times 0.5 \times 0.5 \times 0.5$

(d) 6

(e) $1.9 \times 1.9 \times 1.9 \times 1.9 \times 1.9$

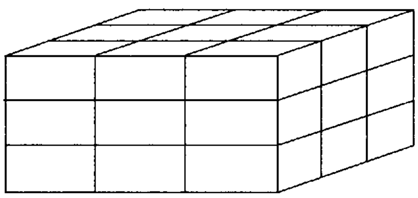
(f) -4×-4

(g) $-6 \times -6 \times -6$

2. The chance of getting 10 heads in a row when tossing a coin, is found by multiplying $\frac{1}{2}$ by itself ten times. Express this in power notation.

3. Seven children from the Avoledo family came to Australia and each of these had seven children and each of these then had seven children. How many people were there in the Avoledo family after three generations?

4. A large piece of cake is cut into smaller pieces, as in the diagram. How many pieces of cake will there be?



5. The art of sword making in Japan was an old and revered profession which was secretly passed from father to son or master to pupil. Since the sword had to be both strong and flexible, it had to be constructed in layers. To do this the craftsmen heated the sword to welding temperature, folded it in half, the heat welding the two halves together, and then reshaped the sword. This process of doubling it over, welding and shaping was done 22 times producing 2^{22} layers of steel.
   Using your calculator, find the number of layers of steel as an ordinary number.

Did you notice in the first question of the last activity that:

$-4 \times -4 = (-4)^2 = 16$ while

$-6 \times -6 \times -6 = (-6)^3 = -216$?

One answer was positive while the other was negative.

Let's now investigate this a little further.

Evaluate the following: Remember that **evaluate** means find the answer.

$(-2)^2 = -2 \times -2 =$

$(-2)^3 = -2 \times -2 \times -2 =$

$(-2)^4 = -2 \times -2 \times -2 \times -2 =$

$(-2)^5 = -2 \times -2 \times -2 \times -2 \times -2 =$

$(-2)^6 = -2 \times -2 \times -2 \times -2 \times -2 \times -2 =$

Can you see a pattern? Look at whether your answer is positive or negative.

Complete the following:

A negative base with an **even** index gives a .... answer.

A negative base with an ....index gives a .... answer.

Did you say:

A negative base with an **even** index gives a **positive** answer.

A negative base with an **odd** index gives a **negative** answer.

Examples

Evaluate $(-3)^6$

$(-3)^6$ The base is –3 and the index is even so the answer will be positive.

$= 729$

Evaluate $(-4)^5$

$(-4)^5$ The base is –4 and the index is odd so the answer will be negative.

$= -1\ 024$

Let's now look at an area of particular importance. This is an area that often causes confusion so make a particular note of it.

The confusion lies in the difference between the following two types of expressions.

$(-2)^6$ and -2^6

In the first expression, $(-2)^6$, the base is –2 and $(-2)^6 = 64$ a positive number as we would expect from the work just covered.

The difference with the second expression, -2^6, is that the base is 2. We could rewrite this expression as $-(2^6)$. We could even think of this expression as -1×2^6. The answer in this case is –64. Your answer depends on your being very clear about what number is the base.

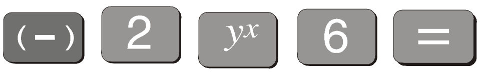
To recap:

$(-2)^6 = 64$ The base is –2

$-2^6 = -(2^6) = -64$ The base is 2

Now try this on your calculator.

Write down the keystrokes you have used if they are different.



To evaluate the second expression on the calculator you must 'know' that the answer will be negative.

3.2 Calculating with powers

While it is great to be able to express numbers as a power, it is no use to us unless we can then move on and calculate with them as we could with ordinary numbers. This section will concentrate on working with numbers that have been expressed using power notation.

3.2.1 Multiplying powers

Consider the expression

$7^2 \times 7^3$

We know that

$7^2 = 7 \times 7$

and $7^3 = 7 \times 7 \times 7$

Therefore we can write $7^2 \times 7^3$

$= 7 \times 7 \times 7 \times 7 \times 7$

$= 7 \times 7 \times 7 \times 7 \times 7$

$= 7^5$

That is $7^2 \times 7^3 = 7^5$

Can you see a quick way to arrive at this answer?

....

Did you say something like add the indices but leave the base the same?

Let's try another example and see if this same rule applies.

Simplify $6^5 \times 6^2$ Remember that simplify means to write in a simpler form but do not evaluate.

What do you think the answer is going to be?..

Let's do it the long way to check your answer.

$6^5 \times 6^2$

$= 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$

$= 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$

$= 6^7$

So, $6^5 \times 6^2 = 6^{5+2} = 6^7$

Was this the answer you worked out before? If not, have another look over these two examples.

We can now generalise this result.

When multiplying powers which have the same base we add the indices.

Let's think of this in symbols as we did before.

$$a^m \times a^n = a^{m+n}$$

In words we can think of this as multiplying *m* numbers each equal to *a* by *n* numbers each equal to *a* to give *m* + *n* numbers each equal to *a*.

Let's check this rule on the following examples.

Example

Simplify $9^7 \times 9^3$

$9^7 \times 9^3$
$= 9^{7+3}$
$= 9^{10}$

Example

Simplify $(1.7)^5 \times (1.7)^3 \times (1.7)^4$

$(1.7)^5 \times (1.7)^3 \times (1.7)^4$
$= (1.7)^{5+3+4}$
$= (1.7)^{12}$

Example

Simplify $(-3)^3 \times (-3)^2$

Note that –3 is in brackets. This means that the base is –3. Therefore $(-3)^2 = (-3) \times (-3)$

$(-3)^3 \times (-3)^2$
$= (-3)^{3+2}$
$= (-3)^5$

Example

Simplify $-3^3 \times -3^2$

$-3^3 \times -3^2$

$= 3^3 \times 3^2$
$= 3^{3+2}$
$= 3^5$

Recall that 2 negatives multiplied together give a positive.

As you can see from the above two examples it is important to take great care when dealing with negative bases. Always put brackets around a negative base.

Example

Simplify $\left(\frac{1}{4}\right)^3 \times \left(\frac{1}{4}\right)^6$

$\left(\frac{1}{4}\right)^3 \times \left(\frac{1}{4}\right)^6$

$= \left(\frac{1}{4}\right)^{3+6}$

$= \left(\frac{1}{4}\right)^9$

Example

Simplify $(0.4)^2 \times 9^5 \times (0.4)^5 \times 9^7$

$(0.4)^2 \times 9^5 \times (0.4)^5 \times 9^7$

$= (0.4)^2 \times (0.4)^5 \times 9^5 \times 9^7$ Group like numbers (those with the same base)

$= (0.4)^{2+5} \times 9^{5+7}$ Use the rule on both pairs

$= (0.4)^7 \times 9^{12}$

We cannot simplify this expression further as we have **different** bases. Remember that the question said to simplify so you must stop here and not go on and evaluate.

How are you feeling now about multiplying numbers involving powers?

Here are some for you to do.

Activity 3.2

1. **Simplify** the following expressions as far as possible.

(a) $5^2 \times 5^7$

(b) $3^8 \times 3^2 \times 3^5$

(c) $(-4)^3 \times (-4)^2 \times (-4)^5$

(d) $(2.6)^7 \times (2.6)^{12} \times (2.6)$

(e) $(-7)^3 \times 6^2 \times (-7)^2 \times 6^2$

(f) $\left(\frac{1}{2}\right)^3 \times (-4)^3 \times \left(\frac{1}{2}\right)^5 \times (-4)^2$

(g) $5^8 \times (0.3)^2 \times (0.3)^5 \times 5^2 \times (0.3)^7$

(h) $-4^3 \times -4^2$

(i) $-9^3 \times -9^2 \times -9^1$

2. **Evaluate** your answers in question 1 using your calculator.

3. A glass of water contains approximately 10^{25} water molecules and the Atlantic ocean contains approximately 10^{21} glasses of water. Approximately how many molecules of water are there in the Atlantic Ocean? Leave your answer expressed as a power.

4. In our galaxy there are 10^{11} (100 billion) stars, and we know of 10^9 (1 billion) galaxies. If each of these galaxies has the same number of stars as ours, how many stars would there be? Leave your answer expressed as a power.

5. The frequency of television waves is 10^8 cycles per second. One type of x-ray wave has a frequency 10^{11} times greater than this. What is the frequency of the x-ray? Leave your answer expressed as a power.

3.2.2 Dividing powers

Consider the expression $7^5 \div 7^2$

Recall that a division can be written as a fraction.

Recall:
Module 2
Section 2.5.2

So, $7^5 \div 7^2$ can be written as $\frac{7^5}{7^2}$

We know that $7^5 = 7 \times 7 \times 7 \times 7 \times 7$

$7^2 = 7 \times 7$

Therefore we can write the above expression as

$7^5 \div 7^2 = \dfrac{7^5}{7^2} = \dfrac{\cancel{7} \times \cancel{7} \times 7 \times 7 \times 7}{\cancel{7} \times \cancel{7}}$ Cancel to simplify the fraction.

$= 7 \times 7 \times 7$

$= 7^3$

That is, $7^5 \div 7^2 = 7^3$

Again it looks like there is a quick way to arrive at this result. What do you think it is?

....

Did you say something like subtract the powers but leave the bases the same?

Let's try another example and see if this same rule applies.

Simplify $3^6 \div 3^4$

What do you think the answer is going to be?...

$3^6 \div 3^4 = \dfrac{3^6}{3^4} = \dfrac{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times 3 \times 3}{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}} = 3 \times 3 = 3^2$

So $3^6 \div 3^4 = 3^{6-4} = 3^2$

Does this agree with the answer you thought of above?

Let's now generalise this result.

When dividing powers which have the same base, we subtract the indices.

Now let's express this in symbols as before.

$a^m \div a^n = a^{m-n}$

In words we can think of this as dividing m numbers each equal to a by n numbers each equal to a to give $m - n$ numbers each equal to a.

Let's check this rule on the following examples.

Example

Simplify $5^3 \div 5^2$

$5^3 \div 5^2$
$= 5^{3-2}$
$= 5^1$
$= 5$

Example

Simplify $(1.5)^9 \div (1.5)^6 \div (1.5)$

$(1.5)^9 \div (1.5)^6 \div (1.5)$
$= (1.5)^9 \div (1.5)^6 \div (1.5)^1$
$= (1.5)^{9-6-1}$
$= (1.5)^2$

Example

Simplify $(-2)^9 \div (-2)^2$

Note that the use of brackets here is the same as for multiplication.

$(-2)^9 \div (-2)^2$
$= (-2)^{9-2}$
$= (-2)^7$

This means that the base is –2

Example

Simplify $-2^9 \div -2^2$

Note: The lack of brackets in this expression means the same here as in multiplication.

$-2^9 \div -2^2$
$= 2^9 \div 2^2$
$= 2^{9-2}$
$= 2^7$

Dividing two negatives gives a positive.

Take care when dealing with negative bases in division as well as in multiplication.

Activity 3.3

1. **Simplify** the following expressions as far as possible.

(a) $5^7 \div 5^2$

(b) $3^8 \div 3^2$

(c) $(-4)^3 \div (-4)^2$

(d) $(2.6)^{12} \div (2.6)^7$

(e) $(-7)^3 \div (-7)$

(f) $\left(\frac{1}{2}\right)^{13} \div \left(\frac{1}{2}\right)^5$

(g) $-5^7 \div -5^2$

(h) $\left(\frac{1}{4}\right)^5 \div \left(\frac{1}{4}\right)^3$

(i) $(1.7)^{10} \div (1.7)^6 \div (1.7)^2$

2. **Evaluate** your answers in question 1 using your calculator.

3. If a radio frequency is 10^8 cycles per second and gamma rays have a frequency of 10^{21} cycles per second, how many times greater is the gamma ray frequency than the radio frequency.

Negative Indices

Consider the following division.

$$7^5 \div 7^9$$

We know from the last section that we subtract the indices because it is a division.

$$7^5 \div 7^9 = 7^{5-9} = 7^{-4}$$

Let's look at this another way

$$7^5 \div 7^9 = \frac{7^5}{7^9} = \frac{\not{7} \times \not{7} \times \not{7} \times \not{7} \times \not{7}}{\not{7} \times \not{7} \times \not{7} \times \not{7} \times \not{7} \times 7 \times 7 \times 7 \times 7} = \frac{1}{7 \times 7 \times 7 \times 7} = \frac{1}{7^4}$$

Let's combine these two results.

$$7^5 \div 7^9 = 7^{-4} = \frac{1}{7^4}$$

So $7^{-4} = \dfrac{1}{7^4}$

Here's another example:

$$6^5 \div 6^8$$

What do you think the answer will be?..

Can you express this answer without the negative index?..

We know that $6^5 \div 6^8 = 6^{5-8} = 6^{-3} = \dfrac{1}{6^3}$

We can now generalise this result.

A power with a negative index is equal to one over the same power with a positive index of the same size.

In symbols this would be:

$$a^{-m} = \frac{1}{a^m}$$

Let's look at some further examples.

Examples

Express the following with no negative indices.

$$5^{-2} = \frac{1}{5^2}$$

$$26^{-4} = \frac{1}{26^4}$$

$$(-4)^{-2} = \frac{1}{(-4)^2}$$

Note here that the base is negative and moves with the power.

$$\frac{1}{5^{-8}} = 5^8$$

This time we used the rule in reverse.

$$-2^{-3} = -1 \times 2^{-3} = -1 \times \frac{1}{2^3} = \frac{-1}{2^3}$$

Be **very careful** when any of the numbers involved is negative. Note that it is only the sign of the index that changes.

$$\frac{2}{3^{-7}} = 2 \times \frac{1}{3^{-7}} = 2 \times 3^7$$

$$\frac{5^{-4}}{6^{-2}} = \frac{6^2}{5^4}$$

Did you notice that in general moving a power from the top to the bottom or the bottom to the top of a fraction merely changes the sign of the index.

To Evaluate Negative Powers on the Calculator

An example of keystrokes to evaluate 5^{-2} are:

5 y^x (−) 2 =

The display should read 0.04

Now try this on your calculator.

Write down the keystrokes you have used if they are different.

Activity 3.4

Simplify, expressing your answer with no negative indices.

1.

(a) 8^{-2}

(b) 6^{-2}

(c) 7^{-1}

(d) $\frac{1}{5^{-2}}$

(e) $\frac{3}{4^{-2}}$

(f) $\frac{2^{-5}}{3^{-2}}$

(g) $\frac{4^{-2}}{3}$

(h) 2×4^{-3}

(i) -3^{-2}

(j) $(-3)^{-2}$

2. Evaluate each of the expressions in question 1 on your calculator.

3.

