Module A8

GENERALISING NUMBERS – GRAPHS

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Introduction

We have already looked at many aspects of graphing. In module 4 we looked at pie charts and in module 5 line graphs and how to construct graphs. In module 6 we looked at bar graphs and histograms. In this module we will investigate graphs similar to those in module 5, but this time from yet another perspective.

We will attempt to analyse these graphs more carefully. Why do graphs look the way they do? You should even be able to look at equations and be able to describe what the picture of that equation will look like.

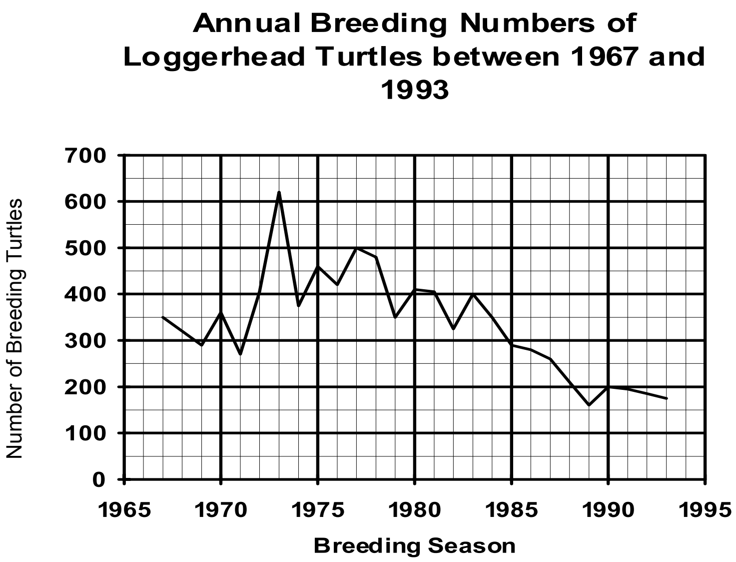
In much of the mathematics you have studied so far, you have looked at the relationships between variables – how one thing changes in relation to another. This module will extend this concept.

On successful completion of this module you should be able to:

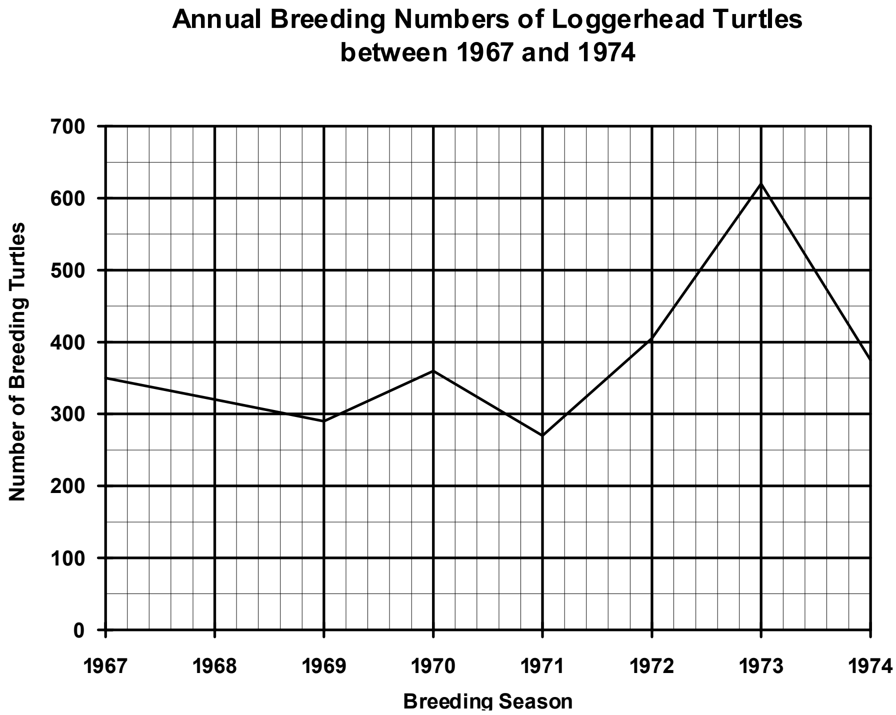
- identify, draw and interpret the graphs of linear, parabolic and exponential equations;
- predict the effects on the above graphs of changes to coefficients and constants in their equations; and
- use graphs to solve simultaneous equations.

8.1 Gradients of line graphs

Consider the following graph, that we have looked at in a previous module, showing the annual breeding numbers of loggerhead turtles at Mon Repos on the Bundaberg coast over about 20 years.



Part of this graph, from 1967 up to 1974 is enlarged and reproduced below.



What was the increase in breeding numbers between 1969 and 1970? ....

What was the increase in breeding numbers between 1972 and 1973? ....

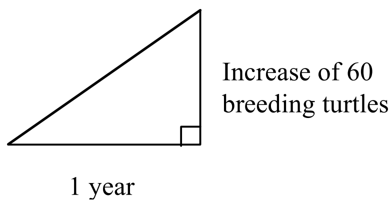
Between 1969 and 1970 turtle numbers increased by about 60, while between 1972 and 1973 numbers increased by about 220.

What do you notice about the steepness of the graph between 1969 and 1970 and between 1972 and 1973? ....

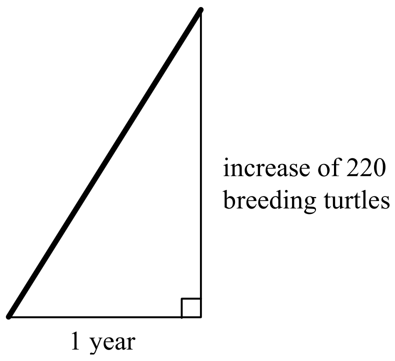
Did you say that the graph is steeper over the second period?

Let's look more closely at these two results. We will look at an even smaller part of the graph this time.

Over the 1 year period between 1969 and 1970 the number of turtles rose by 60.



Between 1972 and 1973, again a 1 year period, the number of turtles rose by 220.



Imagine walking up these two 'hills'. It would be hard work climbing up the second much steeper 'hill'. We say that the second of our diagrams has a much greater **slope** than the first diagram. Another word for the steepness or slope of a line is to talk of its **gradient**.

In fact we can put a value on the steepness or gradient of the line, by putting the value for the change in height over the change in horizontal distance.

That is, gradient = $\dfrac{\text{change in height}}{\text{change in horizontal distance}}$

You might also see this written as:

$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$

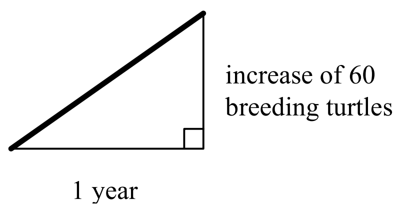
This formula means exactly the same thing as the formula above.

Let's find the gradients of the two lines taken from our graph above.

Over the 1 year period between 1969 and 1970 the number of turtles rose by 60.

gradient = $\dfrac{\text{change in height}}{\text{change in horizontal distance}}$

gradient = $\dfrac{60}{1} = 60$

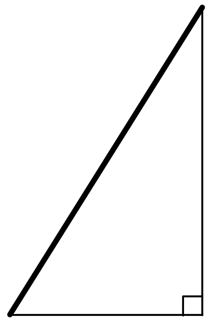


We can say that the **rate of change** in turtle numbers over this period was 60 turtles per year.

Between 1972 and 1973, again a 1 year period, the number of turtles rose by 220.

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

$$\text{gradient} = \frac{220}{1} = 220$$



Similarly we can say that the **rate of change** in turtle numbers over this period was 220 turtles per year.

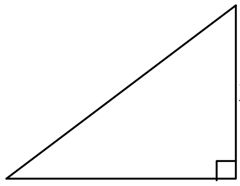
So, the gradient of the graph between 1969 and 1970 is 60 while the gradient of the graph between 1972 and 1973 is 220. It can be seen from these figures that the gradient of the second part of the graph is much steeper than the gradient of the first part of the graph.

Let's now look at some other examples of gradients.

Example

Find the gradient of the following line segment.

A line segment is just a part of a line.



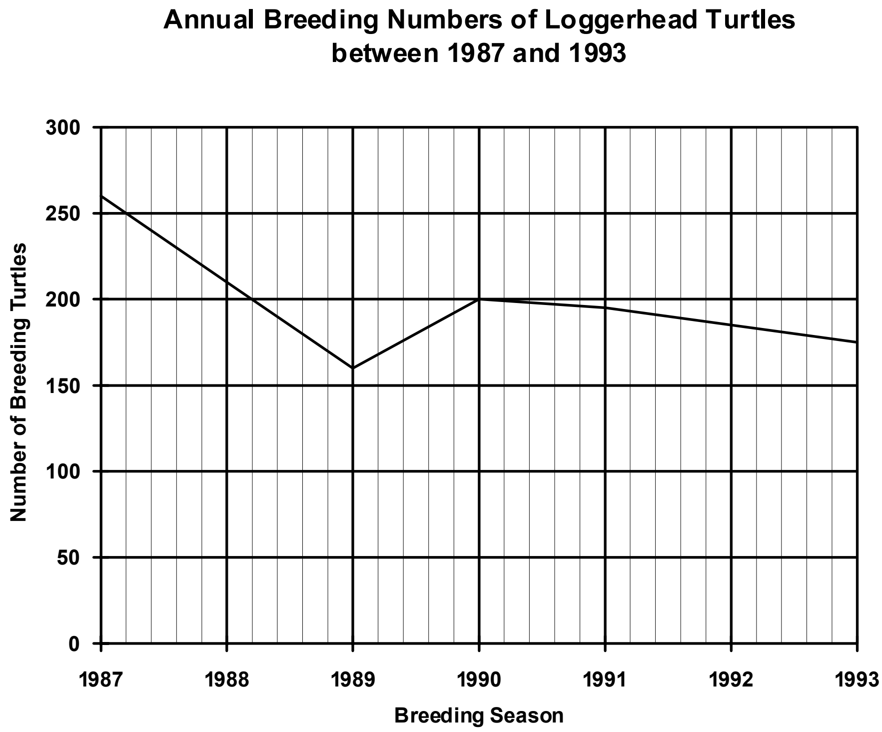
$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

$$\text{gradient} = \frac{2}{8} = \frac{1}{4}$$

What the gradient is telling us is that for every 8 metres we move along in a horizontal distance, the vertical distance rises by 2 metres. We could also say that for every 4 metres in a horizontal direction we rise 1 metre. You may see engineers refer to this as a gradient of 1 : 4 using ratios as we have done in a previous module. If this was your block of land with this slope, the builder, the surveyor and the engineer would all be interested in the gradient of the block of land. If the land is too steep then slippage of the land could occur once the soil is disturbed. If the land is too steep there would need to be a deep cut in the land to make a level piece of ground on which to situate a concrete slab based house. This might mean that alternative methods of construction need to be considered.

In fact the building code prevents anyone building on land with a gradient greater than 1 : 4 due to the danger of land slippage.

Let's return to our turtles at Mon Repos. This time we are only going to look at that part of the graph from 1987 to 1993.



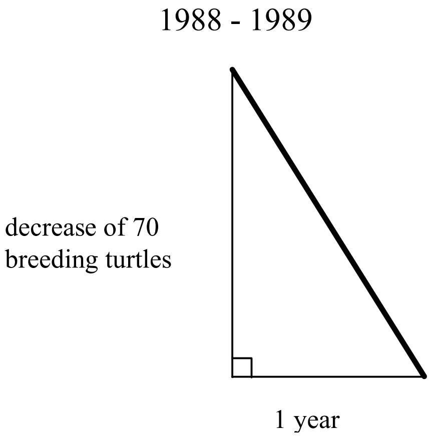
What was happening to the turtle numbers for the periods 1988 to 1989 and from 1991 to 1992? ....

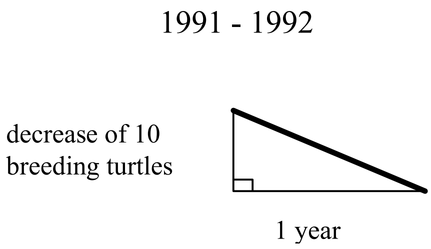
You should have said that the turtle numbers were decreasing.

Have a look at the two sections of graph for the above periods. How do they differ from the two periods that we looked at previously?

....

This time the two segments of graph are falling as we move along the graph from left to right.





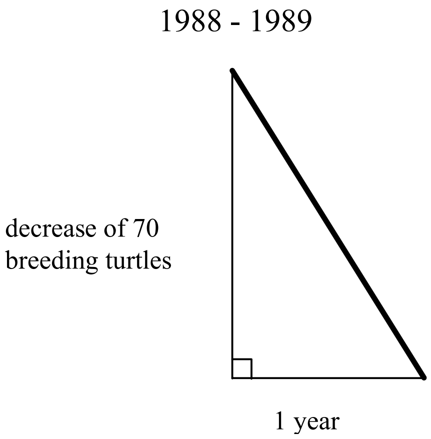
To distinguish a graph that is rising as we move from left to right from a graph that is falling as we move from left to right we give the **falling** graph a **negative** gradient. We use the same formula as before but we must always check for rising or falling lines.

Let's find the gradients for the two periods of falling turtle numbers.

For the period 1988 to 1989:

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

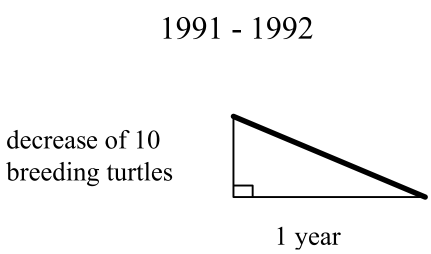
$$\text{gradient} = \frac{-70}{1} = -70$$



For the period 1991 to 1992:

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

$$\text{gradient} = \frac{-10}{1} = -10$$



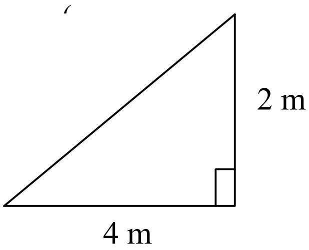
Notice again that the bigger the size of the number (ignoring the negative), the steeper the line.

Ski slopes are given ratings according to their gradients. A green run is a gentle slope suitable for beginners. A blue run is a steeper run suitable for the intermediate skier. The black run is steepest of all and is suitable only for the advanced skier.

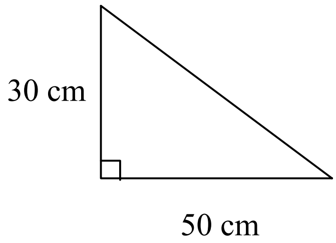
Activity 8.1

1. Find the gradient of the following line segments.

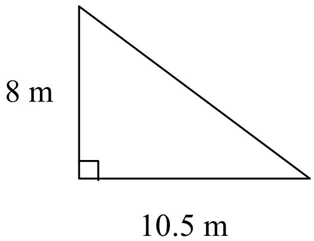
(a)



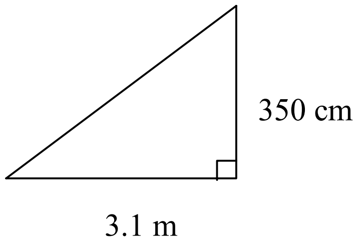
(b)



(c)



(d)



For the following questions, a **diagram** may help to clarify the situation.

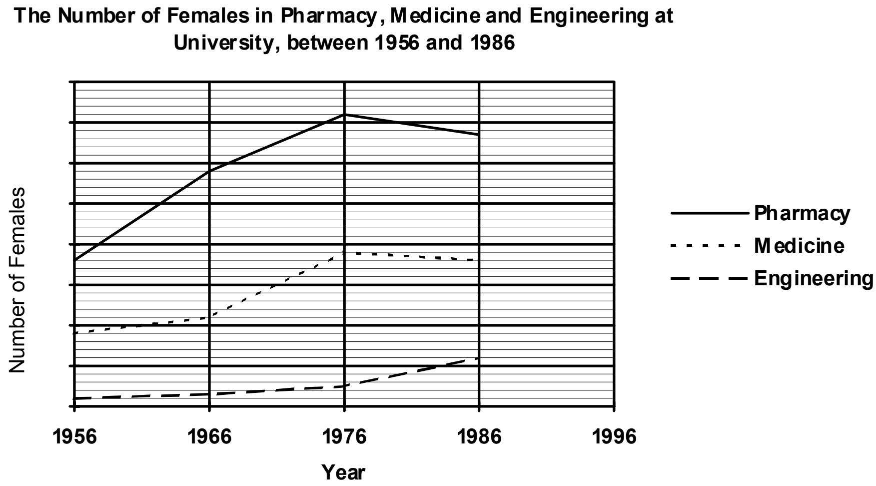
2. Interest rates rise from 3% to 5% in 2 years, what is the gradient? (Hint: think of the interest rate as the ‘rise’ and the years as the ‘run’.)