(a) $\frac{-3}{6^{-5}}$

(b) $\frac{7}{5^{-2}}$

(c) $\frac{2 \times 4^2}{3^{-4}}$

(d) -3^{-4}

(e) -2×5^{-3}

(f) $\frac{2^{-5}}{5^{-3}}$

(g) $\frac{2^4}{3^{-6}}$

(h) $\frac{-3^5}{(-2)^{-4}}$

(i) $\frac{(-4)^{-2}}{2}$

(j) $\frac{-5^{-4} \times 3^4}{2^3}$

(4.Evaluate each of the expressions in question 3 using your calculator.

Zero Index

Consider $5^2 \div 5^2$

You know from your rules for dividing powers that

$$5^2 \div 5^2 = 5^{2-2} = 5^0$$

Let's look at this another way

$$5^2 \div 5^2 = \frac{5^2}{5^2} = \frac{\cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5}} = 1$$

Combining these two results

$$5^2 \div 5^2 = 5^0 = 1$$

In general, any number (except zero) raised to the power of zero is equal to 1

In symbols we can write

$a^0 = 1 \qquad a \neq 0$ The symbol $\neq$ means 'not equal to'.

Activity 3.5

Evaluate the following, check your answers with the calculator.

(a) 5^0

(b) 256^0

(c) 4×65^0

(d) $3^2 \times 55^0$

(e) $(-4)^0$

(f) -4^0

3.2.3 Adding and subtracting powers

While there are quick ways of multiplying and dividing powers there are no simple methods for adding and subtracting.

Example

Evaluate $2^3 + 2^6$

The only way to add these two terms together is to **evaluate** each one and **then add** the results.

$2^3 + 2^6$

$= 8 + 64$

$= 72$

Example

Evaluate $7^4 - 5^3$

$7^4 - 5^3$
$= 2\,401 - 125$
$= 2\,276$

3.2.4 Special powers

Fractional Indices

So far we have looked at indices which have been positive and negative whole numbers and zero, that is, indices that were integers. The other type of numbers that we studied in module 2 were fractions.

Let's look at powers where the index is a fraction.

Consider $9^{\frac{1}{2}} \times 9^{\frac{1}{2}}$

From the previous sections we know that:

$9^{\frac{1}{2}} \times 9^{\frac{1}{2}}$

$= 9^{\frac{1}{2}+\frac{1}{2}}$

$= 9^1$

$= 9$

Note that $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = \left(9^{\frac{1}{2}}\right)^2$

We have shown that:

$$\left(9^{\frac{1}{2}}\right)^2 = 9$$

But we know that $3^2 = 9$. Therefore $\left(9^{\frac{1}{2}}\right)^2 = 3^2$

Therefore $9^{\frac{1}{2}}$ must equal 3.

Now we know that $\sqrt{9} = 3$

Therefore $9^{\frac{1}{2}} = \sqrt{9} = 3$

> 
> Recall:
> Module 2
> Section 2.2.3

Hence, an index of $\frac{1}{2}$ means the **square root** of the number.

Examples

$100^{\frac{1}{2}} = \sqrt{100} = 10$

$49^{\frac{1}{2}} = \sqrt{49} = 7$

Now consider $8^{\frac{1}{3}}$

We can write $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}}$

Note that $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = \left(8^{\frac{1}{3}}\right)^3$

$= 8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$

$= 8^1$

$= 8$

We have shown that $\left(8^{\frac{1}{3}}\right)^3 = 8$

But we also know that $2^3 = 8$. Therefore $\left(8^{\frac{1}{3}}\right)^3 = 2^3$

Therefore $8^{\frac{1}{3}}$ must equal 2.

We call 2 the **cubed root** of 8 since it is the number when multiplied together three times gives you 8.

We write this as $\sqrt[3]{8}$

Therefore $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

An index of $\frac{1}{3}$ means the third root or **cubed root** of the number.

This symbol for the cubed root is from 17th Century France. Earlier than this, in 1525, a German mathematician named Christoph Rudolff used the symbol  for the cubed root of a number.

What do you think $16^{\frac{1}{4}}$ would mean?

....

You should have said something like it means the fourth root of 16. The number that when multiplied together 4 times will give you 16.

Do you know what this number will be? ....

If you said 2 then well done!!

We write $16^{\frac{1}{4}} = \sqrt[4]{16} = 2$

What symbol do you think Rudolff would have used to represent the fourth root of a number?

Did you say  ?

The roots are not always easy to find but again your calculator will help with this calculation.

Let's find $1\ 024^{\frac{1}{5}}$. That is, the fifth root of 1 024. What number when multiplied together 5 times will give you 1 024?

There are a number of ways to do these on your calculator.

The display should read 4.

That is 4 multiplied by itself 5 times will give you 1 024

The other way to do this calculation is to enter the index as a fraction and use the x^y key. Depending on your calculator you may or may not have to add the brackets.

Now try this on your calculator.

Write down the keystrokes you have used if they are different.

An example of keystrokes are:

[1] [0] [2] [4] [y^x] [(] [1] [ab/c] [5] [)] [=]

We can write $1\ 024^{\frac{1}{5}} = \sqrt[5]{1\ 024} = 4$

Example

Evaluate $123^{\frac{1}{3}}$, estimating your answer before you begin.

Estimation under these circumstances can be very difficult, but a general idea of the answer will be helpful.

What you are looking for is the number, that when cubed, will give you 123.

You may know that $4^3 = 64$

and that $5^3 = 125$

So, the answer we are looking for will be between 4 and 5. In fact it should be fairly close to 5.

On the calculator $123^{\frac{1}{3}} = 4.973189833 \ldots\ldots$

≈ 4.97

So our answer is very close to 5 as we had estimated.

Activity 3.6

1. Express the following without using an index. Do not evaluate.

(a) $25^{\frac{1}{2}}$ (d) $7\,776^{\frac{1}{5}}$

(b) $27^{\frac{1}{3}}$ (e) $100^{\frac{-1}{2}}$

(c) $81^{0.25}$

2. Evaluate the following using your calculator. Think about what your answer will be before you calculate.

(a) $25^{\frac{1}{2}}$ (f) $16^{\frac{1}{2}} \times 49^{\frac{1}{2}}$

(b) $27^{\frac{1}{3}}$ (g) $36^{\frac{1}{3}} \times 54^{\frac{1}{2}}$

(c) $81^{0.25}$ (h) $63^{\frac{1}{5}} + 125^{\frac{1}{4}}$

(d) $7\,776^{\frac{1}{5}}$ (i) $123^{\frac{1}{2}} \div 567^{\frac{1}{6}}$

(e) $100^{\frac{-1}{2}}$ (j) $534^{\frac{1}{3}} - 34^{\frac{1}{4}}$

Finding a power of a power

Consider the expression

$(4^2)^3$

Read this as *4 squared all cubed*.

We can rearrange this expression:

$(4^2)^3$ To cube a number multiply it together three times.

$= 4^2 \times 4^2 \times 4^2$

$= 4 \times 4 \times 4 \times 4 \times 4 \times 4$

$= 4^6$

So $(4^2)^3 = 4^6$

Can you see a shortcut way to get to this result?

....

If you said to multiply the indices then you would be correct.

$(4^2)^3 = 4^{2 \times 3} = 4^6$

We can generalise this result.

When raising a power to another power we multiply the indices

In symbols we can write this as

$(a^m)^n = a^{m \times n}$

Check this rule with the following examples.

Example

Simplify $(3^2)^5$

$(3^2)^5$
$= 3^{2 \times 5}$
$= 3^{10}$

Example

Simplify $((-2)^2)^7$

$((-2)^2)^7$
$= (-2)^{2 \times 7}$
$= (-2)^{14}$

Let's consider $\left(8^{\frac{1}{3}}\right)^2$

$\left(8^{\frac{1}{3}}\right)^2$

$= 8^{\frac{1}{3} \times 2}$

$= 8^{\frac{2}{3}}$

Now we know that $8^{\frac{1}{3}}$ can be written $\sqrt[3]{8}$ (the cubed root of 8)

So another way to write $\left(8^{\frac{1}{3}}\right)^2$ is $(\sqrt[3]{8})^2$

and so $8^{\frac{2}{3}} = (\sqrt[3]{8})^2$

To evaluate this

$(\sqrt[3]{8})^2 = 2^2$ The cubed root of 8 is 2

$= 4$

So $8^{\frac{2}{3}} = 4$

We have extended our knowledge of fractional indices.

When the index is a fraction, the denominator tells us the root to take and the numerator tells us the power to take.

To evaluate $8^{\frac{2}{3}}$ on your calculator

The display should read 4.

Let's do another.

Example

Evaluate $16^{\frac{3}{4}}$

The index $\frac{3}{4}$ tells us to take the fourth root of 16 and then to raise this to the power 3

That is

$16^{\frac{3}{4}} = (\sqrt[4]{16})^3$ The fourth root of 16 is 2

$= 2^3$

$= 8$

Now try this on your calculator.

Activity 3.7

1. Simplify the following

(a)	$(2^3)^5$	(e)	$((-5)^3)^3$
(b)	$(3^4)^2$	(f)	$((1.2)^3)^0$
(c)	$(5^6)^2$	(g)	$((\frac{1}{4})^4)^3$
(d)	$((-2)^3)^4$	(h)	$(6^{\frac{3}{4}})^3$

2. Evaluate each of the above expressions with your calculator.

3. Evaluate each of the following expressions with your calculator.

(a)	$64^{\frac{4}{3}}$	(e)	$\dfrac{7^3 \times 5^4}{(35)^2}$
(b)	$8^{\frac{4}{3}} \times 16^{\frac{5}{4}}$	(f)	$\dfrac{(121)^{\frac{5}{2}} \times (49)^{\frac{3}{2}}}{11^3 \times 7^2}$
(c)	$27^{\frac{7}{3}} - 36^{\frac{3}{2}}$	(g)	$(-27)^{\frac{1}{3}} + \sqrt[4]{81}$
(d)	$4^{2.5} + 81^{2.5}$	(h)	$\sqrt[5]{7\,776} \div -4^{\frac{1}{2}}$

Finding a power of a product

Consider the expression

$(3 \times 4)^2$

Let's rearrange this as below

$(3 \times 4)^2$
$= (3 \times 4) \times (3 \times 4)$
$= 3 \times 4 \times 3 \times 4$
$= 3 \times 3 \times 4 \times 4$
$= 3^2 \times 4^2$

That is $(3 \times 4)^2 = 3^2 \times 4^2$

Now try this on your calculator.

The display should again read 144.

Example

Remove the bracket from $(5 \times 2)^3$

$$(5 \times 2)^3 = 5^3 \times 2^3$$

Let's go the other way.

Example

Express $7^5 \times 3^5$ as a single power statement.

$$7^5 \times 3^5 = (7 \times 3)^5$$

We can generalise the rule.

When raising the product of two (or more) numbers to a power we find the product of each of these numbers raised to the same power.

We can also express it in symbols as we have done before.

$$(a \times b)^m = a^m \times b^m$$

Activity 3.8

1. Apply the power of a product rule to the following.

(a)	$(9 \times 5)^2$	(d)	$6^5 \times 3^5$
(b)	$(7 \times 6)^2$	(e)	$12^6 \times 2^6$
(c)	$(8 \times 5)^3$	(f)	$7^3 \times 4^3$

2. Evaluate the above expressions on your calculator.

We have looked at many different ways of manipulating numbers that are expressed as powers.

Here is an activity that combines many of these techniques.

Activity

Activity 3.9

Simplify the following expressions giving your answers with no negative indices and no brackets.

1.

(a) $5^{\frac{4}{3}} \div 5^{\frac{2}{3}}$

(b) $(3^0)^2$

(c) $3^{-1} \times 4^0$

(d) $2^6 \times 2^2 \div 2^7$

(e) $\dfrac{3^4 \times 3}{3^5}$

(f) $\dfrac{3^{-2} \times 3^0}{3^4}$

(g) $\dfrac{5^{-2} \div 5^3}{5^4}$

(h) $\dfrac{2^4 \times 2^5}{(2^2)^{-2} \times 2^{-3}}$

(i) $\dfrac{3^5 \div 3^2}{3^4 \times (3^4)^{-2}}$

(j) $\dfrac{(-2)^3 \times (-2)^5}{(-2)^{10}}$

(k) $\dfrac{5^{-3} \times (5^2)^3}{(5^3)^4 \times 5^{-7}}$

Simplify the following expressions giving your answers with no negative indices and no brackets.

2.

(a) $\dfrac{(3^2 \times 10^6)^{\frac{1}{2}}}{3^4 \times 10^{-2}}$

(b) $\dfrac{(5^{-3} \times 4^2)^3}{(4^{-7} \times 5^2)^2}$

(c) $\dfrac{5^{\frac{1}{3}} \times 5^0}{5^{-1}} \div 5^2$

3.3 Common applications of powers

Remember that we set out to find ways of expressing and manipulating very large and very small numbers. We have found that power notation is a convenient way to express some of these numbers and we have looked at ways of manipulating these numbers expressed in power notation.

We will now consider another shorthand way of expressing any number, but especially very large and very small numbers containing many zeros.

3.3.1 Scientific notation

Quick, which is larger, 381000000000 or 98200000000?

And which is smaller, 0.000000000034 or 0.00000000085?

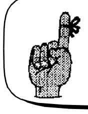
It is very difficult to tell which is larger or smaller without counting zeros.

Scientific notation is a convention by which we can write these very large and very small numbers in such a way that their relative sizes are immediately clear.

It also spares us the effort of having to pronounce them!!!!!!

We will return to these numbers in a moment but firstly let's look a little more closely at powers of 10 which are an important part of being able to express numbers in scientific notation. Recall that we covered these in module 2 when multiplying decimals.

Multiplying by powers of 10



Recall:
Module 2
Section 2.6.2

Consider the following examples where 75 is multiplied by $10 = 10^1$, $100 = 10^2$ and $1\ 000 = 10^3$.

Do the following calculations on your calculator and write in the results.

$75 \times 10^1 = 75 \times 10 =$
$75 \times 10^2 = 75 \times 100 =$
$75 \times 10^3 = 75 \times 1\ 000 =$

Remember that the decimal point in a whole number is understood to be after the last digit.

If you look carefully at them you will notice that we have shifted the decimal point in the 75 to the right in each case – one place when multiplying by ten, two places when multiplying by 100 and three places when multiplying by 1 000.