3. If you were skiing down a hill that fell 3 metres for every 2 metres of horizontal distance that you covered, what is the gradient of this hill?

4. Suppose you were riding the mine train down a tunnel to an underground mine. For every 10 metres of horizontal distance covered, you went 20 metres under the ground. What is the gradient of the mine train tunnel?

5. Suppose that you knew the gradient of a block of land you were looking at purchasing was 2. If you were to walk up the slope until you were 6 metres higher than your starting position, how many metres of horizontal distance would you have covered? A diagram might help you with your calculation.

6. The following graph is again one that you have seen before. This time I have removed the scale on the vertical axis, but you still know that it represents the number of females in each of the given disciplines.

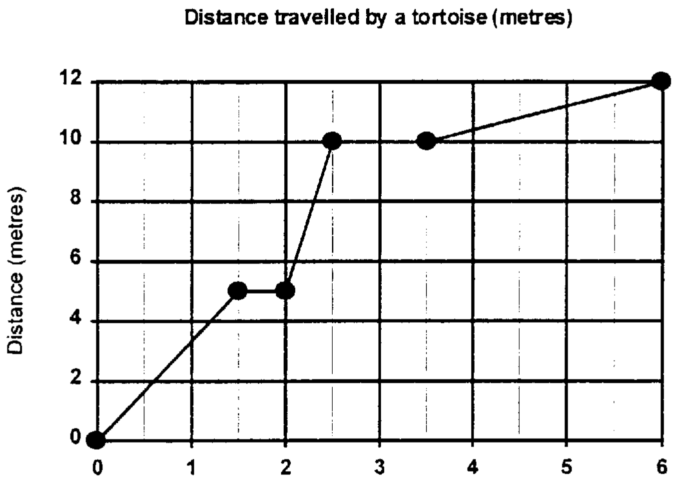


   (a) Between 1956 and 1966, which course showed the greatest increase in numbers of females enrolled? How can you tell this from the graph?

   (b) Over what periods and for what courses was there a decline in the number of females?

   (c) Which course showed the greatest decline in numbers of females?

7. The following graph shows the distance covered by a tortoise over a given time.



   (a) If we look at the gradient of this graph over any particular period, we would again put the change in height over the change in horizontal distance. Let's look at what this is telling us about this graph.

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

$$\text{gradient} = \frac{\text{distance}}{\text{time}}$$

Do you recognise the formula? What is the gradient telling us about this tortoise?

(b) Looking at the graph above, over which period of time is the tortoise travelling at the greatest speed? Calculate the gradient of the graph over this period.

(c) Over which period of time is the tortoise travelling at the least speed? Calculate the gradient of the graph over this period.

(d) Find the gradient of the graph from 2.5 to 3.5 minutes.

(e) Using your answer from part (d) complete the following sentence.

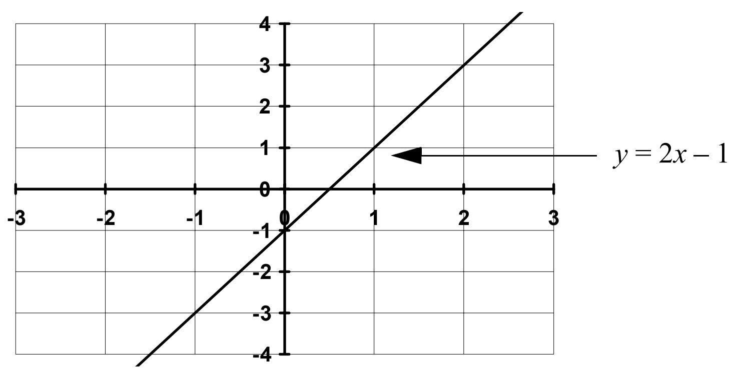
A horizontal line has a gradient of ....

8.1.1 Finding the gradient of a given line

In many situations the relationship between 2 variables can be represented by a formula or equation as we have seen already. It is often possible to tell a lot about the graph of this relationship without actually drawing the graph.

Example

Consider the following graph of the line $y = 2x - 1$ that we looked at in module 5.

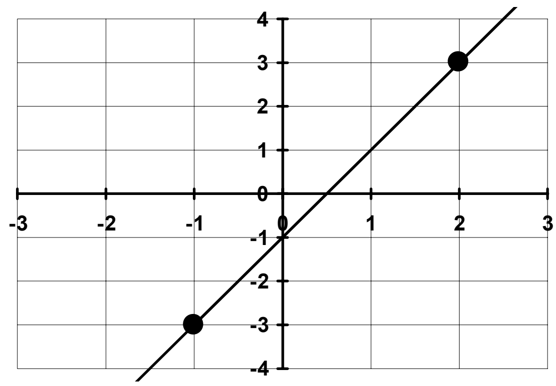


To find the gradient of this line we use the following procedure.

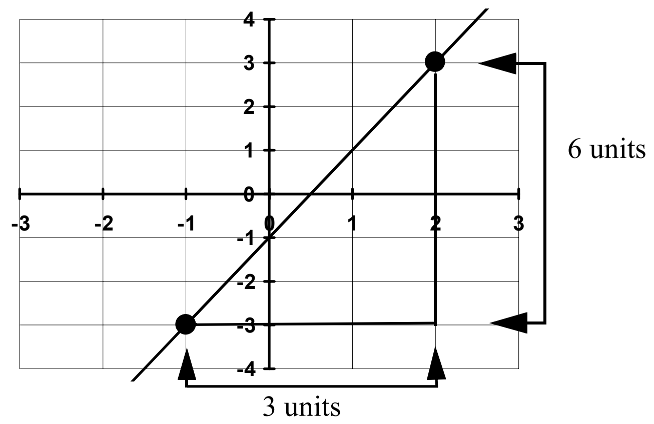
- Choose **any** two points on the line.
- Draw a triangle that shows the change in height over the change in horizontal distance between the two points.
- Calculate the gradient by putting the change in height over the change in horizontal distance. **Don't forget to check for a rising or falling line.**

Let's look at this for the above line.

Step 1. Choose **any** two points on the line.



Step 2. Draw a triangle that shows the change in height over the change in horizontal distance between the two points.



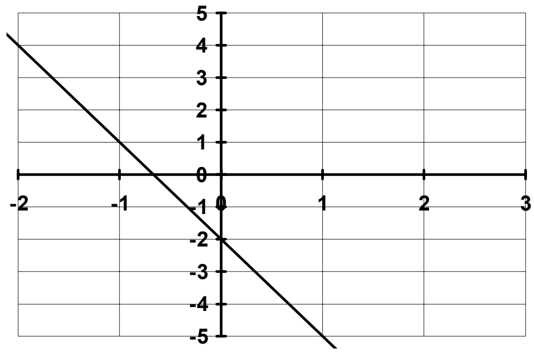
Step 3. Calculate the gradient by putting the change in height over the change in horizontal distance. Check to see if the line is rising or falling. This line is rising as we move from left to right and will thus have a positive gradient.

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

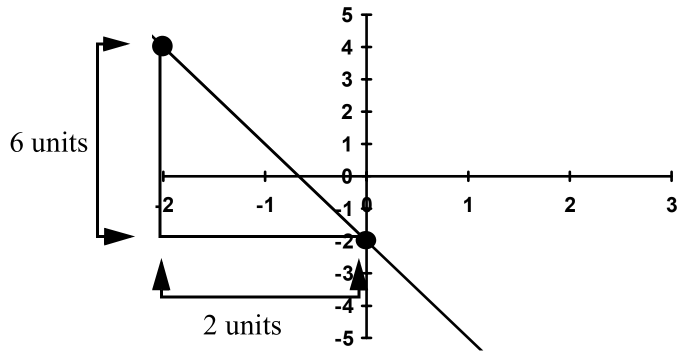
$$\text{gradient} = \frac{6}{3} = 2$$

Example

Find the gradient of the following line with equation $y = -3x - 2$



Two convenient points to choose this time might be (–2,4) and (0,–2)

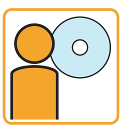


Consider this time that the line is falling as we move from left to right so the gradient will be negative.

Therefore the gradient is:

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

$$\text{gradient} = \frac{-6}{2} = -3$$



Check out the resource CD and in particular 'Worksheet 1: Gradients'. This worksheet is interactive, in that it allows you to interact with the graph; you might find it useful for understanding the concept of gradient.

8.1.2 Drawing a line given the gradient

Using the skills we have just learnt, it is now possible to draw a line given its gradient and one point it passes through.

The steps that we will follow to do this are:

- plot the given point;
- move horizontally and vertically according to the ‘instructions’ given by the gradient, and mark another point onto the Cartesian plane;
- finally draw a line through and beyond these two plotted points.

Example

Let’s follow through these steps and draw a line passing through the point (1,–2) with a gradient of 3.

Step 1 Plot the point (1,–2)

Step 2 Look at the gradient to determine the ‘instructions’ it is providing.

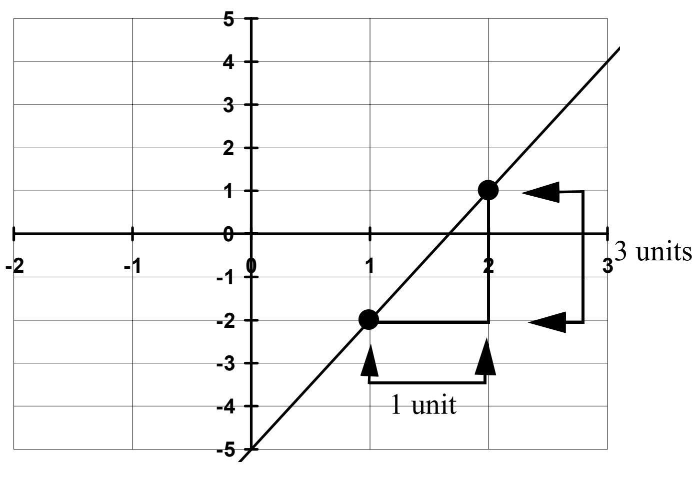
Now, $$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

We know that the gradient for this question is 3, and we can express this as $\frac{3}{1}$, so we can write:

$$\text{gradient} = \frac{3}{1}$$

We take particular note that the gradient in this case is positive. This means that from our plotted point (1,–2), we will move 1 unit horizontally to the right and 3 units vertically upwards. This is the position of the second point.

Step 3 Now draw a line through and beyond the plotted points.



Example

This time we will look at an example where the gradient is negative.

Consider a line that passes through the point (–1,4) with a gradient of $\frac{-3}{2}$

Step 1 Plot the point (–1,4)

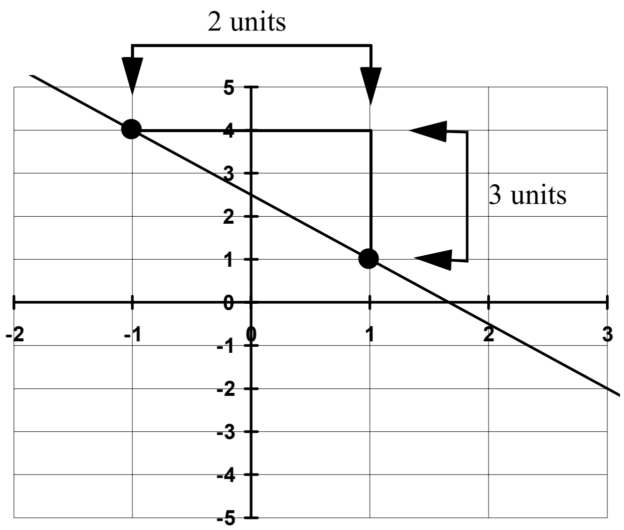
Step 2 Look at the gradient to determine the 'instructions' it is providing.

Now, $\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}}$

We know that the gradient for this question is $\frac{-3}{2}$, that is, the line falls 3 units for every 2 that it moves horizontally.

We take particular note that the gradient in this case is negative. This means that from our plotted point (–1,4), we will move 2 unit horizontally to the right and 3 units vertically downwards to show that it is a falling line. This is the position of the second point.

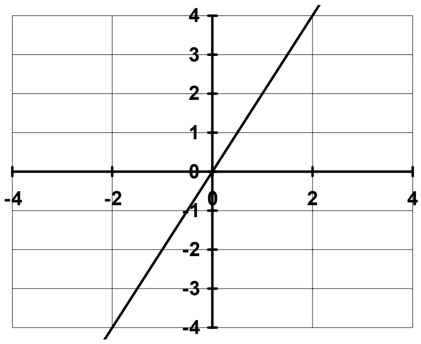
Step 3 Now draw a line through and beyond the plotted points.



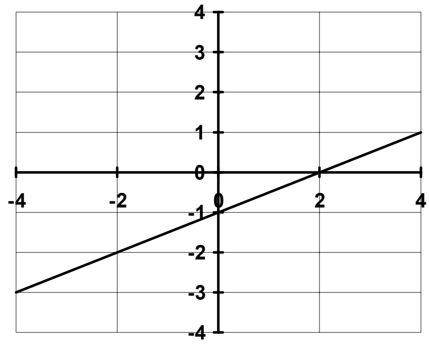
Activity 8.2

1. For each of the following lines, calculate the gradient. Don't forget to consider the scale on the axes and the rising or falling of the line.

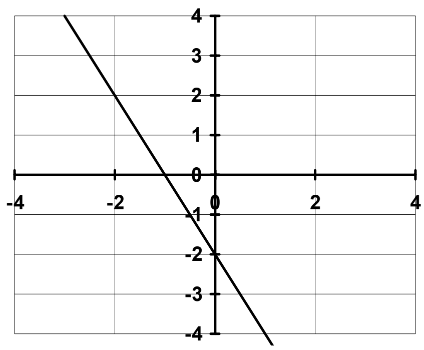
(a)



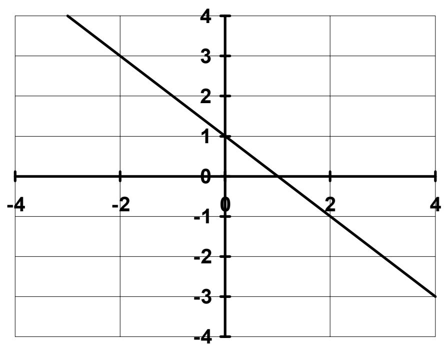
(b)



(c)



(d)



2. Draw a line with a gradient of 2 passing through (3,1)

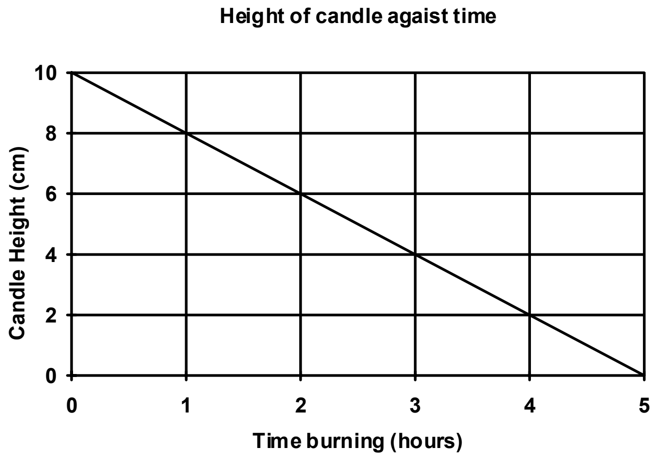
3. Draw a line with a gradient of $\frac{2}{3}$ passing through the point (–1,–2)

4. Before the invention of mechanical clocks, candles were sometimes used to measure the passage of time. A formula for the height of such a candle related to time is given below.

$h = 10 - 2t$ where h equals the height of the candle in centimetres,

and t equals the time in hours that the candle has been burning.

The graph of this relationship as you discovered in module 5 looked like this.



(a) Find the gradient of this line.

(b) What is this graph 'telling' us about the rate at which the candle is burning?

We are now going to move on and look at a variety of equations and their graphical representation. You should be able to look at an equation and tell a lot about what the graph will look like before you draw it. This is an extension of the process of estimation that you are following with numerical calculations. This is a very valuable skill as it allows you to check that your calculating and plotting of values has been correct.

8.2 Linear equations

We call equations that produce straight line graphs, **linear equations**.

Here are some examples of linear equations that we have looked at in previous modules.

$C = 2D$ where C represents the amount of money that Chris earns in dollars, and D represents the amount of money that David earns in dollars.

$J = C + 2$ where J represents Joseph's age in years, and C represents Chris' age in years.

$I = 200 + 50n$ where I represents the total income in dollars, and n represents the number of items sold.

$y = 3x - 2$ where x and y were not defined.

$h = 10 - 2t$ where h equals the height of the candle in centimetres, and t equals the time in hours that the candle has been burning.

Notice that in each case the variables are of power one and no two variables are multiplied together. This is the case for all linear equations.

Linear equations have both variables of power one and no variables multiplied together.

Activity 8.3

Which of the following are linear equations.

1. $y = 4x$
2. $y = 5x^2$
3. $3y + 2x = 6$
4. $xy = 4$
5. $3x = 7 - 2y$
6. $y = -2x$
7. $y = x^2 - 3x + 4$
8. $x = 3 - y$

Let's look more closely at linear equations and how we can draw a graph of them on the Cartesian plane.

Firstly let's recall how to graph a line.

Example

Sketch the graph of the equation $y = 2x - 1$

Before we begin to draw the graph, look at the equation and recognise it as a linear equation, because it has variables of power one and no variables multiplied together. By doing this we know that the graph we draw should be a straight line.

To draw the graph we need to plot a series of points then connect them together with a straight line. To get these points we need to calculate a table of values.

You will need to decide what x values you will choose to put into your table. (It is always best to choose values that are easy to calculate.)

x	–2	0	2
y			

When $x = -2$

$y = 2x - 1$

$y = 2 \times -2 - 1$

$y = -4 - 1$

$y = -5$

When $x = 0$

$y = 2x - 1$

$y = 2 \times 0 - 1$

$y = 0 - 1$

$y = -1$

When $x = 2$

$y = 2x - 1$

$y = 2 \times 2 - 1$

$y = 4 - 1$

$y = 3$

Now complete the table of values.

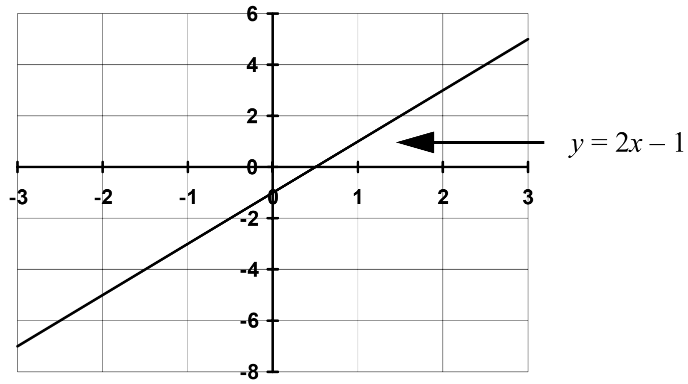
x	–2	0	2
y	–5	–1	3

The ordered pairs that we will plot are:

(–2,–5) (0,–1) (2,3)

When graphing an equation like this it is usual to graph over all four quadrants.

Plot the points, draw a line through and beyond the points. Finally label your graph.



Activity 8.4

1. Calculate a table of values for each of the following lines and graph on the one set of axes on your graph paper.

   (a) $y = x$

   (b) $y = 2x$

   (c) $y = 0.5x$

   (d) $y = 3x$

2. Calculate a table of values for each of the following lines and graph on another set of axes on your graph paper.

   (a) $y = -x$

   (b) $y = -2x$

   (c) $y = -0.5x$

   (d) $y = -3x$

You should be able to see some patterns in these graphs.

Firstly, they all pass through the origin (0,0). We will discuss this point in a moment.

Look at the lines you have drawn in question 1 in the above activity. The coefficient of x in each case is positive and the slope or gradient of the line is also positive (rising as we move from left to right). What do you notice about the value of the coefficient of x in the equation and the gradient of the line?

....

You should have said something about the greater the coefficient of x the steeper the line.

Let's now look at the equations and lines you drew in question 2 of the above activity.

This time the coefficients of x are negative and the gradients of the lines are negative (they fall as we move from left to right).

You should also note that $y = -3x$ is a much steeper line than $y = -1x$ (which may be written as $y = -x$)

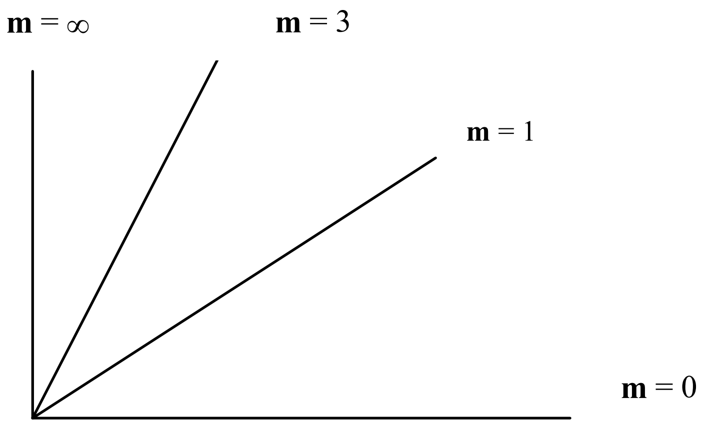
We can summarise this information gained from the last activity as follows:

The coefficient of x in a linear equation, is given the name gradient or slope (when the equation is in the form $y =$ ......, for example $y = 5x$). We will use the letter m to represent the gradient.

Therefore in:

$y = 7x$	m = 7
$y = -3x$	m = –3
$y = 0.5x$	m = 0.5
$y = \frac{x}{5}$	$m = \frac{1}{5}$

Looking again at the solution to question 1 above, if we were to continue to increase the gradient the line will eventually be running vertical as is the y-axis. Such lines have infinite slope. Conversely, if we continue to decrease the slope, eventually the line will be horizontal and have a gradient of zero as we have discovered earlier in the module.



Activity 8.5

Graph the following lines on the one set of axes on your graph paper.

(a) $y = 2x$

(b) $y = 2x + 1$

(c) $y = 2x + 2$

(d) $y = 2x - 1$

(e) $y = 2x - 2$

Once again look for patterns in this set of graphs.

Notice that all the lines are parallel to each other - that is, if you extend them infinitely in either direction, they will never meet. Now look at the value of **m** in each equation. It is always 2 for these examples. This tells us that **lines with the same gradient are parallel.**

Can you see a relationship between the point where each line crosses the *y*-axis and the number on its own in each equation?

....

In fact this number on its own tells you the point at which the graph will cut the *y*-axis. We use the letter **c** to represent the ***y*-intercept** (the point where the line cuts the *y*-axis)

Therefore in:

$y = 7x + 3$	c = 3
$y = -3x + 7$	c = 7
$y = 0.5x - 1$	c = –1

In general one way to write a linear equation is in the form:

$$y = mx + c \qquad \text{where} \quad m = \text{gradient or slope of the line}$$

$$c = \text{the } y\text{-intercept}$$

Let's summarise what we have learnt so far about linear equations.

- If **m** is positive, the gradient is positive and the line rises as we move from left to right.
- If **m** is negative, the gradient is negative and the line falls as we move from left to right.
- The greater the size of **m** the steeper the line.
- Parallel lines have the same gradient.
- The point where the line cuts the y-axis is called the y-intercept and is represented by the letter **c**.
- Lines parallel to the y-axis have infinite slope while lines parallel to the x-axis have zero slope.

This information now gives us a very powerful tool for **estimating** what a linear graph will look like given its equation.

Example

Find the slope and y-intercept of the following equation.

$$y = -3x + 2$$

Firstly, it is a linear equation. We should next check that it is in the form $y = mx + c$

In this case it is in the correct form, therefore we can read off the values for **m** and **c**.

$$\begin{aligned} \text{Gradient} &= -3 \\ y\text{-intercept} &= 2 \end{aligned}$$

Example

Find the slope and y-intercept of the following equation.

$$3x + y = 5$$

Firstly, it is a linear equation. We should next check that it is in the form $y = mx + c$

In this case it is not in the correct form, therefore we must rearrange the equation using the techniques from module 7 before we can read off the values for **m** and **c**.

$$\begin{aligned} 3x + y &= 5 && \text{We want to make } y \text{ the subject of the equation.} \\ 3x + y - 3x &= 5 - 3x \\ y &= 5 - 3x \\ y &= -3x + 5 \end{aligned}$$

It is now in the form $y = mx + c$ and we can read off the required values.

$$\text{Gradient} = -3$$

$$y\text{-intercept} = 5$$

Example

Find the slope and y-intercept of the following equation.

$$3x + 2y = 7$$

Firstly, it is a linear equation. We should next check that it is in the form $y = mx + c$

Again we must rearrange the equation before we can read off the values for **m** and **c**.

$$3x + 2y = 7$$ We want to make y the subject of the equation.

$$3x + 2y - 3x = 7 - 3x$$

$$2y = 7 - 3x$$

$$\frac{2y}{2} = \frac{7}{2} - \frac{3x}{2}$$ Divide everything on both sides by 2. Note that we have done this slightly differently to module 7, but it has exactly the same result.

$$y = \frac{7}{2} - \frac{3x}{2}$$

$$y = \frac{-3x}{2} + \frac{7}{2}$$

It is now in the form $y = mx + c$ and we can read off the required values.

$$\text{Gradient} = \frac{-3}{2}$$

$$y\text{-intercept} = \frac{7}{2}$$

Example

Find the slope and y-intercept of the following equation.

$$5x - 3y = -7$$

Firstly, it is a linear equation. We should next check that it is in the form $y = mx + c$

Again we must rearrange the equation before we can read off the values for **m** and **c**.

$$5x - 3y = -7$$ We want to make y the subject of the equation.

$$5x - 3y - 5x = -7 - 5x$$

$$-3y = -7 - 5x$$ Divide everything on both sides by –3

$$\frac{-3y}{-3} = \frac{-7}{-3} - \frac{5x}{-3}$$

$$y = \frac{-7}{-3} - \frac{5x}{-3}$$

Notice that top and bottom of these fractions are negative. Dividing two negative numbers gives a positive number.

$$y = \frac{7}{3} + \frac{5x}{3}$$

$$y = \frac{5x}{3} + \frac{7}{3}$$

It is now in the form $y = \text{m}x + \text{c}$ and we can read off the required values.

$$\text{Gradient} = \frac{5}{3}$$

$$y\text{-intercept} = \frac{7}{3}$$

Example

Just as we can find the gradient and y-intercept, given the equation of a line, it is also possible to form the equation, given the gradient and y-intercept.

Form an equation for a line with gradient 3 and y-intercept $\frac{-4}{5}$

We have, $\mathbf{m} = 3$

$$\mathbf{c} = \frac{-4}{5}$$

So the equation becomes:

$$y = \text{m}x + \text{c}$$

$$y = 3x - \frac{4}{5}$$

Activity 8.6

1. State the gradient and y-intercept of the graphs of the following linear equations.

   (a) $y = -2x + 3$

   (b) $y = 5 + 3x$

   (c) $y = 2 - 4x$

   (d) $y = x - 3$

   (e) $y = 5x$

   (f) $y + 4 = 6x$

   (g) $3x - y = 6$

   (h) $4y + 6 = -12x$

   (i) $2x + 2y - 7 = 0$

2. Write equations for lines with the following gradients and y-intercepts.

   (a) **m** = 5, **c** = 2

   (b) **m**= 3, **c** = –1

   (c) **m**= 1, **c** = 0

   (d) **m** = $\frac{1}{2}$, **c** = –5

   (e) **m**= $\frac{-3}{4}$, **c** = $\frac{7}{2}$

3. Write a few sentences comparing the graphs of the following pairs of equations.

   (a) $y = 3x + 1$

   $y = 3x + 4$

   (b) $y = 2x + 2$

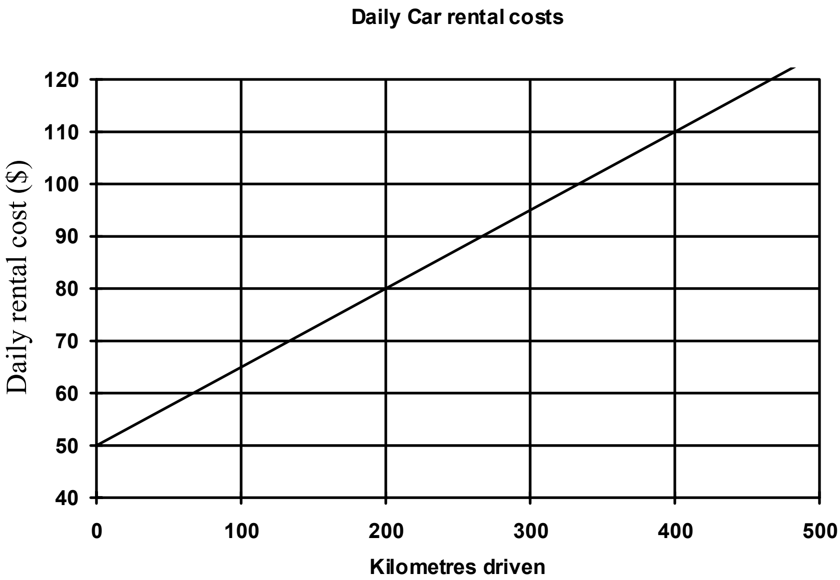
   $y = 3x + 2$

   (c) $y = 3x + 2$

   $y = -3x + 2$

4. Sketch the above pairs of graphs to check your comparisons.

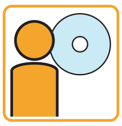
5. A car rental company charges an initial fee of \$50 per day plus 15 cents per kilometre. Following is the graph representing this situation. (We looked at this in module 5)



(a) Calculate the gradient and y-intercept from the above graph.

(b) Can you interpret the meaning of the gradient and y-intercept for this question?

(c) Now form the equation for this relationship.



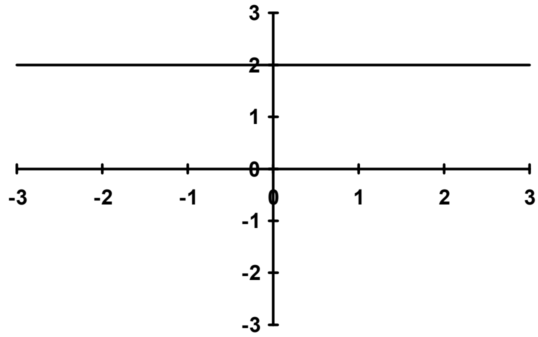
Check out your resource CD and in particular ‘Worksheet 2: Straight line’. This is another interactive worksheet that will help you understand the relationship between the gradient, intercept and equation of a straight line.

8.2.1 Special lines

There are two other graphs that you will need to recognise that do not use the above form.

Horizontal lines

We have already talked about the gradient of a horizontal line. The gradient of a line parallel to the x-axis is zero.

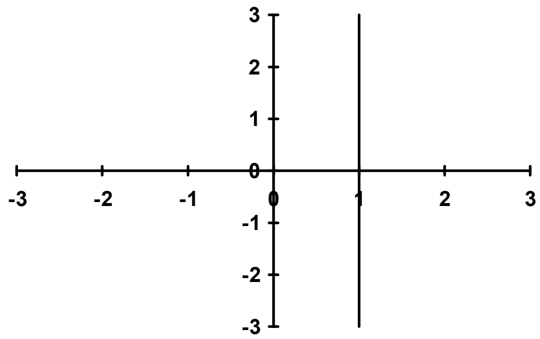


You will notice that the value for y is 2 no matter what x value we choose. We say that the equation of this line is $y = 2$

Any line parallel to the x-axis will have an equation in the form $y = a$

Vertical lines

We have already talked about the gradient of a vertical line. The gradient of a line parallel to the y-axis is infinity.