In fact the number of places moved was the same as the number of zeros in the power of 10 involved. The number of places moved also corresponds to the power of ten that we were multiplying by.

$75 \times 10^1 = 75 \times 10 = 75.0 = 750$
$75 \times 10^2 = 75 \times 100 = 75.00 = 7\,500$
$75 \times 10^3 = 75 \times 1\,000 = 75.000 = 75\,000$

Following this pattern, if we multiply 75 by 10^6 (1 000 000), the result should be

$75 \times 10^6 = 75 \times 1\,000\,000 = 75.000000 = 75\,000\,000$

Let's try this process on some other products involving powers of 10.

$56 \times 10^2 = 56 \times 100 = 56.00 = 5\,600$

$4.6 \times 10^3 = 4.6 \times 1\,000 = 4.600 = 4\,600$

$0.789 \times 10^5 = 0.789 \times 100\,000 = 0.78900 = 78\,900$

We can generalise this.

When **multiplying** by a power of ten, move the decimal point to the **right** by the number of zeros in the power of ten.

Activity 3.10

1. Multiply by the numbers shown to complete the table.

	Number	$\times$ **10(10^1)**	$\times$ **100(10^2)**	$\times$ **1 000(10^3)**	$\times$ **1 000 000(10^6)**
(a)	23				
(b)	–590				
(c)	0.6				
(d)	0.04				
(e)	–0.0305				
(f)	8.9				

2. Five bottles each contain 100 tablets. How many tablets are there?

3. A large company orders embroidered caps to give away as a promotion. If they order 1 000 caps at $4.32, how much does the order cost?

Dividing by powers of 10

This time let's divide 75 by the following powers, $10 = 10^1$, $100 = 10^2$ and $1\,000 = 10^3$.

Do the following calculations on your calculator and write in the results.

$75 \div 10^1 = 75 \div 10 =$
$75 \div 10^2 = 75 \div 100 =$
$75 \div 10^3 = 75 \div 1\,000 =$

Can you now write a rule for dividing by powers of 10?

....

....

Once again, if you look at the pattern you can see that the decimal point has moved, this time to the left – one place when dividing by ten, two places when dividing by 100 and three places when dividing by 1 000.

A similar pattern has emerged to when we multiplied by powers of 10. The number of decimal places shifted depends on the number of zeros in the power of 10. The number of places moved also corresponds to the power of ten that we were dividing by.

That is

$75 \div 10^1 = 75 \div 10 = 7\,5. = 7.5$

$75 \div 10^2 = 75 \div 100 = 0\,7\,5. = 0.75$

$75 \div 10^3 = 75 \div 1\,000 = 0\,0\,7\,5. = 0.075$

Following this pattern, if we divide 75 by 1 000 000, the result should be

$75 \div 1\,000\,000 = 0\,0\,0\,0\,0\,7\,5. = 0.000\,075$ We moved the decimal point 6 places to the left

Let's try this process on some other divisions involving powers of 10

$56 \div 100 = 0\,5\,6. = 0.56$ We moved the decimal point 2 places to the left

$4.6 \div 1\,000 = 0\,0\,0\,4.6 = 0.004\,6$ We moved the decimal point 3 places to the left

$0.789 \div 100\,000 = 0\,0\,0\,0\,0\,0.789 = 0.000\,007\,89$ We moved the decimal point 5 places to the left

We can generalise this.

When **dividing** by a power of ten, move the decimal point to the **left** by the number of zeros in the power of ten.

Activity 3.11

1. Divide by the numbers shown to complete the table

	Number	$\div 10(10^1)$	$\div 100(10^2)$	$\div 1\,000(10^3)$	$\div 1\,000\,000(10^6)$
(a)	23				
(b)	–590				
(c)	0.6				
(d)	0.04				
(e)	–0.0305				
(f)	8.9				

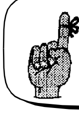
2. One thousand individual doses of medicine are to be drawn from a 5 500 millilitre container. How much medicine will be in each dose?

3. At a sale, Jeremy purchases 10 pairs of socks for $49.50. How much did each pair cost?

Let's look more closely at dividing in terms of the power of 10.

Example

Express $567 \div 10^2$ as a multiplication, without evaluating.



Recall:
Module 2
Section 2.5.3

$567 \div 10^2$

$= 567 \div 100$

$= 567 \times \dfrac{1}{100}$ Multiply by the reciprocal

$= 567 \times \dfrac{1}{10^2}$ Expressing 100 as a power

$= 567 \times 10^{-2}$ Recall that $\dfrac{1}{10^2} = 10^{-2}$

Recall:
Module 3
Section 3.2.2

So $567 \div 10^2 = 567 \times 10^{-2}$

Dividing by a power of ten is the same as multiplying by the same power with the index of the opposite sign.

Can you express $351 \div 10^3$ as a multiplication?

....

Let's try: $351 \div 10^3$

$= 351 \times \frac{1}{10^3}$

$= 351 \times 10^{-3}$

Is this the answer you suggested above?

Writing Numbers in Scientific Notation

Any positive number can be written as a multiplication of a number between 1 and 10 and a power of ten. A number written in this form is said to be written in **scientific notation**. It is very useful for writing very large and very small numbers in a manner that is easy to understand.

For example, 3.2×10^3, 4.69×10^{15} and 9.567×10^{-6} are all positive numbers written in scientific notation.

It is also possible to write negative numbers in scientific notation.

For example, -8.2×10^3, -6.72×10^{15} and -3.201×10^{-23} are all negative numbers written in scientific notation. This time the number at the front is between –1 and –10.

Writing numbers in scientific notation involves two steps:

- moving the decimal point so that it is placed after the first non-zero digit from the left; and
- multiplying by a power of ten that would return the number to its original form.

Example

Convert 365 000 to scientific notation.

Step 1 3.650 00 — Move the decimal point to be after the first non-zero digit.

Step 2 $3.650\ 00 \times 10^5$ — To maintain the initial value of the number we must multiply by 10^5. That is, move the decimal point 5 places to the right. Our index is positive because we need to make the number larger to get back to the original number.

So 365 000 = 3.65×10^5 expressed in scientific notation. Note that we have left off the zeros that came after the last number after the decimal point.

Example

Express 0.003 05 in scientific notation.

Step 1 003.05 — Move the decimal point to be after the first non-zero number.

Step 2 3.05×10^{-3} — To maintain the initial value of the number we must multiply by 10^{-3} (which is the same as dividing by 10^3). That is, move the decimal point 3 places to the left. Our index is negative because we need to make the number smaller to get back to the original number.

So 0.003 05 = 3.05×10^{-3}

Let's now look at those numbers from before.

Which is larger, 381000000000 or 98200000000?

And which is smaller, 0.000000000034 or 0.00000000085?

Consider the first two very large numbers.

Write 381000000000 in scientific notation. ....

Now write 98200000000 in scientific notation. ....

Your answers should have been 3.81×10^{11} and 9.82×10^{10}

It should be clear now that the first number is the larger because it contains the higher power of 10.

Now to those very small numbers.

Write 0.000000000034 in scientific notation..

Now write 0.00000000085 in scientific notation. ....

You should have answered 3.4×10^{-11} and 8.5×10^{-10}.

Which is the smaller number? ....

Did you say that the first number is the smaller of the two? The first number will have the greater number of zeros after the decimal point and so will be smaller than the second number.

Activity 3.12

1. Write the following numbers in scientific notation.

(a)	65 000	(g)	0.000 002 4
(b)	750 000	(h)	825 600
(c)	8 700	(i)	650
(d)	0.003 5	(j)	–0.005 9
(e)	0.04	(k)	7.2
(f)	–65		

2. Write in scientific notation the numbers for the mass of the virus, the breaking strength of steel and the profit of the bank given in the introduction to this module.

3. The number 578×10^6 is not written in scientific notation. Can you explain why not?

4. The number of hairs on your head varies with the colour of your hair.

Colour of hair	**Number of hairs**
Black or Brown	1.05×10^5
Blond	1.4×10^5
Red	9×10^4

(a) Which colour head of hair has the most number of hairs?

(b) How did you determine that this was the largest number?

5. Arrange the following in ascending (from smallest to largest) order.

(a) 1.6×10^3, 2.54×10^3, 9.6×10^3, 2.4×10^3, 1.7×10^3

(b) 5.1×10^6, 5.1×10^{-3}, 5.1×10^4, 5.1, 5.1×10^{-4}

(c) 7.96×10^7, 8.45×10^6, 7.8×10^{-5}, 1.4×10^9, 3.4×10^{-4}

Converting from scientific notation to ordinary form

To convert a number back from scientific notation to its ordinary form, you need to follow the instructions given by the index.

Recall from the section on multiplying by powers of ten:

If the index of the power of 10 is **positive**, move that number of places to the **right**. For example, if the power is 10^8, then move the decimal point 8 places to the right.

If the index of the power of 10 is **negative**, move that number of places to the **left**. For example, if the power is 10^{-8}. Then move the decimal point 8 places to the left.

Example

Write 7.2×10^6 in ordinary form.

Since the power of 10 is positive 6 we move the decimal point 6 places to the right.

7.2×10^6 = 7.2 0 0 0 0 0

= 7 200 000

Example

Write 7.2×10^{-6} in ordinary form.

Since the power of 10 is negative 6 we move the decimal point 6 places to the left.

7.2×10^{-6} = 0 0 0 0 0 0 7.2

= 0.000 007 2

Activity 3.13

1. Write in ordinary form

(a)	6.15×10^3	(e)	-5.76×10^{-5}
(b)	7.24×10^{-3}	(f)	10^4
(c)	9.25×10^4	(g)	10^{-3}
(d)	6.92×10^2		

2. The following concentrations (in milligrams per millilitre of blood) of various chemicals have been found in a blood sample. Write these numbers in ordinary form.

Chemical A	4.56×10^{-4}
Chemical B	3.89×10^{-12}
Chemical C	5.78×10^{-7}

3. The speed of light is 2.998×10^8 metres per second. Express this number in ordinary form.

4. The number of hairs on your head varies with the colour of your hair.

Colour of hair	**Number of hairs**
Black or Brown	1.05×10^5
Blond	1.4×10^5
Red	9×10^4

5. Write each number in ordinary form.

6. Complete the following using one of the symbols, $<$, $>$, or $=$. You may have to change the way some of these numbers are written to be able to make the comparison.

(a) 39.7 ☐ 3.97×10

(b) $\dfrac{1}{1\ 000}$ ☐ 10^{-4}

(c) 6.85×10^5 ☐ $6\ 850$

(a) 39.7 ☐ 3.97×10

(d) 0.000 95 ☐ 7.8×10^{-2}

(e) $\frac{55}{100}$ ☐ 5×10^{-2}

Try this calculation on your calculator.

$9\,876 \times 87\,456 \times 354\,876 =$

What answer did your calculator display? ....

It probably looked something like the following if you are using the same calculator as we have throughout these modules.

3.065118862^{14}

Other calculators will show this as:

3.0651189×10^{14}

What has happened is that there were too many digits to be displayed on your calculator screen so the calculator has automatically gone into scientific notation.

You must be able to recognise this as being a number in scientific notation and write it as such. In this example you would write out the answer as:

$9\,876 \times 87\,456 \times 354\,876 = 3.065118862 \times 10^{14}$

and since the number is an approximation anyway due to the fact that the screen could not display all the digits, it would be quite acceptable to round this answer a bit further.

$$\begin{aligned} 9\,876 \times 87\,456 \times 354\,876 &= 3.065118862 \times 10^{14} \\ &\approx 3.065 \times 10^{14} \end{aligned}$$

In this course we do not have any strict rules for how many decimal places to round to in any given situation but you will find that in other studies you may undertake there could be strict guidelines for rounding to a specific number of decimal places.

Let's do another on your calculator.

Example

$0.2 \div 987\,654\,321 =$ ....

You should check, when keying this calculation on your calculator, that all of these 9 digits are displayed. Different calculators allow for different numbers of digits to be displayed. If all these digits did not appear on the screen the calculator may have ignored the ones that did not appear, giving an incorrect answer. If your calculator did not allow for this many digits, you would need to enter the number in scientific notation as explained earlier.

Did your calculator display show something like 2.025^{-10}? Again this number has been written in scientific notation and you should interpret it as:

$$0.2 \div 987\ 654\ 321 = 2.025 \times 10^{-10}$$

which you know is the ordinary number 0.000 000 000 202 5

It is quite acceptable to give your answer in most of these cases in scientific notation.

Calculations with numbers in scientific notation

We will now use your ability to write numbers in scientific notation, and your knowledge of calculating with powers, to multiply, divide, add and subtract numbers written in scientific notation.

Multiplication

Example

Evaluate $7.25 \times 10^4 \quad \times \quad 9.6 \times 10^2$ — Write your answer in scientific notation.

$7.25 \times 10^4 \quad \times \quad 9.6 \times 10^2$ — Since everything is multiplied we can rearrange in any order.

$= (7.25 \times 9.6) \quad \times \quad (10^4 \times 10^2)$ — Rearrange so that the powers are together.

$= 69.6 \times 10^{4+2}$ — Multiply 7.25 and 9.6 on the calculator.

$= 69.6 \times 10^6$

$= 6.96 \times 10^7$ — Write in scientific notation 69.6×10^6
$= 6.96 \times 10^1 \times 10^6$
$= 6.96 \times 10^7$

Example

Evaluate $6.5 \times 10^{-3} \quad \times \quad 9.2 \times 10^{-7}$ — Answer in scientific notation.

$6.5 \times 10^{-3} \quad \times \quad 9.2 \times 10^{-7}$

$= (6.5 \times 9.2) \quad \times \quad (10^{-3} \times 10^{-7})$ — Rearrange.

$= 59.8 \times 10^{-10}$ — Multiply each group.

$= 5.98 \times 10^{-9}$ — Write in scientific notation
$59.8 \times 10^{-10} = 5.98 \times 10^1 \times 10^{-10}$
$= 5.98 \times 10^{-9}$

It is possible to do these calculations on your calculator. In most instances you will need to show your working for these questions but the calculator is a great tool to check that you were on the right track. Recall that when expressing numbers as powers we used the terms index or **exponent** for the power to which a given base was raised. We have used the word index throughout this module, but exponent is the more precise term. It is this word that the calculator is referring to on the EXP key which we will use to evaluate calculations in scientific notation.

Take the first example above.

$$7.25 \times 10^4 \quad \times \quad 9.6 \times 10^2$$

To evaluate this on the calculator, press the following keys.

[7] [•] [2] [5] [EXP] [4] [×] [9] [•] [6] [EXP]

[2] [=]

Now try this on your calculator.