You will notice that the value for x is 1 no matter what y value we choose. We say that the equation of this line is $x = 1$

Any line parallel to the y-axis will have an equation in the form $x = b$

8.2.2 What if two lines cross?

In module 7 we examined how to solve simultaneous linear equations using algebra. We can now interpret these results graphically. The **simultaneous solution** for a pair of equations is the **point of intersection** of the two graphs.

Consider the following question from an activity in module 7.

Example

Living on Anklebiter Avenue are 26 children. There are two more boys than there are girls. How many girls and how many boys live on Anklebiter Avenue?

Let the number of boys be B and the number of girls be G.

$$B + G = 26 \qquad (1)$$

$$B = G + 2 \qquad (2)$$

If we graph these two equations we will find the point of intersection. To draw our graphs, firstly construct a table of values for each line. Since we don't have any x's and y's this time we will need to plot the B and the G on the axes. But which will go on which axis? The simple answer is that it doesn't really matter in this case, so we will plot the number of girls on the x-axis.

For $B + G = 26$

Many people find it easier to calculate values for the table of values if the equation has one of the variables as the subject, as in equation (2).

We will thus rewrite the above equation as:

$$B = 26 - G$$

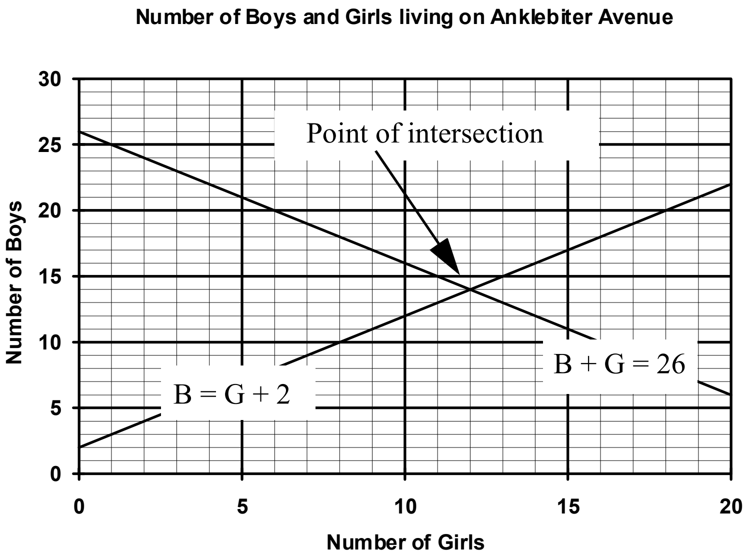
Before choosing values for G we need to think about our answers. We can only have positive numbers of girls and boys so we will only put positive numbers in our table.

G	0	10	20
B	26	16	6

For $B = G + 2$

G	0	10	20
B	2	12	22

Graphing both lines on the one set of axes gives us



The point of intersection is (12,14)

We must interpret this to mean that there are 12 girls and 14 boys living on Anklebiter Avenue.

We should check this answer in each of the original equations as we have done in the past.

Equation (1) LHS = $B + G$
= 14 + 12
= 26
= RHS

Equation (2) LHS = B = 14

RHS = $G + 2$
= 12 + 2
= 14
= LHS

Activity 8.7

1. By graphing the following sets of equations, find the simultaneous solution. Check your answer in both original equations.

   (a) $y = x$

   $y = -4x + 5$

   (b) $y = x + 1$

   $y = 4x - 2$

   (c) $y = -x - 3$

   $y = -2x - 4$

2. Two sales assistants are paid under different systems. Alf is paid \$120 as a wage and \$40 for each item sold while Sally is paid \$80 for each item sold. Using a graphical technique, determine how many items they must sell to earn the same income?

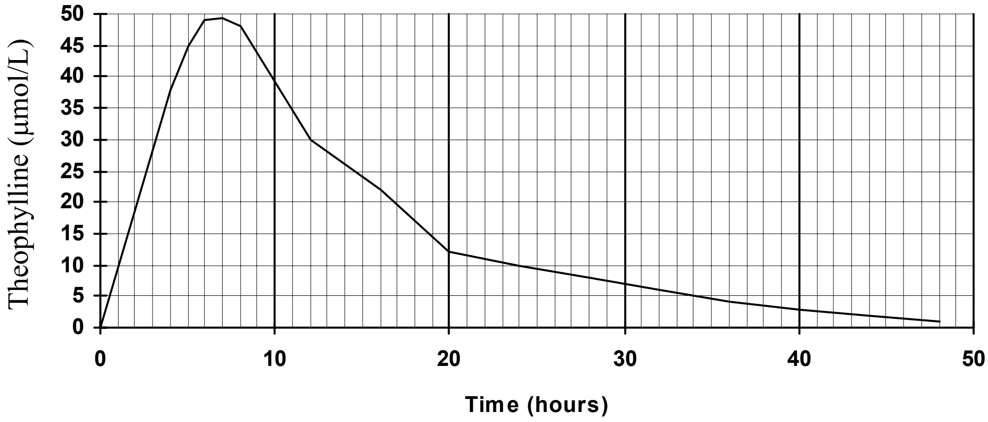
   (a) Form two equations from the given information. Don't forget to define your variables.

   (b) Graph the two lines to find the point of intersection.

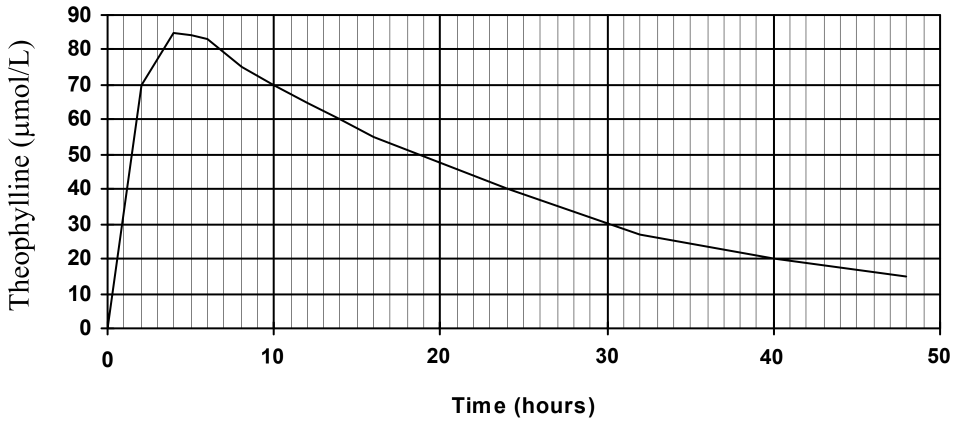
8.3 Introduction to curves

So far we have looked in detail at straight lines. You should now be able to see the relationship between straight lines and their graphs and be able to 'read' straight line graphs by describing the relationship between the two variables involved. But what happens when the graphs are curved? For example in nursing you may look at graphs showing drug levels in the body over time as in the two figures below. In the first figure it is a single dose, in the second it is a Intravenous (IV) infusion.

Theophylline levels with single oral dose



Theophylline levels with IV infusion

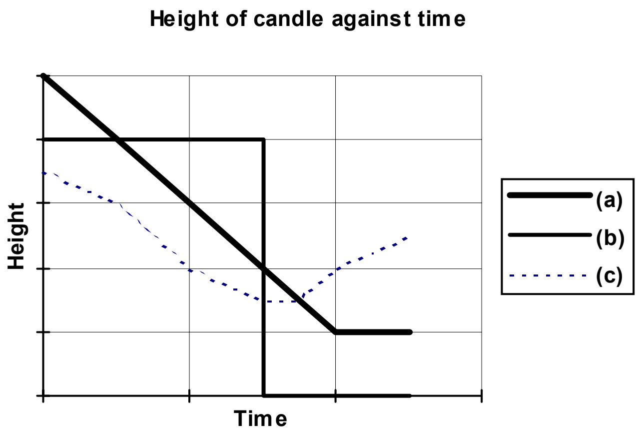


Would you be able to explain these two graphs clearly and would you be able to draw similar graphs if more of the drug were administered? We will return to these graphs at a later stage.

Before you attempt to do that let's look at some simpler curves.

Example

Consider the following graph showing the different ways in which a birthday candle might burn.



Which of the above graphs do you think best represents a birthday candle burning?

First look at the labels on the axes. They are height and time. There is no scale so we can use our imagination – let's say the height ranges from 5 cm to 0 cm, and the time from 0 to 30 seconds.

Next try and tell a story as you look at the lines reading from left to right.

For example:

Graph (a): You could say, as time goes on the height of the candle is **steadily** decreasing. Then suddenly there is no change in height. This could be true – the candle is burning and then someone blows it out.

Graph (b): As time goes on there is no change in height in the candle then suddenly the height goes down to zero. This could be true - maybe someone stole the candle!

Graph (c): As time goes on the candle's height goes down slowly at first then more quickly, then stops and starts growing again! Could this be true?

There is **no** right answer but perhaps (a) is the most probable.

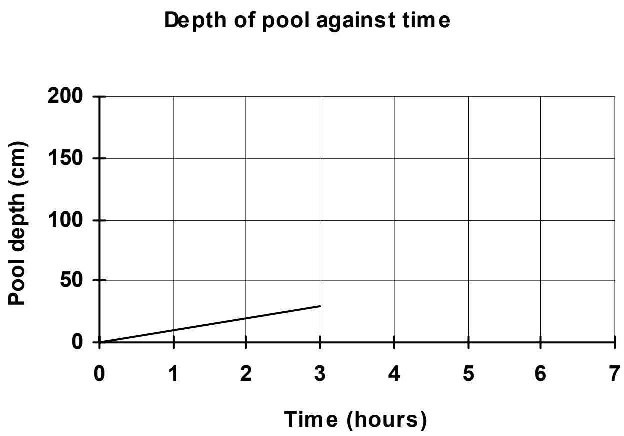
Before going on to some more examples, look at some of the language used in the above descriptions. Why did I use the word **steadily** in answer to (a)?

If you said something like *because it was a straight line* you would be right. There is a steady **rate** of decrease. If you knew the rate - maybe it was 10 mm per minute, that would be the gradient i.e. –10 (remember a negative gradient means the line is falling as we move from left to right).

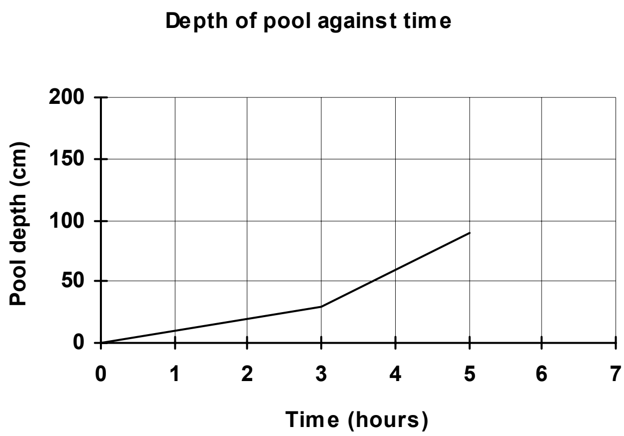
In answer to (c) I said the candle's height goes down slowly then more quickly, then stops. How can you see that on the graph? It has something to do with the gradient but this time there is a curve. Let's look at this in a bit more detail, using your knowledge from the earlier sections of this module.

Example

Suppose I have just built a pool in my back yard. I am now ready to fill the pool. The pool has straight sides and I put the hose on at a steady rate. It works out that the water goes up at a rate of 10 cm per hour. The graph looks like this

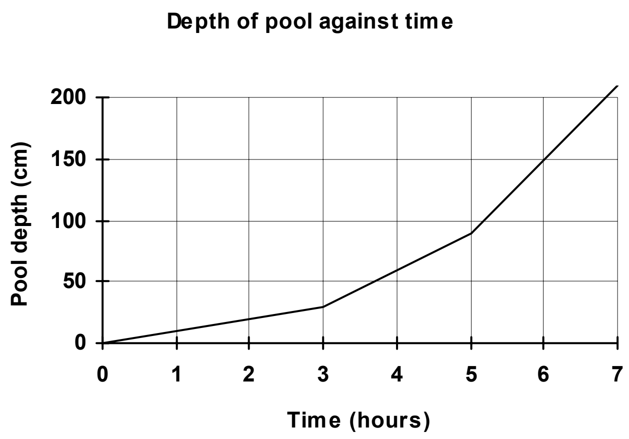


Now after 3 hours, I decided to turn the tap a bit more and it is now filling at 30 cm per hour. After two hours the graph would now look like this

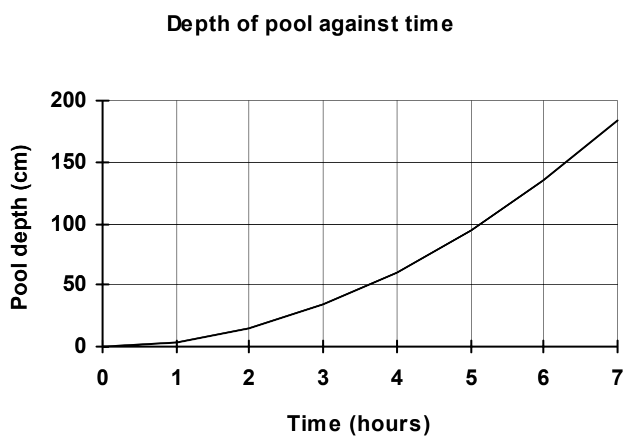


I then get really impatient and get the neighbour's hose (I promised them a swim). The pool now fills twice as fast (60 cm per hour). The graph now looks like this

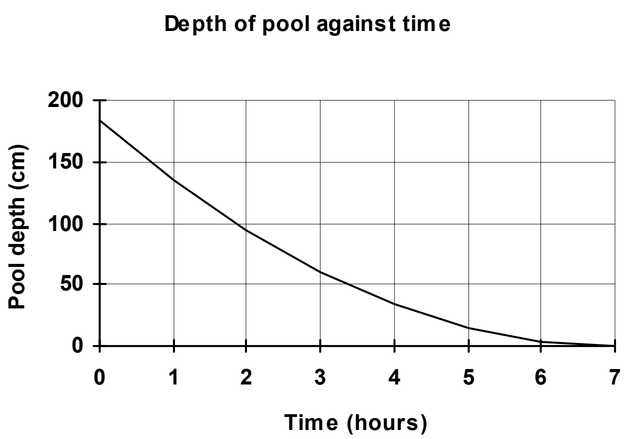
.



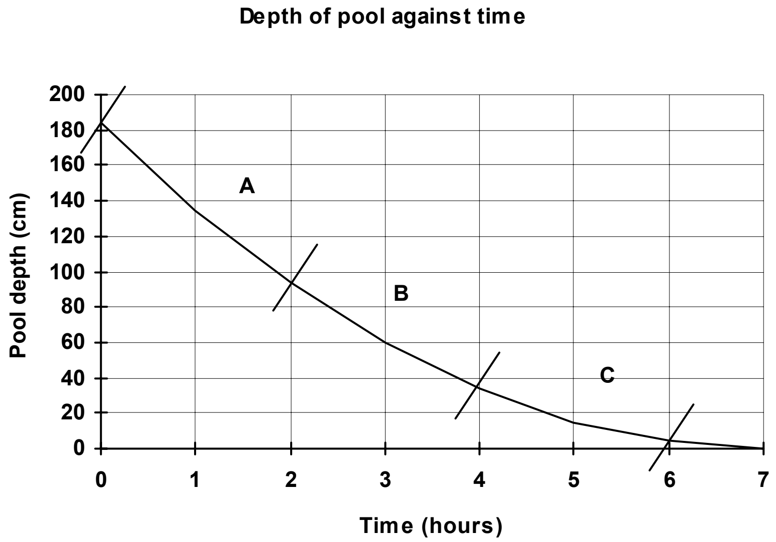
Notice what is happening as the slope of the line changes. The steeper the slope the bigger the rate of change. Imagine what would happen (theoretically) if I stood there next to the hose and gradually turned the hose on more and more. The graph would now be a smooth curve.



At the end of the summer the pool is drained. The graph may look like this.



You could say that as time went on the water drained at a slower rate. How can you see this? Let's take different two hour time slots and compare gradients.



Over period A two hours have passed and about 90 cm has drained. You could say the rate was about 45 cm per hour (this is not quite true - it's just an average).

Over period B the rate was 30 cm per hour (about 60 cm in 2 hours).

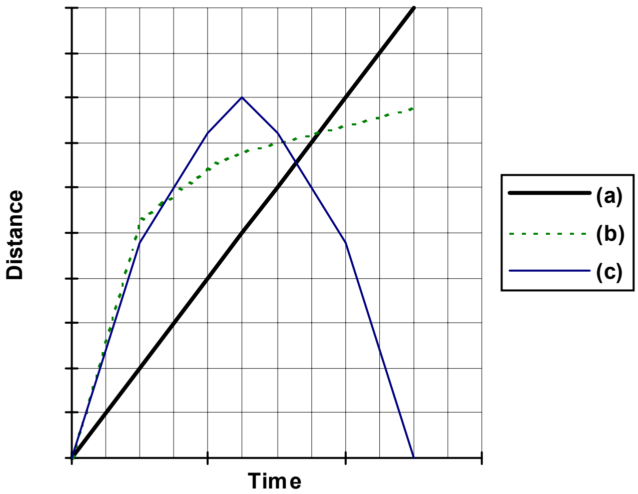
Over period C the rate was 15 cm per hour (about 30 cm in 2 hours).

Over section A of the graph the pool is draining at a much greater rate (45 cm/h) than period B (30 cm/h). Both of these in turn are draining at a much greater rate than over section C (15 cm/h). You might also have noticed that the curve was steeper over period A than over the other two periods.

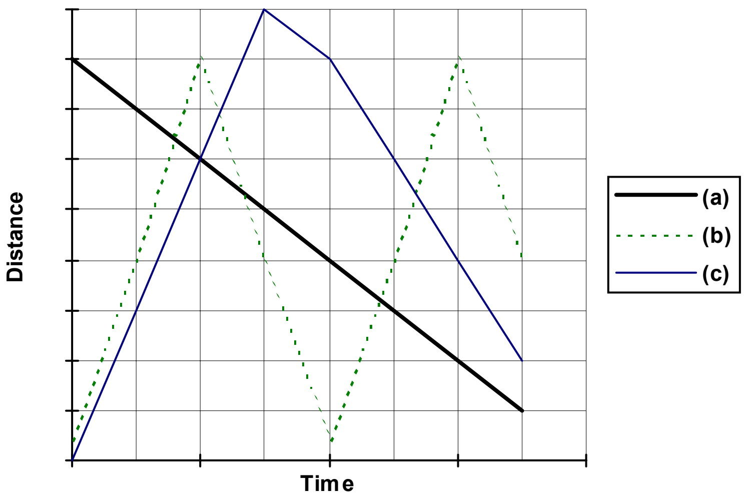
Activity 8.8

1. Match the given phrase with its correct graph. Then write a few sentences explaining your solution.

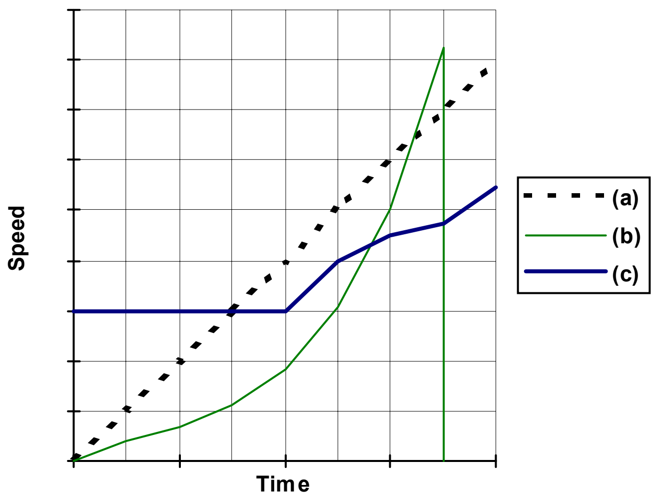
   (a) Running a long distance race



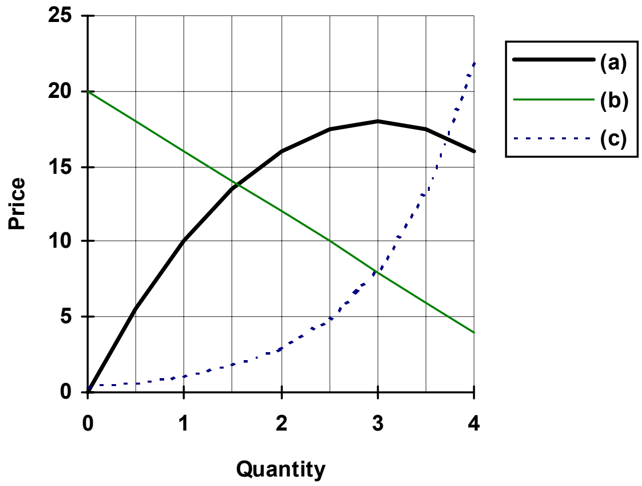
   (b) Height of a ferris wheel



(c) A skier going up in a chairlift then skiing down (note the labels on the axes).



(d) The price of bananas as the quantity demanded changes



2. Sketch the following and justify your answer.

   (a) a person raising a flag

   (b) grass height in your back yard

   (c) you riding a bicycle up a hill

   (d) blowing up a balloon

   (e) the profit that could be taken charging different admission prices to the theatre

Many of the curves in this section so far could be drawn using formulas. But these formulas are more complex than the straight line ones you have met so far. We can, however, look at some simpler curves, that are easier to draw, and that we see around us and in texts. In this next section we will look at parabolas and exponentials.

8.4 Parabolic equations

Suppose that you and your friend find an abandoned well. You wonder how deep it is and decide to drop something down the well and listen for the splash. You find a light stone nearby and your friend a heavy stone. You both drop the stones at the same time into the well. Whose stone will hit the water first?

If you think that the heavy stone will hit the bottom first, you are not on your own. The early Greek scientists thought that the heavy stone would hit the bottom first. They believed that objects fell because they were attracted to the earth, and that the heavier stone would fall more quickly because it was more strongly attracted to the earth. They were in fact wrong!

In the seventeenth century, the Great Italian scientist Galileo discovered that the speed of an object does not depend on its weight. In fact the small and large stones will hit the bottom of the well at the same time.

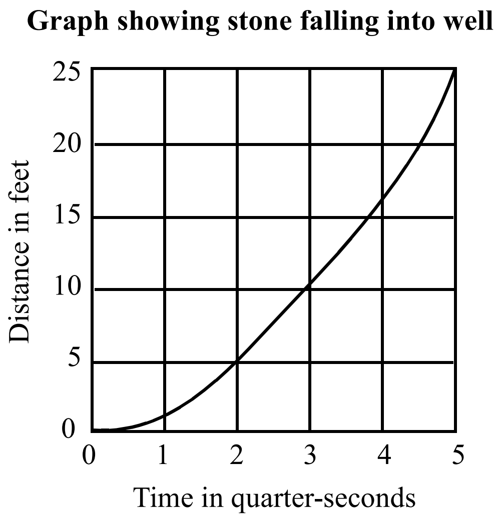
The relationship between the distance the stone falls and the time it takes is represented in the table below.

Time in quarter-seconds t	0	1	2	3	4	5
Distance in feet, d	0	1	4	9	16	25

The formula for this relationship is:

$$d = t^2$$

If we plot these points on a graph it will look like the graph below. Unlike the graphs we have drawn in the past, the points do not lie in a straight line. They can be connected instead with a smooth curve.



In this example we have restricted the time values to positive numbers. Let's look at this equation in a more general form without such restrictions.

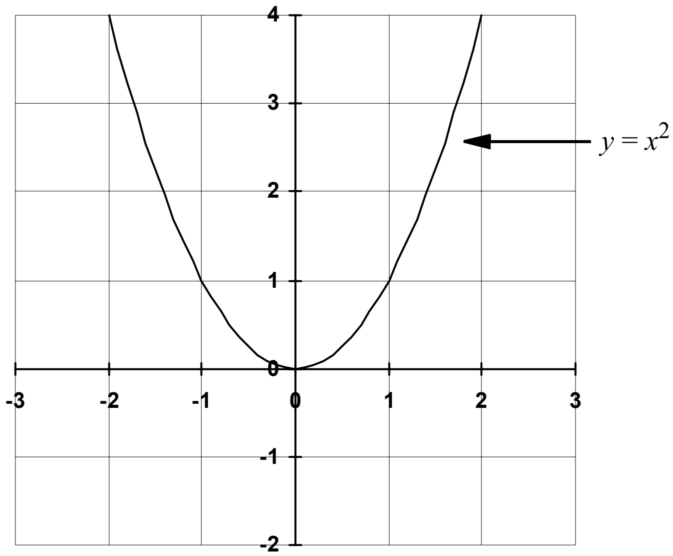
Example

Draw the graph of $y = x^2$

Firstly set up a table of values.

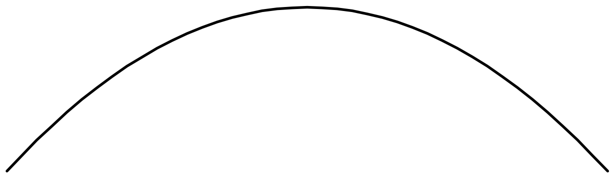
x	–2	–1	0	1	2
$y = x^2$	4	1	0	1	4

Now plot these points and join them up with a smooth curve.



We call this curve a **parabola** (pronounced par/ab/ol/a).

The word parabola comes from the Greek word meaning *thrown*, because that shape was recognized as the path followed by an object in flight. Imagine our parabola turned upside-down



The nature of a parabolic curve makes it ideal for use in head-light protectors and satellite receiving dishes.

You will notice that the equation of the relationship we graphed had one of the variables raised to the power 2. Just as we could recognise a linear equation because the variables had powers of one and no variables multiplied together, we can now recognise a parabolic equation because is has one variable to the power 2 and no variables multiplied together.

Example

Draw the graph of $y = -x^2$

Recall:
Module 3
Section 3.1

You will need to recall here work that we did in module 3.

Problems often arise when students are not sure of the difference between the following two types of expressions.

$(-2)^2$ and -2^2

In the first expression, $(-2)^2$, the base is –2 and $(-2)^2 = -2 \times -2 = 4$

The difference with the second expression, -2^2, is that the base is 2. We could rewrite this expression as $-(2^2)$. We could even think of this expression as -1×2^2. The answer in this case is –4. Your answer depends on your being very clear about what number is the base.

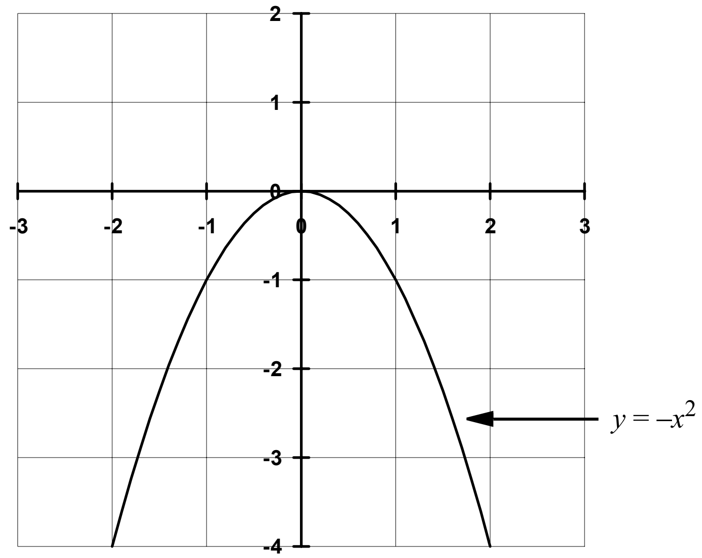
Back to our graph

$$y = -x^2$$

Firstly a table of values.

x	–2	–1	0	1	2
$y = -x^2$	–4	–1	0	–1	–4

Now plot these points and join them up with a smooth curve.



Look carefully at the graphs in the previous two examples. The only difference in the equations was a negative sign in front of the x^2 (that is, $y = –x^2$ instead of $y = x^2$).

The sign in front of the x^2 term (that is, the coefficient of x^2) tells us whether the parabola is going to have a maximum (i.e. of the shape ∩) or a minimum (i.e. of the shape ∪).

If the **coefficient of x^2 is positive** the parabola will open **upwards (∪)** and will have a minimum.

If the **coefficient of x^2 is negative** the parabola will open **downwards (∩)** and will have a maximum.

Examples

The graph of $y = 2x^2$ will be a parabola because one of the variables has a power of 2. The parabola will open upwards because the coefficient of x^2 is positive (2).

The graph of $y = 5x^2 + 3x + 4$ will be a parabola because one of the variables has a power of 2. The parabola will open upwards because the coefficient of x^2 is positive (5).

The graph of $y = –3x^2$ will be a parabola because one of the variables has a power of 2. The parabola will open downwards because the coefficient of x^2 is negative (–3).

The graph of $y = 2 + 3x – 6x^2$ will be a parabola because one of the variables has a power of 2. The parabola will open downwards because the coefficient of x^2 is negative (–6).

Let's now look at graphing some more involved parabolas and the effects on the graph of various changes in its equation. Before we move on to do this we will practice drawing some parabolas.

Example

Graph $y = 5x^2 + 3x + 4$

To draw this we will firstly need to construct a table of values as we have done before.

x	–2	–1	0	1	2
$y = 5x^2 + 3x + 4$					

When $x = -2$

$y = 5x^2 + 3x + 4$

$y = 5 \times (-2)^2 + 3 \times -2 + 4$

$y = 5 \times 4 + -6 + 4$

$y = 20 + -6 + 4$

$y = 18$

When $x = -1$

$y = 5x^2 + 3x + 4$

$y = 5 \times (-1)^2 + 3 \times -1 + 4$

$y = 5 \times 1 + -3 + 4$

$y = 5 + -3 + 4$

$y = 6$

When $x = 0$

$y = 5x^2 + 3x + 4$

$y = 5 \times (0)^2 + 3 \times 0 + 4$

$y = 5 \times 0 + 0 + 4$

$y = 0 + 0 + 4$

$y = 4$

When $x = 1$

$y = 5x^2 + 3x + 4$

$y = 5 \times (1)^2 + 3 \times 1 + 4$

$y = 5 \times 1 + 3 + 4$

$y = 5 + 3 + 4$

$y = 12$

When $x = 2$

$y = 5x^2 + 3x + 4$

$y = 5 \times (2)^2 + 3 \times 2 + 4$

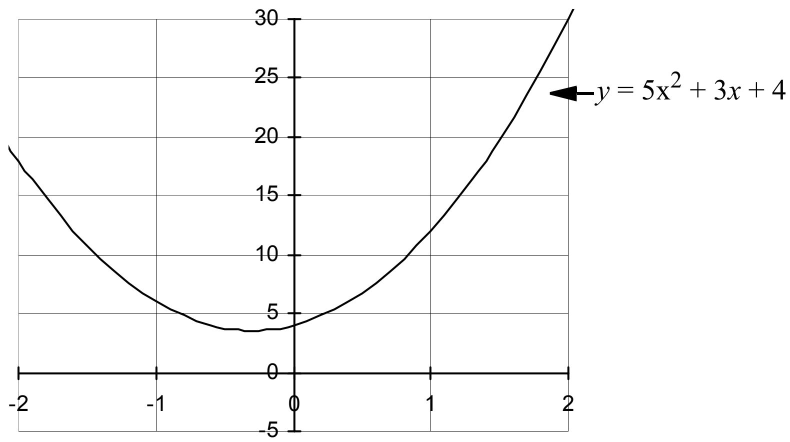
$y = 5 \times 4 + 6 + 4$

$y = 20 + 6 + 4$

$y = 30$

x	–2	–1	0	1	2
$y = 5x^2 + 3x + 4$	18	6	4	12	30

Now you are ready to plot the points on the Cartesian plane and join them up with a smooth curve.