Write down the keystrokes you have used if they are different.

The display should read 69 600 000 which is the same as 6.96×10^{7}

Now try the second example above and see if you come up with the same answer.

$6.5 \times 10^{-3} \quad \times \quad 9.2 \times 10^{-7}$

On your calculator press

[6] [•] [5] [EXP] [(−)] [3] [×] [9] [•] [2] [EXP]

[(−)] [7] [=]

Your answer here will depend on the calculator you are using.

The display could read 0.000 000 005 You will notice that because of the lack of space for digits on the calculator, an error has been introduced. The method by hand has given a more accurate answer. Alternatively the calculator may have given the answer in scientific notation, 5.98×10^{-9} as before.

Activity 3.14

1. Evaluate the following products and answer in scientific notation. Check your answers on the calculator.

   (a) $2.5 \times 10^{2} \quad \times \quad 3.5 \times 10^{3}$

   (b) $6.7 \times 10^{3} \quad \times \quad 5.4 \times 10^{-2}$

   (c) $7.5 \times 10^{-5} \quad \times \quad 1.3 \times 10^{6}$

   (d) $9.25 \times 10^{-2} \quad \times \quad 3.75 \times 10^{-6}$

   (e) $-5.9 \times 10^{3} \quad \times \quad 3.8 \times 10^{-7}$

2. The average adult has about 5 500 millilitres of blood in their body. If each millilitre contains 5×10^{9} red blood cells, how many red blood cells in the average adult body?

3. There are about 4×10^{6} microbes on each square centimetre of your skin, and you have about 2×10^{4} square centimetres of skin. How many microbes would you expect to find on your skin?

4. A light-year, the distance that light travels in one year, is 9.46×10^{12} kilometres. The sun is approximately 2.7×10^{4} light years from the centre of our galaxy. Find this distance in kilometres.

Division

Example

Evaluate $(6.4 \times 10^{5}) \div (3.2 \times 10^{2})$ Write your answer in scientific notation

It is easier to think of this division as a fraction.

$$(6.4 \times 10^{5}) \div (3.2 \times 10^{2}) = \frac{6.4 \times 10^{5}}{3.2 \times 10^{2}}$$

$$= \frac{6.4}{3.2} \times \frac{10^{5}}{10^{2}}$$

$$= 2 \times 10^{5-2}$$ Divide numbers and powers of 10

$$= 2 \times 10^{3}$$

Now try this on your calculator.

Write down the keystrokes you have used if they are different.

6 • 4 EXP 5 ÷ 3 • 2 EXP 2 =

The display should read 2 000

Example

Evaluate $(7.6 \times 10^4) \div (3.9 \times 10^{-2})$ Write your answer in scientific notation.

$$(7.6 \times 10^4) \div (3.9 \times 10^{-2}) = \frac{7.6 \times 10^4}{3.9 \times 10^{-2}}$$

$$= \frac{7.6}{3.9} \times \frac{10^4}{10^{-2}}$$ Divide numbers and powers of 10

$$\approx 1.95 \times 10^{4-(-2)}$$

$$= 1.95 \times 10^6$$

Now try this on your calculator.

Write down the keystrokes you have used if they are different.

7 • 6 EXP 4 ÷ 3 • 9 EXP (−) 2 =

The display should read 1 948 717.949 which is the ordinary form of what we have above.

Activity 3.15

1. Evaluate and write your answers in scientific notation. Check your solutions on the calculator.

   (a) $(5.2 \times 10^3) \div (2.6 \times 10^4)$

   (b) $(7.2 \times 10^5) \div (3 \times 10^{-7})$

   (c) $(9.7 \times 10^{-2}) \div (3.8 \times 10^{-3})$

   (d) $(-5.6 \times 10^9) \div (6.1 \times 10^2)$

   (e) $(7.1 \times 10^2) \div (-8.4 \times 10^7)$

2. The total profits for a company over the last 5 years have been 4.78×10^7 dollars. If profits were the same each year, what was the yearly profit?

3. The earth travels 3.88×10^8 kilometres around the sun each year in approximately 9×10^3 hours. Determine the earth's speed in kilometres per hour, by calculating $(3.88 \times 10^8) \div (9 \times 10^3)$.

4. At one point in *Voyager 1's* journey to Jupiter, its radio waves travelled 2.89×10^8 kilometres to reach earth. These waves travel at a speed of 1.94×10^5 kilometres per second. Determine the number of seconds it took these signals to reach earth by calculating $(2.89 \times 10^8) \div (1.94 \times 10^5)$.

Addition and subtraction

Look at the following expression:

Recall:
Module 2
Section 2.4

$$5.64 \times 10^4 \quad + \quad 3.8 \times 10^4$$

Recall from module 2 that we must do the multiplications before we do the addition. This means that we **cannot** rearrange as before.

What we do have though is the same power in each of the terms. We could use the distributive law to simplify this expression.

$5.64 \times 10^4 + 3.8 \times 10^4$ Taking 10^4 outside the bracket since it is common to both terms

$= (5.64 + 3.8) \times 10^4$

$= 9.44 \times 10^4$

Recall:
Module 2
Section 2.4.1

Now try this on your calculator.

Write down the keystrokes you have used if they are different.

5 • 6 4 EXP 4 + 3 • 8 EXP 4

= The display should read 94 400

Let's look at some other examples.

Example

Evaluate $9.5 \times 10^{-5} - 3.7 \times 10^{-5}$ Write your answer in scientific notation.

$9.5 \times 10^{-5} - 3.7 \times 10^{-5}$
$= (9.5 - 3.7) \times 10^{-5}$ Since 10^{-5} is common to both terms.
$= 5.8 \times 10^{-5}$

Example

Evaluate $3.2 \times 10^2 + 5.7 \times 10^4$ Write your answer in scientific notation.

This time we do not have the same power in both terms so we cannot yet apply the distributive law. There are three methods that you could use to evaluate an expression such as the one above.

Method 1

$3.2 \times 10^2 + 5.7 \times 10^4$	Different powers of 10.
$= 3.2 \times 10^2 + 570 \times 10^2$	Express with the same powers of 10. For 5.7 shift the decimal point 2 places to the right and reduce the power to 10^2.
$= (3.2 + 570) \times 10^2$	
$= 573.2 \times 10^2$	
$= 5.732 \times 10^4$	Rewrite in scientific notation.

Check on your calculator.

Method 2

$3.2 \times 10^2 + 5.7 \times 10^4$	Express with the same powers of 10. For 3.2 shift the decimal point 2 places to the left and increase the power to 10^4.
$= 0.032 \times 10^4 + 5.7 \times 10^4$	
$= (0.032 + 5.7) \times 10^4$	
$= 5.732 \times 10^4$	

Which is the same answer as we calculated before.

Method 3

This final method is only suitable when dealing with numbers with few digits. We can convert each of the numbers to ordinary form and then calculate.

$3.2 \times 10^2 + 5.7 \times 10^4$
$= 320 + 57\,000$
$= 57\,320$
$= 5.732 \times 10^4$

Activity

Activity 3.16

1. Evaluate the following, writing your answer in scientific notation. Check your answers with the calculator.

   (a) $5.9 \times 10^4 + 2.4 \times 10^4$ (d) $4.5 \times 10^3 + 9.6 \times 10^4$

   (b) $7.8 \times 10^{-2} - 3.4 \times 10^{-2}$ (e) $7.6 \times 10^8 - 9.5 \times 10^6$

   (c) $6.1 \times 10^3 - 9.5 \times 10^3$ (f) $1.3 \times 10^{-6} + 8.3 \times 10^{-4}$

2. A part of the brain called the neocortex contains cells called neurons. The numbers of neurons for three different mammals are listed below.

Mammal	**Number of neurons**
Human	3×10^{10}
Gorilla	7.5×10^9
Cat	6.5×10^7

(a) Which mammal has the largest number of neurons?

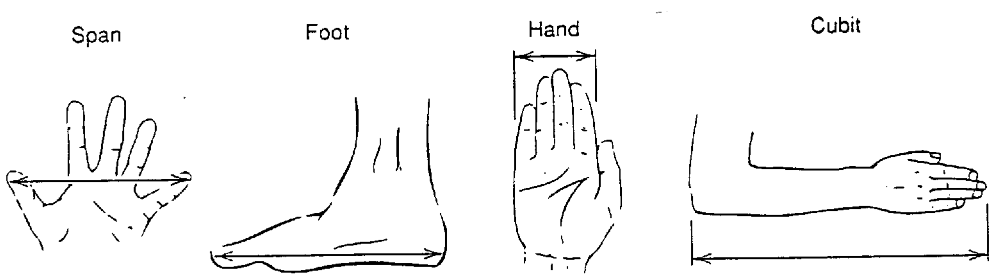
(b) How many more neurons does the neocortex of a human brain have than the neocortex of the brain of the gorilla? (Hint: you will need to subtract)

(c) How many more neurons does the neocortex of a gorilla brain have than the neocortex of the brain of the cat?

3.3.2 The Metric system

In our everyday world we are surrounded by measurements. From the nutritional information on the cereal box at breakfast, to setting the alarm at night. We buy various amounts of meat and vegetables for meals. We travel kilometres in our car and fill it with litres of petrol.

Many of the first units of measurement were parts of the body. Early Babylonian and Egyptian records indicate that the **hand**, the **span**, the **foot**, and the **cubit** were all units of measurement. The hand was a basic unit of measurement used by most ancient civilisations and is the basis of the unit that is used today to measure the height of horses (now standardised as 4 inches or 0.0106 metres).



Since everybody has different measurements for these body parts it was difficult for accurate comparisons to be made. A more formalised system of measurement needed to be developed. This happened in different parts of the world, leading to differing systems of measurement. We have seen two of these in Australia, where we previously used the Imperial system of measures (inches, feet, yards, etc.) and now the **Metric system** of measurement.

Following the introduction of decimal currency in 1966, a program of gradual conversion to the metric system of weights and measures was implemented in Australia. This new system, developed by the French in the 18th century, was based on the decimal system of numbers with the aim of making calculation less complicated.

Here is a table of some of the common units that you might use in your everyday life, together with their symbols.

Measurement	Unit	Symbol
Length	metre	m
Mass	gram	g
Volume	litre	L
Time	second	s

In our everyday life, a convenient unit for measuring such things as the length and width of a block of land, or the height of a building would be the metre.

Think of the distance between Toowoomba and Charleville or the thickness of a 20 cent coin. It is not convenient to write either of these in metres. One is too large at 600 000 metres and one is too small at 0.002 metres.

For this reason a system of prefixes based on the power 10^3 (that is 1 000) has been developed for multiples of units and for parts of units.

You would be familiar with the **kilo**metre which is a much more convenient measure to talk about the distance between Toowoomba and Charleville, a distance of 600 kilometres.

You are probably also familiar with the **milli**metre which would give a more meaningful answer for the thickness of a 20 cent coin at 2 millimetres.

In general it is best to choose a prefix that allows the number to be expressed in the range 1 to 999.

The table below shows the prefixes that are used with metric units.

Prefix	Factor	Numerical factor	Symbol
giga	10^9	1 000 000 000	G
mega	10^6	1 000 000	M
kilo	10^3	1 000	k
	$10^0 = 1$	1	

Prefix	Factor	Numerical factor	Symbol
milli	10^{-3}	0.001	m
micro	10^{-6}	0.000 001	μ*
nano	10^{-9}	0.000 000 001	n
pico	10^{-12}	0.000 000 000 001	p
centi**	10^{-2}	0.01	c

Note: * The small Greek letter mu (pronounced mew as in new)

** Although centi is not really part of the system because it is not a power of 10^3, it is commonly used so we will include it here.

The examples below show how this system of using symbols for units operates.

Examples

96 millilitres = 96 mL

5.2 nanoseconds = 5.2 ns

45 micrograms = 45 μg

Activity 3.17

Write the measurements given here in symbols

(a) 50 milligrams

(b) 65 kilograms

(c) 5 microlitres

(d) 7.2 megalitres

(e) 9 picoseconds

Some of the other commonly used units that you will need to be familiar with are:

Unit	Symbol	Description
tonne	t	another name for megagram 1 000 000 grams
hectare	ha	a measure of area
kilojoule	kJ	a measure of the energy derived from food

Converting between units

Let's look back to the distance between Toowoomba and Charleville, 600 000 metres which we wanted to express in kilometres.

That is, convert 600 000 metres to kilometres.

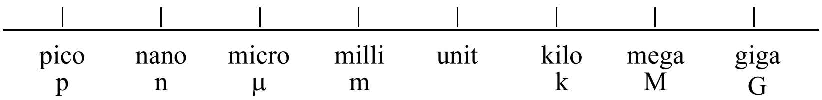
Step 1

Think about the unit you are in (metres) and the unit you are converting to (kilometres). If you are converting from a **larger to a smaller** unit you **multiply**; from a **smaller to a larger** unit you must **divide.**

In our case above we are converting from a smaller to a larger unit so we will divide.

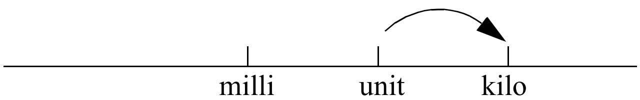
Step 2

We must now decide what to divide by. The metric system is designed in multiples of 1 000. Think of the metric system in the figure below.



Each 'jump' along this line represents 1 000. If we are moving to a larger unit (to the right) we divide by 1 000 for each 'jump' we take. If we are converting to a smaller unit (moving to the left) we will multiply by 1 000 for each 'jump' we take.

So for our conversion 600 000 metres to kilometres, we have taken one 'jump' to the right so we will divide by 1 000



Step 3

So 600 000 metres

$= 600\,000 \div 1\,000$ kilometres

$= 600$ kilometres

$= 600$ km

Recall that dividing by 1 000 is the same as multiplying by 10^{-3}

So 600 000 metres

$= 600\,000 \div 1\,000$ kilometres

$= 600\,000 \times 10^{-3}$ Multiplying by 10^{-3} means move the decimal point to the left 3 places.

$= 600$ km

Look at your answer and see that it looks reasonable. You were moving from a smaller to a larger unit so you had to divide. Dividing by whole numbers means that you should have ended up with a smaller number than the one you started with. Is this the case for this answer?

Yes, it looks reasonable.

Let's look at another example.

Example

Each day, world wide, honey bees collect about 2 700 tonnes of honey. How much would this be in grams.

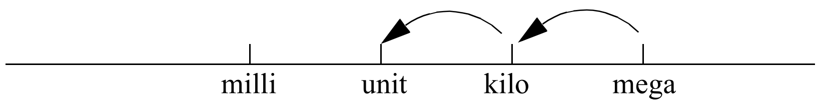
That is, convert 2 700 tonnes to grams.