Example

Graph $y = 2 + 3x - 6x^2$

To draw this we will firstly need to construct a table of values as we have done before.

x	–2	–1	0	1	2
$y = 2 + 3x - 6x^2$					

When $x = -2$

$y = 2 + 3x - 6x^2$

$y = 2 + 3 \times (-2) - 6 \times (-2)^2$

$y = 2 + -6 - 6 \times 4$

$y = 2 + -6 - 24$

$y = -28$

When $x = -1$

$y = 2 + 3x - 6x^2$

$y = 2 + 3 \times (-1) - 6 \times (-1)^2$

$y = 2 + -3 - 6 \times 1$

$y = 2 + -3 - 6$

$y = -7$

When $x = 0$

$y = 2 + 3x - 6x^2$

$y = 2 + 3 \times (0) - 6 \times (0)^2$

$y = 2 + 0 - 6 \times 0$

$y = 2 + 0 - 0$

$y = 2$

When $x = 1$

$y = 2 + 3x - 6x^2$

$y = 2 + 3 \times (1) - 6 \times (1)^2$

$y = 2 + 3 - 6 \times 1$

$y = 2 + 3 - 6$

$y = -1$

When $x = 2$

$y = 2 + 3x - 6x^2$

$y = 2 + 3 \times (2) - 6 \times (2)^2$

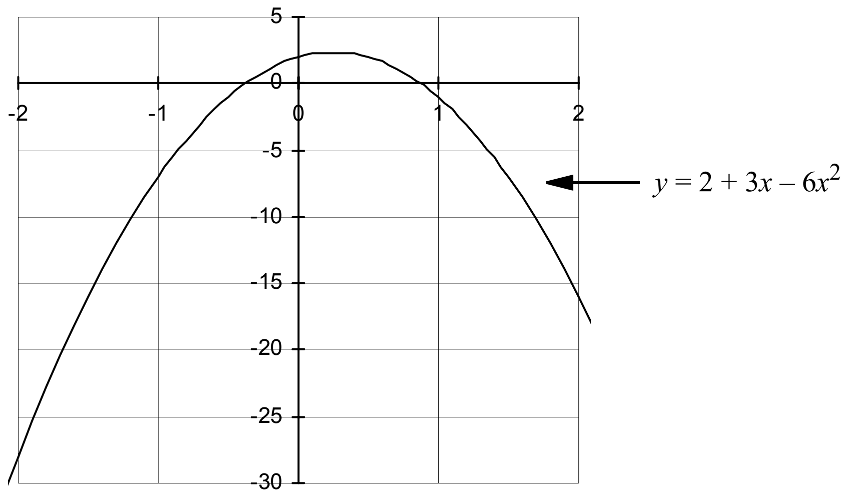
$y = 2 + 6 - 6 \times 4$

$y = 2 + 6 - 24$

$y = -16$

x	–2	–1	0	1	2
$y = 2 + 3x - 6x^2$	–28	–7	2	–1	–16

Now you are ready to plot the points on the Cartesian plane and join them up with a smooth curve.



Activity 8.9

Draw the graphs of the following parabolic equations on the same set of axes.

(a) $y = x^2$

(b) $y = x^2 + 1$

(c) $y = x^2 + 2$

Did you notice that in each case the number on its own is the y-intercept. We call this the **constant** term in the equation because is stays the same (it does not vary as x does).

Let's summarise what we have learnt so far.

- If one of the variables in a two variable equation, is raised to power 2 and no variables are multiplied, the graph will be a parabola.
- Assuming the graph is a parabola, then
  - if the coefficient of the x^2 term is positive, the graph will open upwards and have a minimum.
  - If the coefficient of the x^2 term is negative, the graph will open downwards and have a maximum.
- The parabola will have a y-intercept equal to the value of the constant term.

Activity 8.10

1. For the following, state which graphs will be lines and which will be parabolas.

   (a) $y = 3x$

   (b) $y = 2x^2$

   (c) $y = -4x + 1$

   (d) $y = 2 - 3x^2$

   (e) $y = 2x^2 + x + 1$

2. Describe the graphs associated with the following equations, giving as much detail as possible.

   (a) $y = 4x^2 - 3x + 7$

   (b) $y = -3x^2 - 4$

   (c) $y = 5x + 6$

   (d) $y = 5 + 4x - 3x^2$

   (e) $y = 7 - 5x$

   (f) $y = 4 - 5x + 2x^2$

3. Sketch the following parabolas. Be sure to think about what the graph will look like before you graph it.

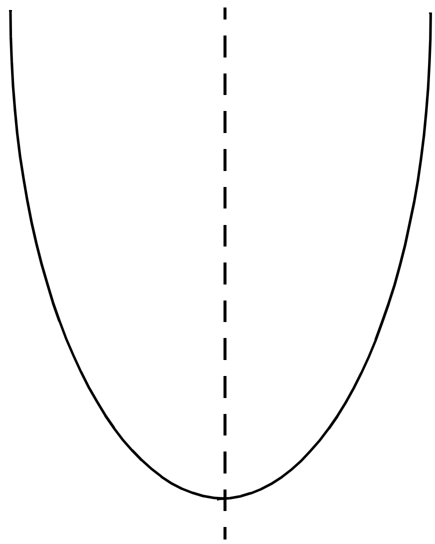
   (a) $y = x^2 + 2x - 3$

   (b) $y = 2 + x - x^2$

8.4.1 The axis of symmetry

Look back to any of the parabolas that you have already drawn. Imagine there is a line drawn down the centre of the parabola. Fold the parabola in half along this line.

You should find that the two sides of the parabola lie exactly on top of each other.



This line that you folded on is called the **axis of symmetry.**

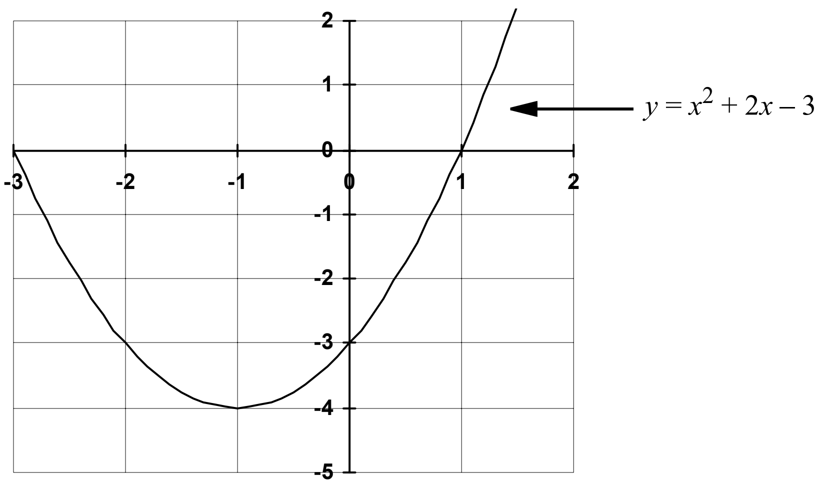
You may also have noticed that the axis of symmetry passes through the maximum or minimum turning point of the parabola.

Since this axis is a vertical line, it will have an equation in the form $x = \mathrm{a}$.

In this unit, we want you to be able to estimate where that axis of symmetry would lie. If you do more maths in the future you will study other methods for finding this equation exactly.

Example

For the parabola $y = x^2 + 2x - 3$, the graph will look like this:



The graph goes down to a minimum of $y = -4$ when $x = -1$. The axis of symmetry is $x = -1$

Being able to find the axis of symmetry also allows us to find the exact turning point. The axis of symmetry has given us the x-value of the turning point and by substituting this into the equation we can find the y-value. For this curve, when $x = -1$, $y = -4$. therefore the minimum turning point will be (–1,–4) as you can see on the above graph.

Notice that the points on the right hand side are the same distance from the axis of symmetry as the points on the left hand side. For example the point (1,0) and (–3,0) are both 2 units away from the axis of symmetry.

Activity 8.11

1. Draw the graph of each of the following parabolas. Label the axis of symmetry and the maximum or minimum turning point.

   (a) $y = 2x^2$

   (b) $y = 2 - 3x^2$

   (c) $y = 2x^2 + x + 1$

2. **Use the graphs** drawn in question 1 to **predict** the y-values when $x = -1.5$ and $x = 3$

3. The height H metres reached by a bullet after t seconds when projected upwards at 200 ms^{-1} is obtained from the formula $H = 200t - 10t^2$

   (a) Draw up a table of values of H and t for $t = 0, 5, 8, 10, 12$, and 15.

   (b) Draw a graph of H against t.

   From your graph

   (c) Find the maximum height reached by the bullet.

   (d) At what times does it reach 800 metres above the ground?

4. A stone is thrown vertically upwards from the ground. The height (in metres) above the ground at any time, t (in seconds) is given by:

   $$H = 3t - 0.3t^2$$

   Using graphical techniques find:

   (a) the greatest height reached.

   (b) the time at which this height is reached

   (c) the height of the stone after 2.5 seconds.

5. The area of a certain rectangle is given by the formula

   $A = -3w^2 + 12w$ where w is the width in metres and A is the area.

   Using graphical techniques find:

   (a) the maximum area of the rectangle.

   (b) the width that gives this maximum area.

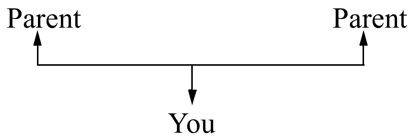
8.5 Exponential equations

Megan Sue Austin, born on 16th May 1982, is listed in the *Guinness Book of Records* because she was born with the greatest number of living ancestors. Still living were a full set of grandparents, a full set of great-grandparents and five of her great-great-grandparents, making 19 direct ascendants (the opposite of descendant).

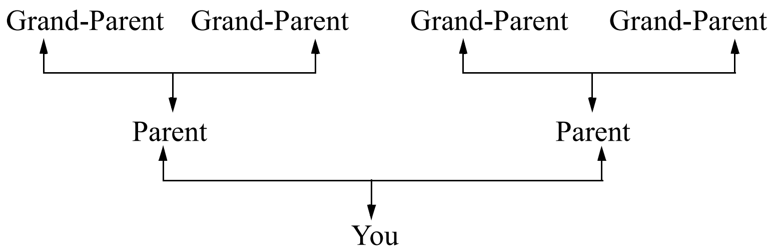
Let's look at the number of ancestors a person has.

Firstly at no generations back there is you, one person.

At one generation back there are two people, your mother and your father.



At two generations back there will be 4 people, your grandparents.



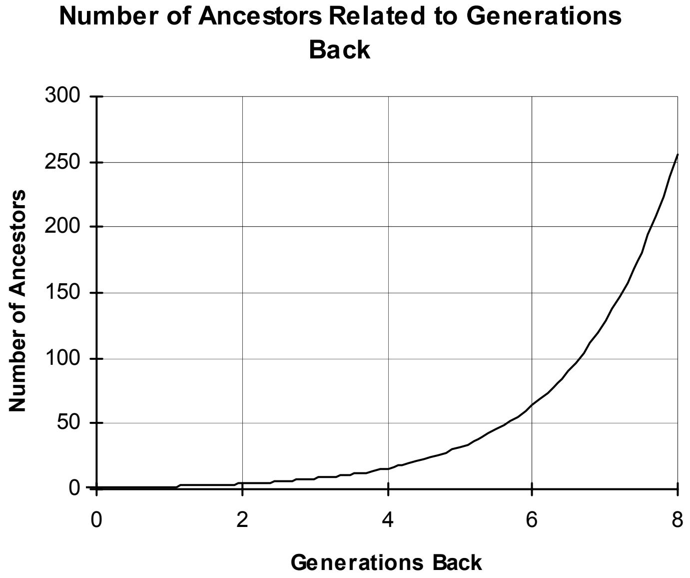
We could continue this indefinitely, but instead let's look at this information in a table.

Generations back (x)	0	1	2	3	4	5	6	7	8
Number of ancestors (y)	1	2	4	8	16	32	64	12 8	25 6

Let's now plot these points on a Cartesian plane, plotting the generations back on the horizontal axis and the number of ancestors on the vertical axis. Go ahead and do this on some graph paper.

It is a little difficult to plot because of the large variation in the values to be plotted on the vertical axis.

Your graph should look something like the one below.



The equation to this curve is $y = 2^x$. This type of graph is called an **exponential growth curve** (pronounced ex/po/nen/shal) and is often used in population growth studies. The name refers to the position of the x in the exponent of the equation. As the number of generations back (x) increases, the number of ancestors (y) gets greater and greater. The curve is getting steeper and steeper.

In the above question we have only looked at positive values for the x variable. In other examples we will need to include the negative x values as well.

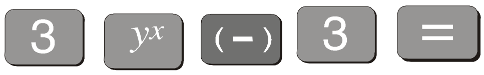
Example

Use your calculator to complete the following table of values and then draw the graph of $y = 3^x$ Round your answers to one decimal place.

x	–3	–2	–1	0	1	2	3
$y = 3^x$							

$x = -3$	$x = -2$	$x = -1$	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$y = 3^x$	$y = 3^x$	$y = 3^x$	$y = 3^x$	$y = 3^x$	$y = 3^x$	$y = 3^x$
$y = 3^{-3}$	$y = 3^{-2}$	$y = 3^{-1}$	$y = 3^0$	$y = 3^1$	$y = 3^2$	$y = 3^3$
$y \approx 0.04$	$y \approx 0.1$	$y \approx 0.3$	$y = 1$	$y = 3$	$y = 9$	$y = 27$

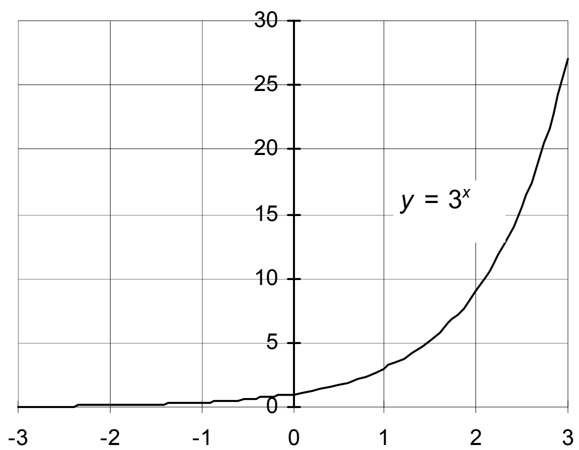
Reminder: To evaluate 3^{-3} on your calculator press:



Now try this on your calculator.

Write down your calculator steps in the space below.

x	–3	–2	–1	0	1	2	3
$y = 3^x$	0.04	0.1	0.3	1	3	9	27



As x takes on more negative values, the curve comes closer and closer to the x-axis (the y values get closer to zero) but **never** touches it.

No matter what the value for x, no matter how large or how small, the value for y is always going to be positive.

Activity 8.12

Draw the following graphs. Use your calculator to find the values for y.

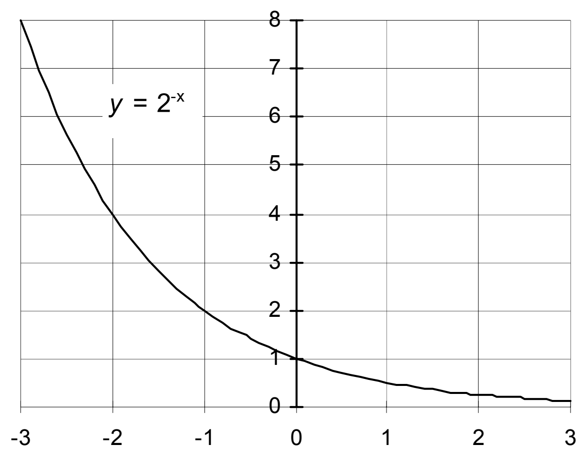
1. $y = 2^x$
2. $y = 4^x$
3. $y = 2^{-x}$
4. $y = 3^{-x}$

Let's look more closely at the last two graphs that you drew.

What did you notice about the exponent in these two questions? ....

Did this effect the look of your graph? ....

For these two questions the exponent was negative and this meant that the graph fell as you moved from left to right.



We call this type of graph an **exponential decay curve**. This time as x takes on more positive values, the curve comes closer and closer to the x-axis but **never** touches it. Exponential decay curves occur in such areas as science when we are talking about radio active decay and in business when we are talking about depreciation.