Step 1

We are converting from a larger to a smaller unit so we will multiply.

Step 2

Remember that tonnes is the same as megagrams.



We have moved 2 'jumps' to the left so we will multiply by 1 000 twice.

Step 3

$$\begin{aligned} 2\,700 \text{ t} &= 2\,700 \times 1\,000 \times 1\,000 \text{ grams} \\ &= 2\,700 \times 10^3 \times 10^3 \text{ grams} \\ &= 2\,700 \times 10^6 \text{ grams} \\ &= 2\,700\,000\,000 \text{ grams} \\ \text{or} \quad &= 2.7 \times 10^9 \text{ g} \end{aligned}$$

That's a lot of honey each day!!!!

Look at your answer and see that it looks reasonable. You were moving from a larger to a smaller unit so you had to multiply. Multiplying by whole numbers means that you should have ended up with a larger number than the one you started with. Is this the case for this answer?

Yes, the answer looks reasonable.

Activity 3.18

1. Convert the following:

(a) 5 600 grams into kilograms
(b) 0.032 litres into millilitres
(c) 0.000 000 25 grams into nanograms
(d) 35 000 milligrams into grams
(e) 70 250 centimetres into metres
(f) 1 600 kilograms into tonnes
(g) 0.021 megalitres into millilitres
(h) 206 micrometres into metres

2. Choose the most realistic measure for each of the following.

(a) Length of a car:500 mm	500 cm	500 m
(b) Weight of a person:75 mg	75 g	75 kg
(c) Volume of a car's petrol tank:48 mL	48 L	48 kL
(d) Weight of a toothpick:450 mg	450 g	450 kg
(e) Height of the Sydney Harbour Bridge above water:135 cm	135 m	135 km
(f) Amount of blood in the human body:5.5 mL	5.5 L	5.5 kL

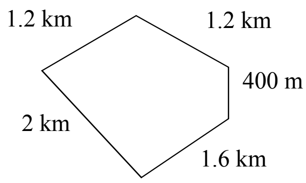
Calculating with metric units

Rather than simply converting from one unit to another your every day need for metric measurements may be a little more involved.

Working with Length

Example

A farmer wishes to renew the fencing around the boundary of the property. The following measurements were obtained (not drawn to scale):



To determine the total length of fencing required we will need to have all measurements in the **same units**. We could convert everything to kilometres or everything to metres. Since you would probably order the fencing material in metres we will use this conversion.

Distance around the property = 1.2 km + 1.2 km + 2 km + 1.6 km + 400 m

= 1 200 m + 1 200 m + 2 000 m + 1 600 m + 400 m

= (1 200 + 1 200 + 2 000 + 1 600 + 400) m

= 6 400 m

This distance around a figure is often called its **perimeter**. In our example the perimeter of the property measures 6 400 metres. This is the amount of fencing that is required.

Example

If 235 centimetres of fabric is required to make a laboratory coat, how much fabric is required to make six of these coats.

To make six coats we require six times as much fabric.

$235 \text{ cm} \times 6 = 1\,410 \text{ cm}$

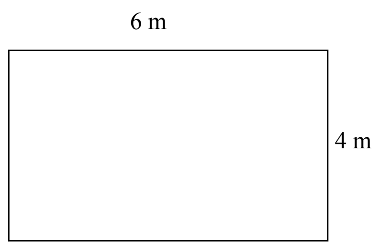
It would be more convenient to express this in metres.

The amount of fabric required is 14.1 metres.

It is also possible to multiply two numbers that both involve lengths.

Example

A kitchen floor measures 4 metres by 6 metres as shown below.

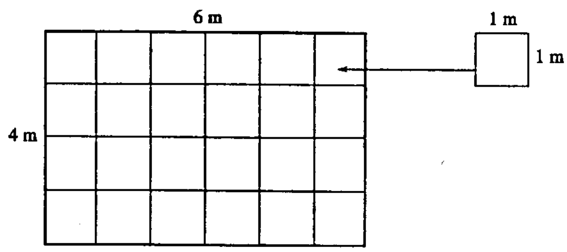


If we needed to order tiles to cover the kitchen floor we need to know the **area** of floor that is to be covered. To do this for a room shaped as in the diagram above (we call this a **rectangle**) we would multiply the length by the width.

Area of the kitchen floor

$= 4 \text{ m} \times 6 \text{ m}$	The units are the same so we can multiply.
$= 4 \times 6 \times \text{m} \times \text{m}$	Note that we must multiply the units as well as the numbers.
$= 24 \text{ m}^2$	We say this as *24 square metres.*

To think of the meaning of a square metre, think of something measuring 1 metre by 1 metre. You would need 24 of these to cover the kitchen floor.



You would need to order 24 square metres of tiles to cover the kitchen floor.

We can also divide lengths which both have units attached.

Example

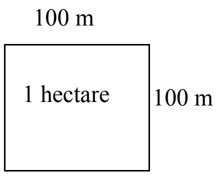
A small crop farmer owns 93.5 ha of land. If the farmer purchased 1.5 km^2 of the neighbour's land, what area, in hectares, is the farm now?

Firstly, let's look more closely at the meaning of a hectare.

A **hectare** is a measure of area. A piece of land 100 m by 100 m gives an **area** of 1 hectare.

That is, 1 ha = 100 m × 100 m

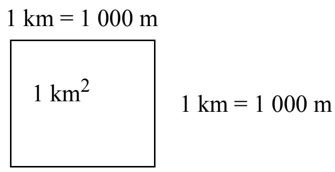
1 ha = 10 000 m^2



The farmer has added a piece of land measured in square kilometres. One square kilometre is a piece of land 1 km by 1 km which we could express in metres as 1 000 m by 1 000 m

1 km^2 = 1 000 m × 1 000 m

= 1 000 000 m^2



How many hectares are there in a square kilometre?

Since 1 km^2 = 1 000 000 m^2

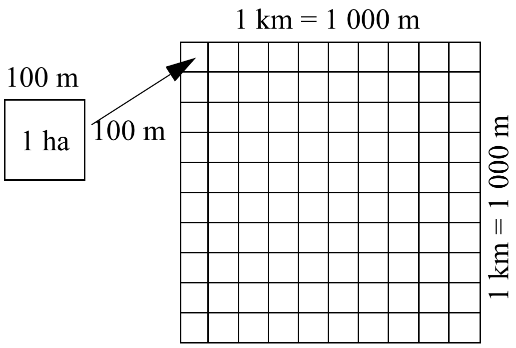
and 1 ha = 10 000 m^2

1 000 000 m^2 ÷ 10 000 m^2

= 100

So, there are 100 hectares in one square kilometre.

Let's picture this.



You can see that ten hectares must fit along each row, giving us 10 rows of 10 hectares, a total of 100 hectares.

Back to the question.

The farmer owned 93.5 hectares and bought a further 1.5 km^2. We cannot add these until they are both in the same units.

$1.5\ km^2 = 1.5 \times 100$ Recall that there are 100 hectares in each square kilometre.

$= 150$ ha

The farmer now has 93.5 ha + 150 ha = (93.5 + 150) ha = 243.5 ha

This was rather a large purchase of land!!

Example

A hospital patient's waist measures 85 cm. Suppose a roll of gauze contains 9.4 metres. Approximately how many times could the gauze be completely wrapped around the patient's waist?

We need to divide 9.4 m by 85 cm to find out how many times around the waist measurement the gauze will fit.

That is 9.4 m ÷ 85 cm

Again we cannot do this calculation while we have differing units. We could change to metres or centimetres, so this time let's try centimetres so as not to have any decimal points.

$9.4\ m \div 85\ cm$	The units are different so we must make the same.
$= 940\ cm \div 85\ cm$	Estimate $900 \div 90 = 10$
$= \dfrac{940\ \cancel{cm}}{85\ \cancel{cm}}$	We must cancel the cm on the top and bottom.
≈ 11.06	Note that no units are left.

So the gauze will go around the patient's waist about 11 times.

Working with Mass

We often use the word weight as meaning the same thing as mass, but scientists distinguish between the two. We can think of mass as the amount of matter that makes up an object. Weight, on the other hand, is the force that gravity exerts on an object. This is why astronauts appear 'weightless' in space. The amount of matter that makes up the astronauts (mass) hasn't changed, but the effect of gravity is not as great. At sea level on earth, mass and weight are essentially equal. In these modules we will keep our feet on the ground and consider mass and weight to be the same.

Working with mass is much the same as working with length except that there is no meaning associated with multiplying together two masses.

Example

To make one batch of tomato relish requires 3.25 kg of brown sugar. How much brown sugar would be needed for four batches of tomato relish?

We need four times as much sugar.

$3.25\ kg \times 4 = 13\ kg$

We would need to have 13 kg of brown sugar to make four batches of tomato relish.

Example

A major pharmaceutical company has been able to get a bulk order of 5 kilograms of a particular type of tablet. Each tablet weighs 2.5 grams, so how many tablets did the pharmaceutical company receive?

To find the answer to this problem we must divide.

5 kg ÷ 2.5 g	
= 5 000 g ÷ 2.5 g	Change to the same units.
= 2 000	Remember that the g 's will divide out.

The pharmaceutical company has received 2 000 tablets.

We can calculate with most of the units in the metric system in much the same manner as the examples we have looked at.

Following is an activity that shows some other applications of metric measures.

Activity 3.19

1. To get to University a student walks 1 450 metres to the bus stop and then travels 7.4 kilometres by bus to the University. What is the total distance travelled by the student?
2. Colin ran 2 750 metres every morning before breakfast. How many kilometres did he run in a week?
3. Water in the flooding creek rose, 1.6 m in the first hour, 1.16 m in the second hour and 65 cm in the third hour. How far did the creek rise in the first three hours?
4. Tom's father is 1.85 m tall. If Tom is 92 cm shorter than his father, how tall is he?
5.
   (a) A doctor orders Keflex 1 g every 6 hours. If the tablets are 500 mg each, what is the correct dose?

   (b) A patient if to receive Eltroxin at 0.1 mg daily. The tablets are labelled 0.05 mg each. What is the correct dosage?
6. A land developer divided his land into 18 blocks each 550 m^2. If he also had to allow 920 m^2 for roads and footpaths, how many hectares did he develop?
7. A wealthy farming couple gave 750 hectares of land to each of their five children. How many square kilometres does the couple now own if they originally owned 150 km^2?
8. (a) A crate of bread loaves weighs 49.9 kg. If the crate weighs 2.3 kg, how many 680 g loaves of bread does the crate contain?

   (b) A bakery supplies 31.025 tonnes of bread in a year. What number of 680 g loaves are baked each day (assuming they bake the same number of loaves every day of the year)?

9. One of the largest swarms of locusts ever seen was estimated to contain 4×10^{10} insects. A locust is capable of consuming as much as 15 grams of grain in a week.

    (a) How many grams of grain could be consumed in a week by this swarm of locusts? (Write your answer in scientific notation)

    (b) Express your answer in (a) in tonnes.

You should now be ready to attempt questions 1, 2, 3, 4 and 5 of Assignment 2A (see your Introductory Book for details). If you have any questions, please refer them to your course tutor.

3.4 A taste of things to come

1. The table below gives the nutritional information for Kellogg's Special K.

NUTRITION INFORMATION

Servings per package – 20
Serving size – 30g (1 cup)

	PER 30g SERVE	PER 30g WITH ½ CUP SKIM MILK	PER 100g
ENERGY	461kJ	649kJ	1536kJ
	(114 Cal)	(157 Cal)	(379 Cal)
PROTEIN	6.4g	10.9g	21.4g
FAT	0.2g	0.3g	0.5g
CARBOHYDRATE			
- TOTAL	21.6g	28.0g	72.1g
- SUGARS	4.3g	8.8g	14.4g
DIETARY FIBRE	0.9g	0.9g	2.9g
SODIUM	142mg	212mg	475mg
POTASSIUM	59mg	276mg	196mg
THIAMINE (VIT B_1)	0.28mg	0.34mg	0.93mg
(% Aust R.D.I.*)	(25%)	(31%)	(84%)
RIBOFLAVIN (VIT B_2)	0.4mg	0.6mg	1.3mg
(% Aust R.D.I.*)	(25%)	(37%)	(81%)
NIACIN	2.8mg	2.8mg	9.3mg
(% Aust R.D.I.*)	(25%)	(25%)	(84%)
CALCIUM	175mg	330mg	583mg
(% Aust R.D.I.*)	(25%)	(44%)	(83%)
IRON	5.0mg	5.0mg	16.6mg
(% Aust R.D.I.*)	(50%)	(50%)	(166%)

(*Recommended Dietary Intake)

INGREDIENTS: WHITE RICE, WHEAT GLUTEN, WHEAT, ICING SUGAR MIXTURE, SUGAR, WHEAT FLOUR, WHEATGERM, SALT, MALT EXTRACT, VEGETABLE OILS (SOYBEAN AND/OR COTTONSEED AND/OR SUNFLOWER), VITAMINS (THIAMINE, RIBOFLAVIN, NIACIN), MINERALS (CALCIUM, IRON).

Kellogg's

THE BEST TO YOU EACH MORNING

(a) How many micrograms of Thiamine are contained in a 30 g serve of Special K with $\frac{1}{2}$ cup of skim milk?

(b) For breakfast, a truck driver eats an amount of Special K equivalent to 4 serves, each with $\frac{1}{2}$ cup skim milk. How many grams of Sodium are consumed by the truck driver?

(c) How many milligrams of Calcium are contained in $\frac{1}{2}$ cup of skim milk?

(d) How many grams of Special K are required to provide 3.94 megajoules of energy?

2. A hospital storage container holds 9.750 litres of ethyl alcohol.

   (a) If 150 mL, 200 mL, 100 mL, 1 000 mL and 2.5 L are removed from the container, how many litres of alcohol remain?

   (b) On one morning the 9.750 litre container had 4.250 litres in it. If 3.125 litres are added, what is the total?

   (c) How much alcohol could be stored in 7 of these containers?

   (d) If the container is $\frac{2}{3}$ full, how much alcohol is left?

3. Studying courses in psychology or history you may refer to data on ethnic backgrounds as presented by W S Sherman in the book ***Australian Organisational Behaviour***. In Australia today we have approximately 140 different ethnic backgrounds, 90 different languages spoken at home and 40 different religions practised. The main ethnic backgrounds are set out in the table below.