Now that you have drawn several exponential curves, try to answer this question.

What point is common to all the exponential curves that you have drawn so far?

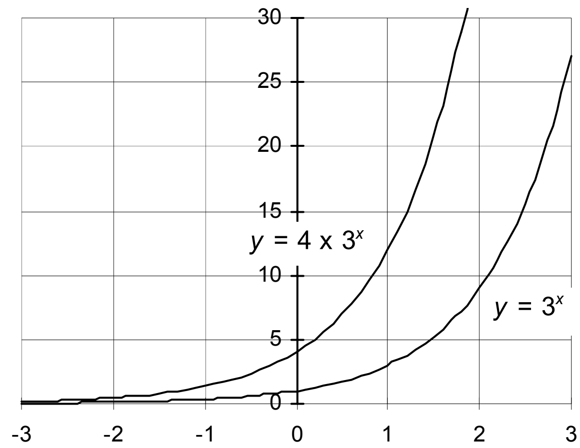
....

In fact, every graph that you draw in the form $y = a^x$ and $y = a^{-x}$ where a is a positive real number, will pass through the point (0,1)

However there are some exponential graphs that do not pass through this point.

Look at the graph of $y = 3^x$ that we looked at before. We will now graph $y = 4 \times 3^x$ on these same axes.

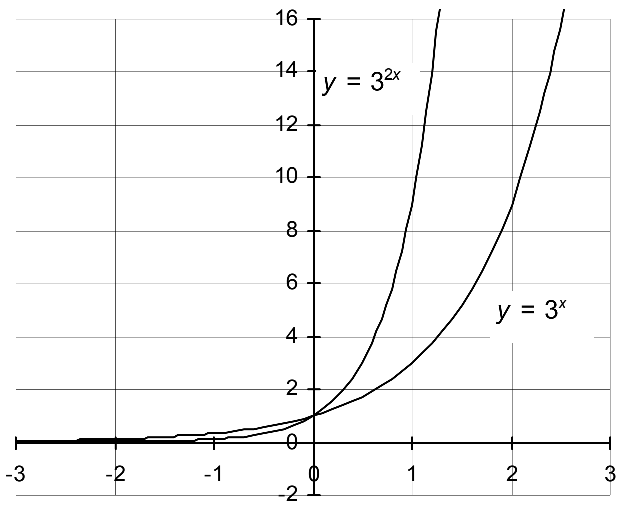
x	–3	–2	–1	0	1	2	3
$y = 4{\times}3^x$	0.16	0.4	1.2	4	12	36	108



Notice that this time the graph $y = 4 \times 3^x$ cuts the y-axis at 4 while the graph $y = 3^x$ cuts the y-axis at 1 as in the past. This coefficient at the front of the expression has given us the y-intercept.

Let's now look at a change in the exponent.

Again look at the graph of $y = 3^x$ Let's compare it to $y = 3^{2x}$ and you can see the effect of doubling the exponent.



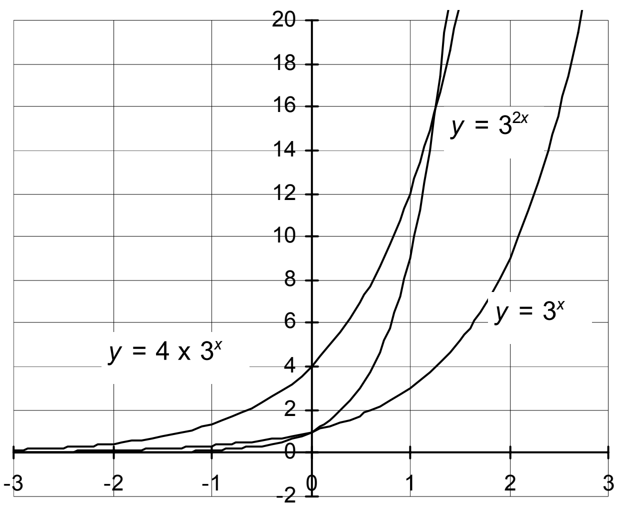
This time the values of y have risen very quickly as the x-values increased. The curve is much steeper for positive values of x. Doubling the exponent has meant a marked increase in the steepness of the graph.

Even in the previous graph where we were comparing the y-intercepts there was an increase in the steepness of the curve at any particular x-value.

Finally let's compare all the three graphs $y = 3^x$, $y = 4 \times 3^x$ and $y = 3^{2x}$

Let's look at all the values set out in the following table.

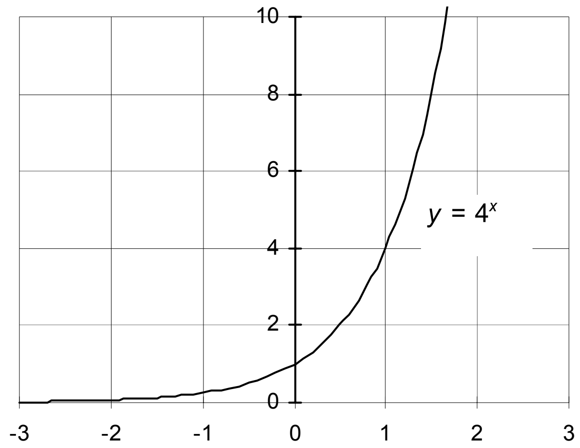
x	–3	–2	–1	0	1	2	3
$y = 3^x$	0.04	0.1	0.3	1	3	9	27
$y = 3^{2x}$	0.001	0.01	0.11	1	9	81	729
$y = 4{\times}3^x$	0.15	0.44	1.33	4	12	36	108



Have a look at the graph when $x = 1$. Here the value of y (y = 9) in $y = 3^{2x}$ is smaller than the value of y (y = 12) in $y = 4 \times 3^x$. When $x = 3$, the value of y in $y = 3^{2x}$ is 729 compared with 108 in $y = 4 \times 3^x$. The values of y have increased more rapidly in $y = 3^{2x}$ than in $y = 4 \times 3^x$

Activity 8.13

1. The graph of $y = 4^x$ is given below.

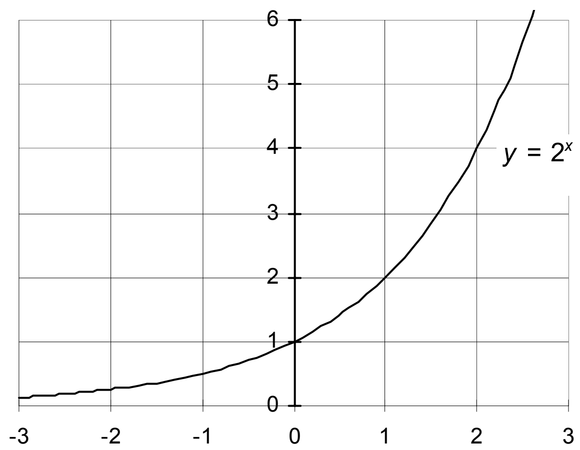


Remembering that the coefficient of the right hand term gives the y-intercept, sketch the following graphs onto the above diagram. Do **not** plot points.

(a) $y = 0.5 \times 4^x$

(b) $y = 2 \times 4^x$

2. The graph of $y = 2^x$ is given below.



Sketch the following graphs onto the above diagram. Do **not** plot points.

(a) $y = 2^{0.5x}$

(b) $y = 2^{2x}$

8.5.1 A special number

There is one more example that you need to look at. That is using the base 2.718281..... This may look a little strange but there is a name for this number. It is an irrational number **e**. (remember π was another irrational number)

The main thing to remember is that **e** is just another number. It is a number lying between 2 and 3 on the number line. We can use **e** in formulas and equations and we can in turn graph these on the Cartesian plane.

Let's firstly look at evaluating some powers of **e**

Example

Evaluate $\mathbf{e}^3$ to 4 decimal places.

On your calculator press

The display should read 20.08553692

Rounded to 4 decimal places $\mathbf{e}^3 \approx 20.0855$

Example

Evaluate $\mathbf{e}^{-2.1}$ to 4 decimal places.

Now try this on your calculator.

Write down your calculator steps in the space below.

The display should read 0.122456428

Rounded to 4 decimal places $\mathbf{e}^{-2.1} \approx 0.1225$

We will now look at graphing two exponential equations involving **e**.

Activity 8.14

1. Complete the following table of values for $y = \mathbf{e}^x$ (round your answers to 2 decimal places).

x	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2
$y = \mathbf{e}^x$									

2. Complete the following table of values for $y = \mathbf{e}^{-x}$ (round your answers to 2 decimal places).

x	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2
$y = \mathbf{e}^{-x}$									

3. Now draw the above two graphs on the one set of axes on 2 mm graph paper.

You should have found that $y = \mathbf{e}^x$ has the same shape as the other exponential growth curves and that $y = \mathbf{e}^{-x}$ has the same shape as the exponential decay curves.

Let's look at an exponential equation involving **e** in a practical example.

When we drink alcohol the body eliminates it slowly but this varies from person to person. It can, however, be expressed as an equation involving **e**.

Example

At a pub, Chris has drunk quite a bit of alcohol. Her blood alcohol concentration may be 0.1 (remember the legal limit in Australia is 0.05). To look at the elimination of the alcohol from her body, the formula could be:

$$y = 0.1\mathbf{e}^{-0.4t}$$

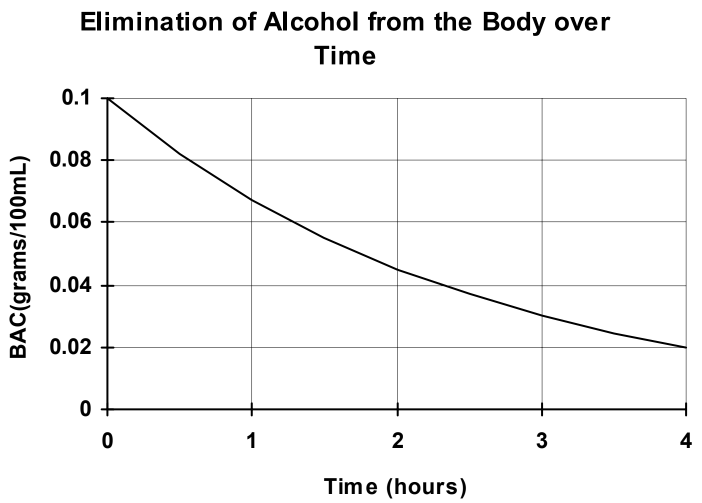
where y represents the blood alcohol concentration in grams per 100 millilitres of blood and t represents the time in hours.

Calculating a table of values gives us:

t	0	0.5	1	1.5	2	2.5	3	3.5	4
$y = 0.1e^{-0.4t}$	0.1	0.08	0.07	0.05	0.04	0.04	0.03	0.02	0.02

Now plot these values on some 2 mm graph paper.

Your graph should look something like this:



Let’s continue this example in the following activity.

Activity 8.15

1. If Chris had drunk a lot more alcohol, her BAC may be 0.2 (she would be very drunk and quite ill).

   Now plot the graph of $y = 0.2\mathbf{e}^{-0.4t}$ on the graph you drew in the previous example.

2. Since the elimination of alcohol varies from person to person, the graphs above may not be suitable for everyone.

   (a) If Frank had a slower metabolic rate (i.e. the body gets rid of the alcohol more slowly), how would the graph above change for someone who had a BAC of 0.2?

   (b) Sketch this on your graph.

   (c) The graph of the curve you just drew could look something like

   $y= 0.2\mathbf{e}^{-0.3t}$ Draw this on your graph.

Note the differences between the graph you drew in 2(c) and the graph you drew in 1. After 3 hours, Chris has a BAC of 0.04, but Frank still has a BAC of 0.08. He is still not sober enough to drive home.

Summarising what we have learnt about exponential equations and their graphs.

- Graphs of equations in the form $y = a^x$ with a **positive** exponent are called exponential **growth** curves.
- Graphs of equations in the form $y = a^{-x}$ with a **negative** exponent are called exponential **decay** curves.
- Changing parts of the equation can effect the curves in two ways.

  •A change in the coefficient of a^x changes the y-intercept. e.g. $4a^x$ and $3a^x$

  •A change in the coefficient of the exponent changes the steepness of the graph. e.g. a^{2x} and a^{6x}

You should now be able to describe the graphs shown at the beginning of section 8.3. Here is a sample paragraph explaining the drug Theophylline levels in the body with oral dose.

For the first 5 hours the amount of drug in the body was increasing fairly steadily (about 9 µmol/L per hour). In the next two hours it was still increasing, but at a slower rate. After 7 hours, the drug levels in the body started to decrease, quickly at first, then more slowly. About 50% of the drug (25 µmol/L) had gone after 15 hours, and another 25% had gone after about 20 hours After 50 hours almost all of the drug had gone from the body.

See if you can write a similar paragraph for the IV infusion.

You might have said something like the following.

Theophylline levels with IV infusion.

For the first 2 hours the drug levels increased steadily at about 35 µmol/L per hour. For the next two hours the drug increased to 85 µmol/L (about 7.5 µmol/L per hour). After 4 hours the drug levels started to decrease, quickly at first then more slowly. After about 11.5 hours there was 50% of the drug left (43 µmol/L) and after a further 19 hours there was only 25% of the drug left.

You have practiced your graphing skills throughout the previous activities, but what happens when graphs intersect (meet)? We can use our knowledge of graphing to help us find the point/s of intersection of two graphs. That is, we can solve these different types of equations simultaneously.

8.6 When two graphs meet

We looked at solving simultaneous equations involving straight lines, earlier in this module. We will now look at an example where we can graphically find the simultaneous solution of a line and a curve.

Example

Solve the following pair of simultaneous equations. That is, find the point/s of intersection of the two graphs.

$$y = x^2 + 4$$

$$y = 2x + 4$$

You will notice that the first equation represents a parabolic graph. It will be an upward opening graph which cuts the y-axis and 4.

The second graph is a straight line which has a gradient of 2 and cuts the y-axis at 4.

To get the graphs for these two equations we must firstly construct a table of values.

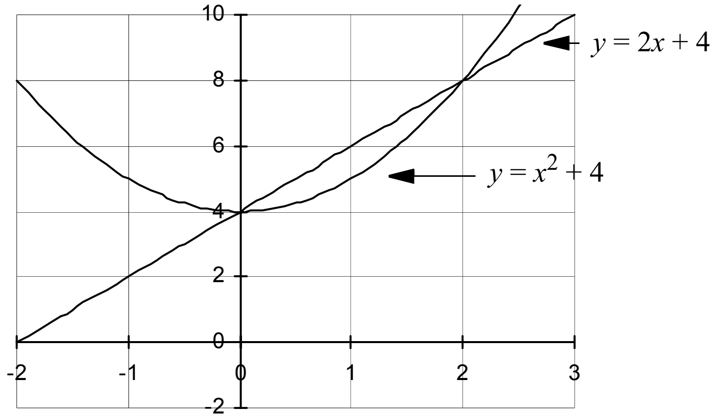
Let's do the line first. Remember that for a line we only need two points but that we choose three as a check.

x	–2	0	2
$y = 2x + 4$	0	4	8

Now for the parabola.

x	–2	–1	0	1	2
$y = x^2 + 4$	8	5	4	5	8

Now on the same piece of graph paper draw the two graphs.



As you can see from the above diagram, the points of intersection are (0,4) and (2,8).

We should check these two results by substituting them into the original equations.

When $x = 0$	$y = x^2 + 4$	$y = 2x + 4$
	$y = 0^2 + 4$	$y = 2 \times 0 + 4$
	$y = 0 + 4$	$y = 0 + 4$
	$y = 4$	$y = 4$
When $x = 2$	$y = x^2 + 4$	$y = 2x + 4$
	$y = 2^2 + 4$	$y = 2 \times 2 + 4$
	$y = 4 + 4$	$y = 4 + 4$
	$y = 8$	$y = 8$

Sometimes you will get small errors in the answers when checking due to inaccuracies when graphing and reading the points of intersection.

Activity 8.16

Solve the following pairs of simultaneous equations using a graphical technique. Check your solutions in the original equations.