Ethnic Background	**Number of Australians**
British	4.7 million
Irish	3.5 million
German/Austrian	1.2 million
Italian	1 million
Greek	6.5×10^5
Maltese	4×10^5
Yugoslav	2.5×10^5
Dutch	2×10^5
Aboriginal	1.6×10^5
Spanish Speaking	1.5×10^5
Arabic Speaking	1.5×10^5
Polish	1×10^5

(Source: Ainsworth, WM & Willis, QF 1985, *Australian organisational behaviour*: *readings,* 2nd edn, Macmillan, Melbourne)

In order to write a paragraph on Australia's ethnic background you might consider some of the following types of questions.

(a) Write the first four numbers as ordinary numbers and then convert to scientific notation.

(b) How many more people come from a British background than from a Greek background?

(c) Find the total number of people from the first five backgrounds in the table .

Now write a few sentences about the ethnic backgrounds of Australians.

3.5 Post-test

1. Write $1.4 \times 1.4 \times 1.4$ in power notation.

2. Simplify the following, expressing your answers with no negative indices.

   (a) $5^4 \times 5^8 \times 5^{-3}$ (c) $12^{27} \div 12^5$

   (b) $(-2)^5 \times (-2)^{-6}$ (d) $4^{-3} \times (4^2)^3 \div 4^{-1}$

3. Evaluate the following on your calculator. Round your answers to two decimal places if necessary.

   (a) $243^{\frac{2}{5}}$ (d) $\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^5$

   (b) $196^{\frac{1}{2}}$ (e) $4^{-3} \times (4^2)^3 + 4^{-1} \times 4^5$

   (c) $3^5 + 3^4$

4. The recipe for a party punch has the following ingredients.

   3.5 L unsweetened pineapple juice
   400 mL orange juice
   300 mL lemon juice
   4 L ginger ale
   2.5 L soda water
   500 mL mashed strawberries
   800 mL mixture of sugar, water and mint leaves.

   (a) How much punch will the recipe make in litres?

   (b) If 30 people are at the party, how much punch will there be for each person?

   (c) This recipe was used at the local fete to raise money. Each drink of 80 mL cost 25 cents. What was the profit on the sale of punch if the ingredients cost $12.50?

5. The following table gives the diameter, and mass in terms of the earth's mass, of the planets.

Planet	**Diameter* (km)**	**Mass (in terms of the Earth's mass)**
Mercury	5×10^3	0.04
Venus	1.24×10^4	0.82
Earth	1.27×10^4	1.00
Mars	6.87×10^3	0.11
Jupiter	1.4×10^5	318.3
Saturn	1.14×10^5	95.3
Uranus	5.1×10^4	14.7
Neptune	5×10^4	17.3
Pluto	1.27×10^4	0.0021

* Diameter is the distance from one side of the planet to the other through the centre.

(a) (i) List the planets in order of size by diameter.

(ii) Convert each diameter to ordinary form.

(b) (i) Arrange the planets in order of size by mass.

(ii) The mass of the earth is 5.983×10^{24} kg. Determine the mass of Venus, Saturn and Pluto.

(c) The sun has a diameter of 1.39×10^6 km and has a mass 3.29×10^6 times the mass of the Earth.

(i) What is the mass of the Sun?

(ii) What is the difference between the Earth's diameter and the Sun's diameter?

(Adapted from Shield & Wallace, *Investigating Maths 10*)

3.6 Solutions

Solutions to activities

Activity 3.1

1. (a) $2^3 = 8$
   (b) $3^2 = 9$
   (c) $(0.5)^4 = 0.0625$
   (d) $6^1 = 6$
   (e) $(1.9)^5 = 24.760\,99$
   (f) $(–4)^2 = 16$
   (g) $(–6)^3 = –216$

2. $\left(\frac{1}{2}\right)^{10}$

3. After three generations there would have been 7^3 people. This gives 343 people.

4. There will be 3 × 3 × 3 pieces of cake. That is 3^3 which equals 27 pieces of cake.

5. As an ordinary number 2^{22} is 4 194 304.

Activity 3.2

1. (a) $5^2 \times 5^7$
   $= 5^{2+7}$
   $= 5^9$

   (b) $3^8 \times 3^2 \times 3^5$
   $= 3^{8+2+5}$
   $= 3^{15}$

   (c) $(–4)^3 \times (–4)^2 \times (–4)^5$
   $= (–4)^{3+2+5}$
   $= (–4)^{10}$

   (d) $(2.6)^7 \times (2.6)^{12} \times (2.6)$
   $= (2.6)^{7+12+1}$
   $= (2.6)^{20}$

   (e) $(–7)^3 \times 6^2 \times (–7)^2 \times 6^2$
   $= (–7)^3 \times (–7)^2 \times 6^2 \times 6^2$
   $= (–7)^{3+2} \times 6^{2+2}$
   $= (–7)^5 \times 6^4$

   (f) $\left(\frac{1}{2}\right)^3 \times (–4)^3 \times \left(\frac{1}{2}\right)^5 \times (–4)^2$

   $= \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^5 \times (–4)^3 \times (–4)^2$

   $= \left(\frac{1}{2}\right)^{3+5} \times (–4)^{3+2}$

   $= \left(\frac{1}{2}\right)^8 \times (–4)^5$

(g) $5^8 \times (0.3)^2 \times (0.3)^5 \times 5^2 \times (0.3)^7$
$= 5^8 \times 5^2 \times (0.3)^2 \times (0.3)^5 \times (0.3)^7$
$= 5^{8+2} \times (0.3)^{2+5+7}$
$= 5^{10} \times (0.3)^{14}$

(h) $-4^3 \times {}^-4^2$
$= +4^{3+2}$ Two negatives multiplied together give a positive.
$= 4^5$

(i) $-9^3 \times {}^-9^2 \times {}^-9^1$
$= 9^{3+2} \times {}^-9^1$ Two negatives multiplied together give a positive.
$= -9^{3+2+1}$ A negative times a positive gives a negative.
$= -9^6$

2. (a) $5^2 \times 5^7 = 5^9 = 1\,953\,125$

(b) $3^8 \times 3^2 \times 3^5 = 3^{15} = 14\,348\,907$

(c) $(-4)^3 \times (-4)^2 \times (-4)^5 = (-4)^{10} = 1\,048\,576$

(d) $(2.6)^7 \times (2.6)^{12} \times (2.6) = (2.6)^{20} = 199\,281\,489$

(e) $(-7)^3 \times 6^2 \times (-7)^5 \times 6^2 = (-7)^5 \times 6^4 = -21\,781\,872$

(f) $\left(\frac{1}{2}\right)^3 \times (-4)^3 \times \left(\frac{1}{2}\right)^5 \times (-4)^2 = \left(\frac{1}{2}\right)^8 \times (-4)^5 = -4$

(g) $5^8 \times (0.3)^2 \times (0.3)^5 \times 5^2 \times (0.3)^7 = 5^{10} \times (0.3)^{14} \approx 0.467\,086\,816$

(h) $-4^3 \times {}^-4^2 = 4^5 = 1\,024$

(i) $-9^3 \times {}^-9^2 \times {}^-9^1 = -9^6 = -531\,441$

3. There would be approximately $10^{25} \times 10^{21} = 10^{46}$ molecules of water in the Atlantic Ocean.

4. There would be $10^{11} \times 10^9 = 10^{20}$ stars in all the galaxies.

5. The x-ray would have a frequency of $10^8 \times 10^{11} = 10^{19}$ cycles per second.

Activity 3.3

1. (a) $5^7 \div 5^2$
$= 5^{7-2}$
$= 5^5$

(b) $3^8 \div 3^2$
$= 3^{8-2}$
$= 3^6$

(c) $(-4)^3 \div (-4)^2$
$= (-4)^{3-2}$
$= (-4)^1$
$= -4$

(d) $(2.6)^{12} \div (2.6)^7$
$= (2.6)^{12-7}$
$= (2.6)^5$

(e) $(-7)^3 \div (-7)$
$= (-7)^{3-1}$
$= (-7)^2$

(f) $\left(\frac{1}{2}\right)^{13} \div \left(\frac{1}{2}\right)^5$
$= \left(\frac{1}{2}\right)^{13-5}$
$= \left(\frac{1}{2}\right)^8$

(g) $-5^7 \div {}^-5^2$
$= +5^{7-2}$
$= 5^5$

(h) $\left(\frac{1}{4}\right)^5 \div \left(\frac{1}{4}\right)^3$
$= \left(\frac{1}{4}\right)^{5-3}$
$= \left(\frac{1}{4}\right)^2$

(i) $(1.7)^{10} \div (1.7)^6 \div (1.7)^2$
$= (1.7)^{10-6-2}$
$= (1.7)^2$

2. (a) $5^7 \div 5^2 = 5^5 = 3\ 125$

(b) $3^8 \div 3^2 = 3^6 = 729$

(c) $(-4)^3 \div (-4)^2 = -4$

(d) $(2.6)^{12} \div (2.6)^7 = (2.6)^5 = 118.813\ 76$

(e) $(-7)^3 \div (-7) = (-7)^2 = 49$

(f) $\left(\frac{1}{2}\right)^{13} \div \left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^8 \approx 0.003\ 906\ 25$

(g) $-5^7 \div {}^-5^2 = 5^5 = 3\ 125$

(h) $\left(\frac{1}{4}\right)^5 \div \left(\frac{1}{4}\right)^3 = \left(\frac{1}{4}\right)^2 = 0.062\ 5$

(i) $(1.7)^{10} \div (1.7)^6 \div (1.7)^2 = (1.7)^2 = 2.89$

3. Gamma frequency is $10^{21} \div 10^8 = 10^{13}$ times greater than the radio frequency.

Activity 3.4

1. (a) $8^{-2} = \frac{1}{8^2}$

(b) $6^{-2} = \frac{1}{6^2}$

(c) $7^{-1} = \frac{1}{7^1} = \frac{1}{7}$

(d) $\frac{1}{5^{-2}} = 5^2$

(e) $\frac{3}{4^{-2}} = 3 \times 4^2$

(f) $\frac{2^{-5}}{3^{-2}} = \frac{3^2}{2^5}$

(g) $\frac{4^{-2}}{3} = \frac{1}{3 \times 4^2}$

(h) $2 \times 4^{-3} = \frac{2}{4^3}$

(i) $-3^{-2} = \frac{-1}{3^2}$

(j) $(-3)^{-2} = \frac{1}{(-3)^2}$

2. (a) 0.015625

(b) $0.02\dot{7}$

(c) $0.\dot{1}4285\dot{7}$

(d) 25

(e) 48

(f) 0.28125

(g) $0.0208\dot{3}$

(h) 0.03125

(i) $-0.\dot{1}$

(j) $0.\dot{1}$

3. (a) $\frac{-3}{6^{-5}} = -3 \times 6^5$

(b) $\frac{7}{5^{-2}} = 7 \times 5^2$

(c) $\frac{2 \times 4^2}{3^{-4}} = 2 \times 4^2 \times 3^4$

(d) $-3^{-4} = \frac{-1}{3^4}$

(e) $-2 \times 5^{-3} = \frac{-2}{5^3}$

(f) $\frac{2^{-5}}{5^{-3}} = \frac{5^3}{2^5}$

(g) $\frac{2^4}{3^{-6}} = 2^4 \times 3^6$

(h) $\frac{-3^5}{(-2)^{-4}} = -3^5 \times (-2)^4$

(i) $\frac{(-4)^{-2}}{2} = \frac{1}{2 \times (-4)^2}$

(j) $\frac{-5^{-4} \times 3^4}{2^3} = \frac{-3^4}{2^3 \times 5^4}$

4. (a) –23 328

(b) 175

(c) 2592

(d) $-0.\dot{0}1234567\dot{9}$

(e) –0.016

(f) 3.90625

(g) 11 664

(h) –3 888

(i) 0.03125

(j) –0.0162

Activity 3.5

(a) $5^0 = 1$

(b) $256^0 = 1$

(c) $4 \times 65^0 = 4 \times 1 = 4$

(d) $3^2 \times 55^0 = 9 \times 1 = 9$

(e) $(-4)^0 = 1$

(f) $-4^0 = -1 \times 4^0 = -1 \times 1 = -1$

Activity 3.6

1. (a) $25^{\frac{1}{2}} = \sqrt{25}$

(b) $27^{\frac{1}{3}} = \sqrt[3]{27}$

(c) $81^{0.25} = 81^{\frac{1}{4}} = \sqrt[4]{81}$

(d) $7\,776^{\frac{1}{5}} = \sqrt[5]{7\,776}$

(e) $100^{\frac{-1}{2}} = \dfrac{1}{100^{\frac{1}{2}}} = \dfrac{1}{\sqrt{100}}$

2. (a) $25^{\frac{1}{2}} = 5$

(b) $27^{\frac{1}{3}} = 3$

(c) $81^{0.25} = 3$

(d) $7\,776^{\frac{1}{5}} = 6$

(e) $100^{\frac{-1}{2}} = \dfrac{1}{10} = 0.1$

(f) $16^{\frac{1}{2}} \times 49^{\frac{1}{2}} = 4 \times 7 = 28$

(g) $36^{\frac{1}{3}} \times 54^{\frac{1}{2}} \approx 24.264$

(h) $63^{\frac{1}{5}} + 125^{\frac{1}{4}} \approx 5.634$

(i) $123^{\frac{1}{2}} \div 567^{\frac{1}{6}} \approx 3.855$

(j) $534^{\frac{1}{3}} - 34^{\frac{1}{4}} \approx 5.698$

Activity 3.7

1. (a) $(2^3)^5 = 2^{15}$

(b) $(3^4)^2 = 3^8$

(c) $(5^6)^2 = 5^{12}$

(d) $((-2)^3)^4 = (-2)^{12}$

(e) $((-5)^3)^3 = (-5)^9$

(f) $((1.2)^3)^0 = (1.2)^0 = 1$

(g) $\left(\left(\frac{1}{4}\right)^4\right)^3 = \left(\frac{1}{4}\right)^{12}$

(h) $\left(6^{\frac{3}{4}}\right)^3 = 6^{\frac{9}{4}}$

2. (a) $(2^3)^5 = 2^{15} = 32\,768$

(b) $(3^4)^2 = 3^8 = 6\,561$

(e) $((-5)^3)^3 = (-5)^9 = -1\,953\,125$

(f) $((1.2)^3)^0 = (1.2)^0 = 1$

(c) $(5^6)^2 = 5^{12} = 244\,140\,625$

(d) $((-2)^3)^4 = (-2)^{12} = 4\,096$

(g) $\left(\left(\frac{1}{4}\right)^4\right)^3 = \left(\frac{1}{4}\right)^{12} \approx 0.000\,000\,059$

(h) $\left(6^{\frac{3}{4}}\right)^3 = 6^{\frac{9}{4}} \approx 56.343$

3. (a) $64^{\frac{4}{3}} = 256$

(b) $8^{\frac{4}{3}} \times 16^{\frac{5}{4}} = 16 \times 32 = 512$

(c) $27^{\frac{7}{3}} - 36^{\frac{3}{2}} = 2\,187 - 216 = 1\,971$

(d) $4^{2.5} + 81^{2.5} = 32 + 59\,049 = 59\,081$

(e) $\dfrac{7^3 \times 5^4}{(35)^2} = \dfrac{343 \times 625}{1\,225} = \dfrac{214\,375}{1\,225} = 175$

(f) $\dfrac{(121)^{\frac{5}{2}} \times (49)^{\frac{3}{2}}}{11^3 \times 7^2} = \dfrac{161\,051 \times 343}{1\,331 \times 49} = \dfrac{55\,240\,493}{65\,219} = 847$

(g) $(-27)^{\frac{1}{3}} + \sqrt[4]{81} = -3 + 3 = 0$

(h) $\sqrt[5]{7\,776} \div {}^{-}4^{\frac{1}{2}} = 6 \div {}^{-}2 = -3$

Activity 3.8

1. (a) $(9 \times 5)^2 = 9^2 \times 5^2$

(b) $(7 \times 6)^2 = 7^2 \times 6^2$

(c) $(8 \times 5)^3 = 8^3 \times 5^3$

(d) $6^5 \times 3^5 = (6 \times 3)^5$

(e) $12^6 \times 2^6 = (12 \times 2)^6$

(f) $7^3 \times 4^3 = (7 \times 4)^3$

2. (a) $(9 \times 5)^2 = 45^2 = 2\,025$

(b) $(7 \times 6)^2 = 42^2 = 1\,764$

(c) $(8 \times 5)^3 = 40^3 = 64\,000$

(d) $6^5 \times 3^5 = 7\,776 \times 243 = 1\,889\,568$

(e) $12^6 \times 2^6 = 2\,985\,984 \times 64 = 191\,102\,976$

(f) $7^3 \times 4^3 = 343 \times 64 = 21\,952$

Activity 3.9

1. (a) $5^{\frac{4}{3}} \div 5^{\frac{2}{3}}$

$= 5^{\frac{4}{3} - \frac{2}{3}}$

$= 5^{\frac{2}{3}}$

(b) $(3^0)^2$

$= 3^0$

$= 1$

(c) $3^{-1} \times 4^0$

$= \frac{1}{3} \times 1$

$= \frac{1}{3}$

(d) $2^6 \times 2^2 \div 2^7$

$= 2^{6+2-7}$

$= 2^1$

$= 2$

(e) $\frac{3^4 \times 3}{3^5}$

$= \frac{3^{4+1}}{3^5}$

$= \frac{3^5}{3^5}$

$= 1$

(f) $\frac{3^{-2} \times 3^0}{3^4}$

$= \frac{3^{-2+0}}{3^4}$

$= \frac{3^{-2}}{3^4}$

$= 3^{-2-4}$

$= 3^{-6}$

$= \frac{1}{3^6}$

(g) $\frac{5^{-2} \div 5^3}{5^4}$

$= \frac{5^{-2-3}}{5^4}$

$= \frac{5^{-5}}{5^4}$

$= 5^{-5-4}$

$= 5^{-9}$

$= \frac{1}{5^9}$

(h) $\frac{2^4 \times 2^5}{(2^2)^{-2} \times 2^{-3}}$

$= \frac{2^9}{2^{-4} \times 2^{-3}}$

$= \frac{2^9}{2^{-7}}$

$= 2^{9--7}$

$= 2^{16}$

(i)

$$\frac{3^5 \div 3^2}{3^4 \times (3^4)^{-2}}$$

$$= \frac{3^3}{3^4 \times 3^{-8}}$$

$$= \frac{3^3}{3^{-4}}$$

$$= 3^{3--4}$$

$$= 3^7$$

(j)

$$\frac{(-2)^3 \times (-2)^5}{(-2)^{10}}$$

$$= \frac{(-2)^8}{(-2)^{10}}$$

$$= (-2)^{8-10}$$

$$= (-2)^{-2}$$

$$= \frac{1}{(-2)^2}$$

(k)

$$\frac{5^{-3} \times (5^2)^3}{(5^3)^4 \times 5^{-7}}$$

$$= \frac{5^{-3} \times 5^6}{5^{12} \times 5^{-7}}$$

$$= \frac{5^3}{5^5}$$

$$= 5^{3-5}$$

$$= 5^{-2}$$

$$= \frac{1}{5^2}$$

2. (a)

$$\frac{(3^2 \times 10^6)^{\frac{1}{2}}}{3^4 \times 10^{-2}}$$

$$= \frac{3^1 \times 10^3}{3^4 \times 10^{-2}}$$

$$= 3^{-3} \times 10^5$$

$$= \frac{10^5}{3^3}$$

(b)

$$\frac{(5^{-3} \times 4^2)^3}{(4^{-7} \times 5^2)^2}$$

$$= \frac{5^{-9} \times 4^6}{4^{-14} \times 5^4}$$

$$= \frac{4^{14} \times 4^6}{5^9 \times 5^4}$$

$$= \frac{4^{20}}{5^{13}}$$

(c)

$$\frac{5^{\frac{1}{3}} \times 5^0}{5^{-1}} \div 5^2$$

$$= \frac{5^{\frac{1}{3}}}{5^{-1}} \div 5^2$$

$$= \frac{5^{\frac{1}{3}}}{5^{-1}} \div \frac{5^2}{1}$$

$$= \frac{5^{\frac{1}{3}}}{5^{-1}} \times \frac{1}{5^2}$$

Remember from module 2.
Change division to multiplication as in fractions.

$$= \frac{5^{\frac{1}{3}}}{5^1}$$

$$= 5^{\frac{1}{3} - 1}$$

$$= 5^{\frac{1}{3} - \frac{3}{3}}$$

$$= 5^{\frac{-2}{3}}$$

$$= \frac{1}{5^{\frac{2}{3}}}$$

Activity 3.10

1.

	Number	$\times$ 10(10^1)	$\times$ 100(10^2)	$\times$ 1 000(10^3)	$\times$ 1 000 000(10^6)
(a)	23	230	2 300	23 000	23 000 000
(b)	–590	–5 900	–59 000	–590 000	–590 000 000
(c)	0.6	6	60	600	600 000
(d)	0.04	0.4	4	40	40 000
(e)	–0.0305	–0.305	–3.05	–30.5	–30 500
(f)	8.9	89	890	8 900	8 900 000

2. There will be 5 × 100 tablets. That is, 500 tablets.
3. The total cost of the order will be $4.32 × 1 000 = $4 320

Activity 3.11

1.

	Number	÷ 10(10^1)	÷ 100(10^2)	÷ 1 000(10^3)	÷ 1 000 000(10^6)
(a)	23	2.3	0.23	0.023	0.000 023
(b)	–590	–59	–5.9	–0.59	–0.000 59
(c)	0.6	0.06	0.006	0.000 6	0.000 000 6
(d)	0.04	0.004	0.000 4	0.000 04	0.000 000 04
(e)	–0.0305	–0.00305	–0.000305	–0.000 030 5	–0.000 000 030 5
(f)	8.9	0.89	0.089	0.008 9	0.000 008 9

2. Each dose will contain 5 500 millilitres ÷ 1 000 = 5.5 millilitres.
3. Each pair of socks will cost $49.50 ÷ 10 = $4.95.

Activity 3.12

1. (a) $65\,000 = 6.5 \times 10^4$
   (b) $750\,000 = 7.5 \times 10^5$
   (c) $8\,700 = 8.7 \times 10^3$
   (d) $0.003\,5 = 3.5 \times 10^{-3}$
   (e) $0.04 = 4 \times 10^{-2}$
   (f) $-65 = -6.5 \times 10^1$
   (g) $0.000\,002\,4 = 2.4 \times 10^{-6}$
   (h) $825\,600 = 8.256 \times 10^5$
   (i) $650 = 6.5 \times 10^2$
   (j) $-0.005\,9 = -5.9 \times 10^{-3}$
   (k) $7.2 = 7.2 \times 10^0$

2. Mass of a virus = 0.000 000 000 000 000 003 kilograms = 3×10^{-18} kilograms
   Breaking stress of steel = 430 000 000 Pascals = 4.3×10^{8} Pascals
   Bank profit = $1 119 000 000 = 1.119×10^{9} dollars

3. The number 578×10^{6} is not written in scientific notation because the decimal point is not after the first non-zero digit. If we re-wrote the number to be 5.78×10^{8} it would now be in scientific notation.

4. (a) Blond heads have the greatest number of hairs.

   (b) The number for blond heads, 1.4×10^{5}, had the highest power of 10 along with black or brown heads at 1.05×10^{5}, but the number out the front of 1.4×10^{5} is greater than the number at the front of 1.05×10^{5}.

5. (a) 1.6×10^{3}, 1.7×10^{3}, 2.4×10^{3}, 2.54×10^{3}, 9.6×10^{3}.

   (b) 5.1×10^{-4}, 5.1×10^{-3}, 5.1, 5.1×10^{4}, 5.1×10^{6}.

   (c) 7.8×10^{-5}, 3.4×10^{-4}, 8.45×10^{6}, 7.96×10^{7}, 1.4×10^{9}

Activity 3.13

1. (a) $6.15 \times 10^{3} = 6\,150$

   (b) $7.24 \times 10^{-3} = 0.007\,24$

   (c) $9.25 \times 10^{4} = 92\,500$

   (d) $6.92 \times 10^{2} = 692$

   (e) $-5.76 \times 10^{-5} = -0.000\,057\,6$

   (f) $10^{4} = 1 \times 10^{4} = 10\,000$

   (g) $10^{-3} = 1 \times 10^{-3} = 0.001$

2.

Chemical A	4.56×10^{-4}	= 0.000 456
Chemical B	3.89×10^{-12}	= 0.000 000 000 003 89
Chemical C	5.78×10^{-7}	= 0.000 000 578

3. The speed of light is 2.998×10^{8} metres per second = 299 800 000 metres per second.

4.

Colour of hair	Number of hairs	
Black or Brown	1.05×10^{5}	105 000
Blond	1.4×10^{5}	140 000
Red	9×10^{4}	90 000

5. (a) $39.7 = 3.97 \times 10$ since $39.7 = 39.7$

   (b) $\frac{1}{1\,000} > 10^{-4}$ since $0.001 > 0.000\,1$

(c) $6.85 \times 10^5 > 6\,850$ since $685\,000 > 6\,850$

(d) $0.000\,95 < 7.8 \times 10^{-2}$ since $0.000\,95 < 0.078$

(e) $\frac{55}{100} > 5 \times 10^{-2}$ since $0.55 > 0.05$

Activity 3.14

1\. (a) $2.5 \times 10^2 \times 3.5 \times 10^3$
$= 2.5 \times 3.5 \times 10^2 \times 10^3$
$= 8.75 \times 10^5$

(b) $6.7 \times 10^3 \times 5.4 \times 10^{-2}$
$= 6.7 \times 5.4 \times 10^3 \times 10^{-2}$
$= 36.18 \times 10^1$
$= 3.618 \times 10^2$

(c) $7.5 \times 10^{-5} \times 1.3 \times 10^6$
$= 7.5 \times 1.3 \times 10^{-5} \times 10^6$
$= 9.75 \times 10^1$

(d) $9.25 \times 10^{-2} \times 3.75 \times 10^{-6}$
$= 9.25 \times 3.75 \times 10^{-2} \times 10^{-6}$
$= 34.687\,5 \times 10^{-8}$
$= 3.468\,75 \times 10^{-7}$

(e) $-5.9 \times 10^3 \times 3.8 \times 10^{-7}$
$= -5.9 \times 3.8 \times 10^3 \times 10^{-7}$
$= -22.42 \times 10^{-4}$
$= -2.242 \times 10^{-3}$

2\. Number of red blood cells $= 5\,500 \times 5 \times 10^9$
$= 27\,500 \times 10^9$
$= 2.7500 \times 10^4 \times 10^9$
$= 2.75 \times 10^{13}$

3\. Number of microbes $= 4 \times 10^6 \times 2 \times 10^4$
$= 4 \times 2 \times 10^6 \times 10^4$
$= 8 \times 10^{10}$

4\. Distance of the sun from the centre of our galaxy
$= 9.46 \times 10^{12} \times 2.7 \times 10^4$ kilometres
$= 9.46 \times 2.7 \times 10^{12} \times 10^4$ kilometres
$= 25.542 \times 10^{16}$ kilometres
$= 2.554\,2 \times 10^{17}$ kilometres

Activity 3.15

1. (a) $(5.2 \times 10^3) \div (2.6 \times 10^4)$

$= \dfrac{(5.2 \times 10^3)}{(2.6 \times 10^4)}$

$= 2 \times 10^{3-4}$

$= 2 \times 10^{-1}$

(b) $(7.2 \times 10^5) \div (3 \times 10^{-7})$

$= \dfrac{(7.2 \times 10^5)}{(3 \times 10^{-7})}$

$= 2.4 \times 10^{5-(-7)}$

$= 2.4 \times 10^{12}$

(c) $(9.7 \times 10^{-2}) \div (3.8 \times 10^{-3})$

$= \dfrac{(9.7 \times 10^{-2})}{(3.8 \times 10^{-3})}$

$\approx 2.552\,63 \times 10^{-2-(-3)}$

$\approx 2.55 \times 10^1$

(d) $(-5.6 \times 10^9) \div (6.1 \times 10^2)$

$= \dfrac{(-5.6 \times 10^9)}{(6.1 \times 10^2)}$

$\approx -0.918\,03 \times 10^{9-2}$

$\approx -0.91803 \times 10^7$

$\approx -9.180\,3 \times 10^6$

(e) $(7.1 \times 10^2) \div (-8.4 \times 10^7)$

$= \dfrac{(7.1 \times 10^2)}{(-8.4 \times 10^7)}$

$\approx -0.845\,238 \times 10^{2-7}$

$\approx -0.845238 \times 10^{-5}$

$\approx -8.452\,38 \times 10^{-6}$

$\approx -8.452 \times 10^{-6}$

2. Yearly profit $= (4.78 \times 10^7) \div 5$ dollars

$= \dfrac{(4.78 \times 10^7)}{5}$ dollars

$= 0.956 \times 10^7$ dollars

$= 9.56 \times 10^6$ dollars

3. Earth's speed $= (3.88 \times 10^8) \div (9 \times 10^3)$ kilometres per hour

$= \dfrac{(3.88 \times 10^8)}{(9 \times 10^3)}$ kilometres per hour

$\approx 0.431\,11 \times 10^{8-3}$

$\approx -0.43111 \times 10^5$

$= 4.311\,1 \times 10^4$

4. Time for signals to reach earth $= (2.89 \times 10^8) \div (1.94 \times 10^5)$ seconds

$= \dfrac{(2.89 \times 10^8)}{(1.94 \times 10^5)}$ seconds

$= 1.489\,69 \times 10^{8-5}$ seconds

$= 1.49 \times 10^3$ seconds

Activity 3.16

1. You may have done these using any of the three methods discussed.

(a) $5.9 \times 10^4 + 2.4 \times 10^4$
$= (5.9 + 2.4) \times 10^4$
$= 8.3 \times 10^4$

(b) $7.8 \times 10^{-2} - 3.4 \times 10^{-2}$
$= (7.8 - 3.4) \times 10^{-2}$
$= 4.4 \times 10^{-2}$

(c) $6.1 \times 10^3 - 9.5 \times 10^3$
$= (6.1 - 9.5) \times 10^3$
$= {}^{-}3.4 \times 10^3$

(d) $4.5 \times 10^5 + 9.6 \times 10^4$
$= 0.45 \times 10^4 + 9.6 \times 10^4$
$= (0.45 + 9.6) \times 10^4$
$= 10.05 \times 10^4$
$= 1.00\,5 \times 10^5$

(e) $7.6 \times 10^8 - 9.5 \times 10^6$
$= 760 \times 10^6 - 9.5 \times 10^6$
$= (760 - 9.5) \times 10^6$
$= 750.5 \times 10^6$
$= 7.505 \times 10^8$

(f) $1.3 \times 10^{-6} + 8.3 \times 10^{-4}$
$= 0.013 \times 10^{-4} + 8.3 \times 10^{-4}$
$= (0.013 + 8.3) \times 10^{-4}$
$= 8.313 \times 10^{-4}$

2. (a) Humans have the largest number of neurons

(b) The difference $= 3 \times 10^{10} - 7.5 \times 10^9$
$= 30 \times 10^9 - 7.5 \times 10^9$
$= (30 - 7.5) \times 10^9$
$= 22.5 \times 10^9$
$= 2.25 \times 10^{10}$

(c) The difference $= 7.5 \times 10^9 - 6.5 \times 10^7$
$= 750 \times 10^7 - 6.5 \times 10^7$
$= (750 - 6.5) \times 10^7$
$= 743.5 \times 10^7$
$= 7.435 \times 10^9$

Activity 3.17

(a) 50 milligrams = 50 mg

(b) 65 kilograms = 65 kg

(c) 5 microlitres = 5 μL

(d) 7.2 megalitres = 7.2 ML

(e) 9 picoseconds = 9 ps

Activity 3.18

1. (a) 5 600 g = 5 600 × 10^{-3} kg = 5.6 kg

   (b) 0.032 L = 0.032 × 10^{3} mL = 32 mL

   (c) 0.000 000 25 g = 0.000 000 25 × 10^{9} ng = 250 ng

   (d) 35 000 mg = 35 000 × 10^{-3} g = 35 g

   (e) 70 250 cm = 70 250 × 10^{-2} m = 702.5 m

   (f) 1 600 kg = 1 600 × 10^{-3} t = 1.6 t

   (g) 0.021 ML = 0.021 × 10^{9} mL = 21 000 000 mL

   (h) 206 μm = 206 × 10^{-6} m = 0.000 206 m

2. (a) Length of a car: 500 cm

   (b) Weight of a person: 75 kg

   (c) Volume of a car's petrol tank: 48 L

   (d) Weight of a toothpick: 450 mg

   (e) Height of the Sydney Harbour Bridge above water: 135 m

   (f) Amount of blood in the human body: 5.5 L

Activity 3.19

1. Total distance travelled by the student = 1 450 m + 7.4 km
   = 1.45 km + 7.4 km
   = 8.85 km

   The student travels 8.85 km to University.

2. Colin ran 2 750 m × 7 = 19 250 m = 19.25 km

   Colin runs 19.25 kilometres each week.

3. The creek rose 1.6 m + 1.16 m + 65 cm
   = 1.6 m + 1.16 m + 0.65 m
   = 3.41 m

   The creek rose 3.41 metres in the first three hours.

4. Tom's height $= 1.85$ m – 92 cm
$= 185$ cm – 92 cm
$= 93$ cm
Tom is 93 centimetres tall.

5. (a) Doctor orders 1 gram every 6 hours. That is 1 000 mg every 6 hours.
If tablets are 500 mg each, the patient must take:
1 000 mg ÷ 500 mg tablets every 6 hours
= 2 tablets every 6 hours.

(b) The patient should receive 0.1 mg ÷ 0.05 mg tablets daily
= 2 tablets daily

6. Recall that 1 ha = 10 000 m^2
Land developed $= (18 \times 550 + 920)\ m^2$
$= (9\,900 + 920)\ m^2$
$= 10\,820\ m^2$
$= (10\,820 \div 10\,000)$ ha
$= 1.082$ ha
The land developed amounted to 1.082 hectares.

7. Land given to children $= 750$ ha $\times 5$
$= 3\,750$ ha
$= 37.5\ km^2$ Recall that 1 km^2 = 100 ha
Remaining land $= (150 - 37.5)\ km^2$
$= 112.5\ km^2$
The couple now own 112.5 km^2 of land.

8. (a) Number of loaves of bread in the crate
= (49.9 kg – 2.3 kg) ÷ 680 g
= (49 900g – 2 300 g) ÷ 680 g
= 47 600 g ÷ 680 g
= 70
There are 70 loaves of bread in the crate.

(b) Number of loaves supplied in a year = 31.025 t ÷ 680 g
= 31 025 000 g ÷ 680 g
= 45 625
Number of loaves supplied each day = 45 625 ÷ 365
= 125

Therefore, 125 loaves of bread are supplied each day of the year.

9. (a)This swarm of locusts could consume $4 \times 10^{10} \times 15$ grams of grain in one week
$= 60 \times 10^{10}$ grams of grain in one week
$= 6 \times 10^{11}$ grams of grain in one week

(b) 6×10^{11} g $= 6 \times 10^{11} \times 10^{-6}$ t
$= 6 \times 10^{5}$ t

The locusts could consume 6×10^{5} tonnes of grain in one week.

Solutions to a taste of things to come

1. (a) 30 g serve with $\frac{1}{2}$ cup skim milk = 0.34 mg of Thiamine

$= 0.34 \times 10^3$ μμg

$= 340$ μg

A 30 g serve of Special K with $\frac{1}{2}$ cup skim milk contains 340 μg of Thiamine.

(b) 30 g serve with $\frac{1}{2}$ cup skim milk has 212 mg of sodium

Truck driver has 4×212 mg sodium

$= 848$ mg

$= 848 \times 10^{-3}$ g

$= 8.48 \times 10^{-1}$ g

The truck driver consumes 0.848 g of sodium for breakfast.

(c) 30 g serve of Special K contains 175 mg of calcium.

30 g serve of Special K with $\frac{1}{2}$ cup skim milk contains 330 mg of calcium.

Therefore $\frac{1}{2}$ cup skim milk must contain 330 mg – 175 mg

$= 155$ mg

Therefore $\frac{1}{2}$ cup skim milk contains 155 mg of calcium.

(d) 100 grams of Special K provides 1 536 kJ of energy.

Therefore 1 gram of Special K provides 15.36 kJ of energy.

Now 15.36 kilojoules $= 15.36 \times 10^{-3}$ megajoules

$= 0.015\,36$ MJ

To receive 3.94 MJ of energy you would need 3.94 MJ ÷ 0.015 36 MJ

≈ 256.51

You would need to have about 256.5 grams of Special K to receive 3.94 megajoules of energy.

2. (a) Litres remaining $= 9.75 \text{ L} - (150 \text{ mL} + 200 \text{ mL} + 100 \text{ mL} + 1\,000 \text{ mL} + 2.5 \text{ L})$
$= 9.75 \text{ L} - (0.15 \text{ L} + 0.2 \text{ L} + 0.1 \text{ L} + 1.0 \text{ L} + 2.5 \text{ L})$
$= 9.75 \text{ L} - 3.95 \text{ L}$
$= 5.8 \text{ L}$
Therefore 5.8 litres remain.

(b) New amount $= 4.250 \text{ L} + 3.125 \text{ L}$
$= 7.375 \text{ L}$
After the addition the total amount in the container is 7.375 litres.

(c) Seven containers will hold $7 \times 9.75 \text{ L}$
$= 68.25 \text{ L}$
Seven containers will hold 68.25 litres.

(d) Amount left $= \frac{2}{3} \times 9.75 \text{ L}$
$= 6.5 \text{ L}$
If the container is $\frac{2}{3}$ full it contains 6.5 litres.

3. (a) 4.7 million = 4 700 000 = 4.7×10^6
3.5 million = 3 500 000 = 3.5×10^6
1.2 million = 1 200 000 = 1.2×10^6
1 million = 1 000 000 = 1×10^6

(b) British – Greek
$= 4.7 \times 10^6 - 6.5 \times 10^5$
$= 47 \times 10^5 - 6.5 \times 10^5$
$= (47 - 6.5) \times 10^5$
$= 40.5 \times 10^5$
$= 4.05 \times 10^6$
∴ about 4 million more people come from British backgrounds as from Greek backgrounds.

(c) Total of first five backgrounds:
$= 4.7 \times 10^6 + 3.5 \times 10^6 + 1.2 \times 10^6 + 1 \times 10^6 + 6.5 \times 10^5$
$= 4.7 \times 10^6 + 3.5 \times 10^6 + 1.2 \times 10^6 + 1 \times 10^6 + 0.65 \times 10^6$
$= (4.7 + 3.5 + 1.2 + 1 + 0.65) \times 10^6$
$= 11.05 \times 10^6$
$= 1.105 \times 10^7$
The total number of people represented by the first five ethnic backgrounds is 1.105×10^7

Following is an example of the sort of paragraph that you may have written.

From the data given in the table, the majority of Australia's people from ethnic backgrounds have come from European countries. The ethnic group most represented in Australia today are the British. There are a bit over 4 million more British as Greek people in Australia. The impact of this diversity of ethnic background needs to be understood, learnt from and catered for both socially and in the work place.

Solutions to post-test

1. $1.4 \times 1.4 \times 1.4 = (1.4)^3$

2. (a) $5^4 \times 5^8 \times 5^{-3}$
$= 5^{4+8-3}$
$= 5^9$

(b) $(-2)^5 \times (-2)^{-6}$
$= (-2)^{5-6}$
$= (-2)^{-1}$
$= \dfrac{1}{(-2)^1}$

(c) $12^{27} \div 12^5$
$= 12^{27-5}$
$= 12^{22}$

(d) $4^{-3} \times (4^2)^3 \div 4^{-1}$
$= 4^{-3} \times 4^6 \div 4^{-1}$
$= 4^{-3+6-(-1)}$
$= 4^4$

3. (a) $243^{\frac{2}{5}} = 9$

(b) $196^{\frac{1}{2}} = 14$

(c) $3^5 + 3^4 = 243 + 81 = 324$

(d) $\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^5 = 0.125 - 0.031\ 25 = 0.093\ 75 \approx 0.09$

(e) $4^{-3} \times (4^2)^3 + 4^{-1} \times 4^5$
$= 4^{-3} \times 4^6 + 4^{-1} \times 4^5$
$= 4^{-3+6} + 4^{-1+5}$
$= 4^3 + 4^4$
$= 64 + 256$
$= 320$

4. (a) Amount of punch:
= 3.5 L + 400 mL + 300 mL + 4 L + 2.5 L + 500 mL + 800 mL
= 3.5 L + 0.4 L + 0.3 L + 4 L + 2.5 L + 0.5 L + 0.8 L
= 12 L
Therefore 12 litres of punch will be made from this recipe.

(b) Each person would be able to have:

$12 \text{ L} \div 30 = 12\ 000 \text{ mL} \div 30$

$= 400 \text{ mL}$

Each person could have 400 mL of punch.

(c) If 12 000 mL of punch were made, this gives 12 000 mL ÷ 80 mL = 150 cups of drink to sell at the fete.

If each drink sells for 25 cents this gives income = 150×25 cents

= 3 750 cents

= $37.50

Profit on the sale of punch would be $37.50 – $12.50 = $25.00

5. (a)

	(i)	(ii)
Mercury	5×10^3	= 5 000 km
Mars	6.87×10^3	= 6 870 km
Venus	1.24×10^4	= 12 400 km
Earth	1.27×10^4	= 12 700 km
Pluto	1.27×10^4	= 12 700 km
Neptune	5×10^4	= 50 000 km
Uranus	5.1×10^4	= 51 000 km
Saturn	1.14×10^5	= 114 000 km
Jupiter	1.4×10^5	= 140 000 km

(b) (i) Pluto, Mercury, Mars, Venus, Earth, Uranus, Neptune, Saturn, Jupiter

(ii) Mass of Venus $= 5.983 \times 10^{24} \text{ kg} \times 0.82 = 4.906\ 06 \times 10^{24} \text{ kg}$

Mass of Saturn $= 5.983 \times 10^{24} \text{ kg} \times 95.3 = 570.179\ 9 \times 10^{24} \text{ kg}$

$\approx 5.702 \times 10^{26} \text{ kg}$

Mass of Pluto $= 5.983 \times 10^{24} \text{ kg} \times 0.0021 \text{ kg}$

$\approx 0.01256 \times 10^{24} \text{ kg}$

$= 1.256 \times 10^{22} \text{ kg}$

(c) (i) Mass of the sun $= 5.983 \times 10^{24} \text{ kg} \times 3.29 \times 10^6$

$= 5.983 \times 3.29 \times 10^{24} \times 10^6 \text{ kg}$

$= 19.684\ 07 \times 10^{30} \text{ kg}$

$= 1.968 \times 10^{31} \text{ kg}$

(ii) Sun's diameter – Earth's diameter

$= 1.39 \times 10^6 \text{ km} - 1.27 \times 10^4 \text{ km}$

$= 1.39 \times 10^6 - 0.012\ 7 \times 10^6 \text{ km}$

$= 1.377\ 3 \times 10^6 \text{ km}$

The Sun's diameter is approximately $1.377\ 3 \times 10^6$ km larger than the Earth's diameter.