1. $y = x + 1$ $\quad$ $y = x^2 - x - 2$
2. $y = x^2$ $\quad$ $y = -2x^2 + 12$
3. $y = 2^x$ $\quad$ $2y = 3x + 2$
4. $y = \mathbf{e}^{-x}$ $\quad$ $y = -3x$ $\quad$ Hint: plot values of x between –2 and 0.5

You have now completed all the modules that make up this unit. You have been following a pathway that has taken you through many different environments, each of which has contributed to your overall understanding of mathematics.

We hope that you have found your journey fruitful and are now ready to tackle any mathematics that you will encounter in the future, be that in a degree, in everyday life or in further levels of mathematics.

You should now be ready to attempt all questions of Assignment 5A (see your Introductory Book for details). If you have any questions, please refer them to your course tutor.

8.7 A taste of things to come

1. In studying chemistry which may be associated with nursing, scientists are interested in bacterial cultures and how quickly they are reproducing.

   In a certain bacterial culture the number (N) of bacteria present is given by the formula:

   $N = 10\mathbf{e}^{3t}$ where t is the time in hours that the bacterial culture has been growing.

   (a) By substituting into the formula, find the number of bacteria present at the start. To do this you should substitute $t = 0$.

   (b) By substituting again, find the number of bacteria present after 4 hours.

   (c) Draw the graph for this equation.

2. In the last module we looked at the algebraic solution of these two simultaneous equations. Often in Economics they will also use the graphs of these relationships to determine other information.

   Consider a small company making pottery bowls. They find that the equation for the quantity of bowls supplied is given by:

   $Q = 10P - 10$ where Q represents the quantity supplied.

   and P represents the price of each bowl in dollars.

   They also find that the equation for the quantity of bowls demanded is given by:

   $Q = -20P + 65$ where Q represents the quantity demanded.

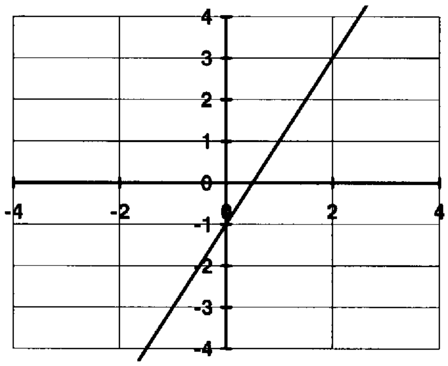
   and P represents the price of each bowl in dollars.

   By graphing the above two equations, find the point of equilibrium between supply and demand. Check that your answer agrees with the result you obtained in module 7.

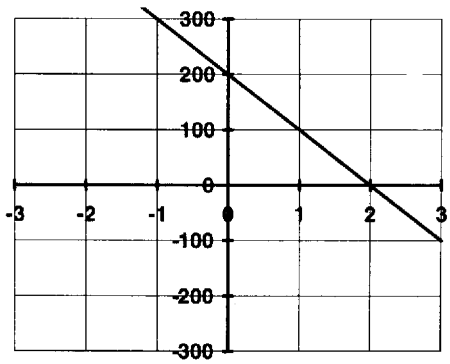
8.8 Post-test

1. Find the equations of the following lines.

(a)



(b)



2. Which of the following equations will give linear graphs and which will give parabolic graphs. For those that will be lines state the gradient and y-intercept and for those that will be parabolas, describe what the parabola will look like.

(a) $y = 3x - 2$

(b) $y = 5x^2 - 9x + 3$

(c) $y = -6x^2 - 2x + 5$

(d) $y = 5x$

3. An arrow is fired by an archer at a target. If the arrow was projected upwards the height obtained (in metres) after t seconds is given by:

$$H = -3t^2 + 18t$$

Using graphical techniques find:

(a) the maximum height of the arrow.

(b) the time taken to reach this height.

4. Find the points of intersection of the following two graphs.

$y = \mathbf{e}^x$

$y = 3x + 4$

8.9 Solutions

Solutions to activities

Activity 8.1

1.

(a) gradient $= \dfrac{\text{change in height}}{\text{change in horizontal distance}} = \dfrac{2}{4} = \dfrac{1}{2}$

(b) gradient $= \dfrac{\text{change in height}}{\text{change in horizontal distance}} = \dfrac{-30}{50} = \dfrac{-3}{5}$

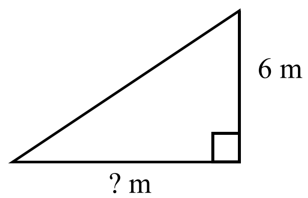
(c) gradient $= \dfrac{\text{change in height}}{\text{change in horizontal distance}} = \dfrac{-8}{10.5} = \dfrac{-80}{105} = \dfrac{-16}{21}$

(d) gradient $= \dfrac{\text{change in height}}{\text{change in horizontal distance}} = \dfrac{350}{310} = \dfrac{35}{31}$

Did you remember to change the measurements so that they are in the same units?

2. The rise is 2% and the run is 2 years so the gradient is $\dfrac{2}{2} = 1$
3. The gradient of the hill would be $\dfrac{-3}{2}$
4. The gradient of the tunnel would be $\dfrac{-20}{10} = -2$
5. A diagram should help with this situation.

   To have a gradient of 2, the horizontal distance covered must have been 3 metres.



6. (a) Between 1956 and 1966, Pharmacy showed the greatest increase in numbers. This can be seen because the Pharmacy line is steeper than the other two lines.

   (b) Between 1976 and 1986 there was a decline in the number of women in Pharmacy and Medicine

   (c) Pharmacy has shown the greatest decline in numbers of women, because it is a steeper line than the Medicine line over the same period.

7. (a) This formula is the formula for speed that we have looked at previously. The gradient is telling us the speed of the tortoise in metres/minute.

   (b) Between 2 and 2.5 minutes the tortoise is travelling at the greatest speed.

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}} = \frac{5}{0.5} = 10$$

   (c) The least speed (apart from when stopped) was over the period 3.5 to 6 minutes.

$$\text{gradient} = \frac{\text{change in height}}{\text{change in horizontal distance}} = \frac{2}{2.5} = 0.8$$

   You can see from this result that the tortoise is indeed much slower over this period.

   (d) $\text{gradient} = \dfrac{\text{change in height}}{\text{change in horizontal distance}} = \dfrac{0}{1} = 0$

   (e) A horizontal line has a gradient of zero.

Activity 8.2

1. For these questions you might have chosen a variety of different point with which to work. With this in mind only the final gradient is given.

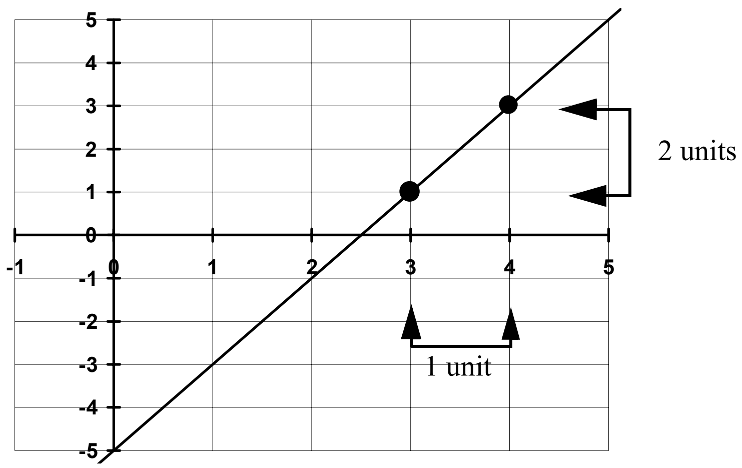
   (a) gradient = 2

   (b) gradient = $\frac{1}{2}$

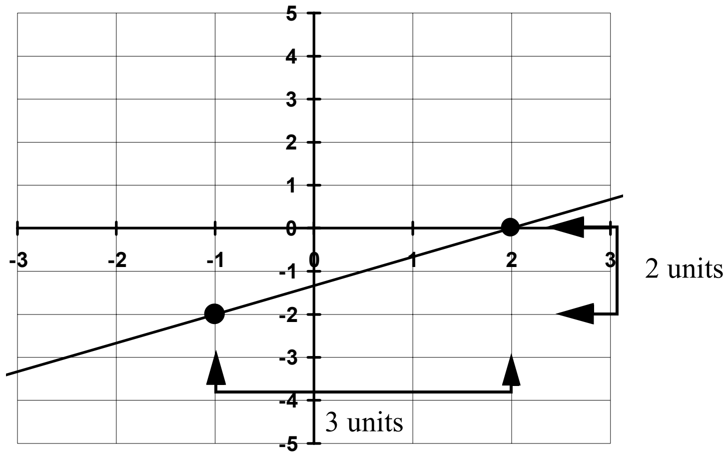
   (c) gradient = –2

   (d) gradient = –1

2. A gradient of 2 can be written $\frac{2}{1}$



3.



4. (a) The gradient of this line between any two points that you might have chosen is –2.

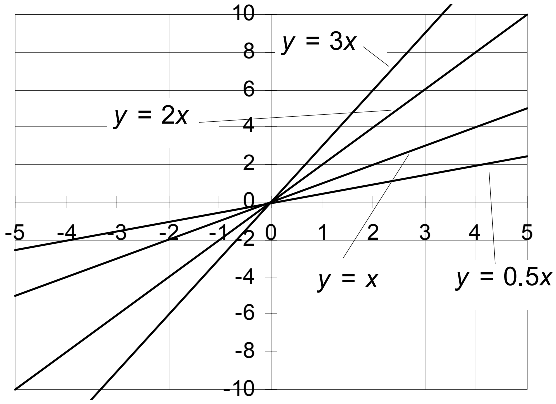
   (b) This gradient 'tells' us that the candle burns (decreases in height) at a rate of 2 cm for every hour.

Activity 8.3

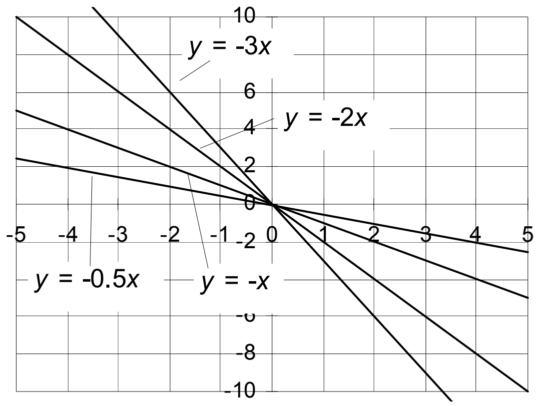
1. $y = 4x$ Linear
2. $y = 5x^2$ Not linear – power greater than one.
3. $3y + 2x = 6$ Linear
4. $xy = 4$ Not linear – two variables multiplied together.
5. $3x = 7 - 2y$ Linear
6. $y = -2x$ Linear
7. $y = x^2 - 3x + 4$ Not linear – power greater than one.
8. $x = 3 - y$ Linear

Activity 8.4

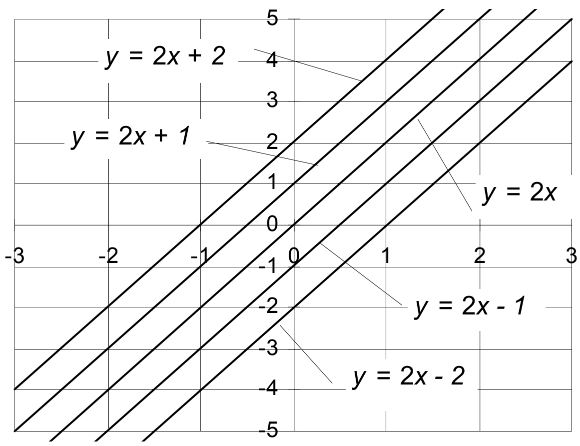
1.



2.



Activity 8.5



Activity 8.6

1.

(a) $y = -2x + 3$ gradient = –2 y-intercept = 3

(b) $y = 5 + 3x$ gradient = 3 y-intercept = 5

(c) $y = 2 - 4x$ gradient = –4 y-intercept = 2

(d) $y = x - 3$ gradient = 1 y-intercept = –3

(e) $y = 5x$ gradient = 5 y-intercept = 0

(f) $y + 4 = 6x$

$y = 6x - 4$ gradient = 6 y-intercept = –4

(g) $3x - y = 6$

$-y = -3x + 6$

$y = 3x - 6$ gradient = 3 y-intercept = –6

(h) $4y + 6 = -12x$

$4y = -12x - 6$

$y = -3x - \frac{3}{2}$ gradient = –3 y-intercept = $-\frac{3}{2}$

(i) $2x + 2y - 7 = 0$

$2y = -2x + 7$

$y = -x + \frac{7}{2}$ gradient = –1 y-intercept = $\frac{7}{2}$

2.

(a) $\mathbf{m} = 5$, $\mathbf{c} = 2$ $y = 5x + 2$

(b) $\mathbf{m} = 3$, $\mathbf{c} = -1$ $y = 3x - 1$

(c) $\mathbf{m} = 1$, $\mathbf{c} = 0$ $y = x + 0$ or $y = x$

(d) $\mathbf{m} = \frac{1}{2}$, $\mathbf{c} = -5$ $y = \frac{1}{2}x - 5$

(e) $\mathbf{m} = -\frac{3}{4}$, $\mathbf{c} = \frac{7}{2}$ $y = -\frac{3}{4}x + \frac{7}{2}$

3.

(a) $y = 3x + 1$

$y = 3x + 4$

These two lines have the same gradient and are therefore parallel. The first graph will cut the y-axis at 1 and the second graph will cut the y-axis at 4.

(b) $y = 2x + 2$

$y = 3x + 2$

These two lines cut the y-axis at the same point, $y = 2$. The second graph will be slightly steeper than the first graph owing to the fact that it has a larger gradient.

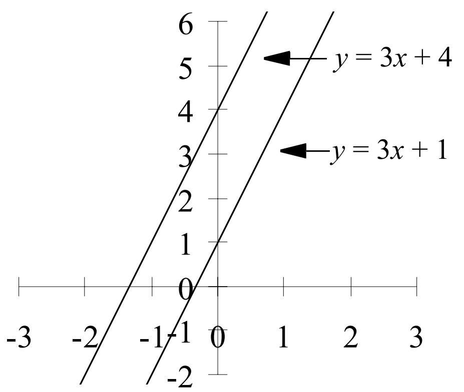
(c) $y = 3x + 2$

$y = -3x + 2$

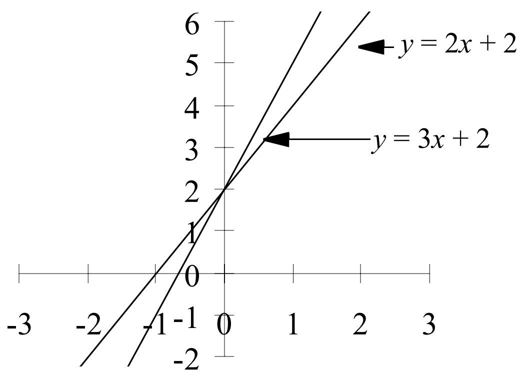
These two lines again cut the y-axis at the same place, $y = 2$. This time though, the first line has a positive gradient and the line rises as we move from left to right, while the second line has a negative gradient and falls as we move from left to right.

4.

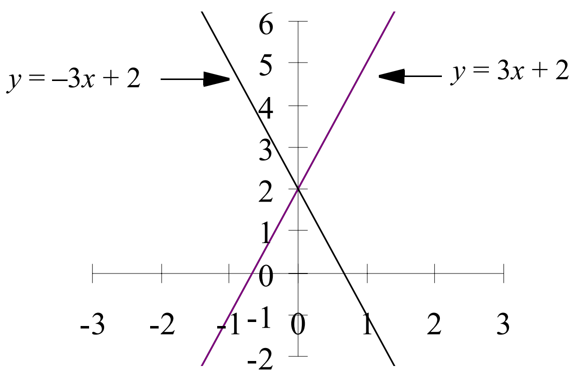
(a)



(b)



(c)



5. (a) gradient = 0.15

   y-intercept = 50

   (b) The gradient of 0.15 tells us that for every 1 kilometre we travel the cost will be $0.15.

   The y-intercept tells us that distance zero or at the starting position the cost is $50. It is usual in questions like this for the y-intercept to tell us what is happening at the **start** of a certain event.

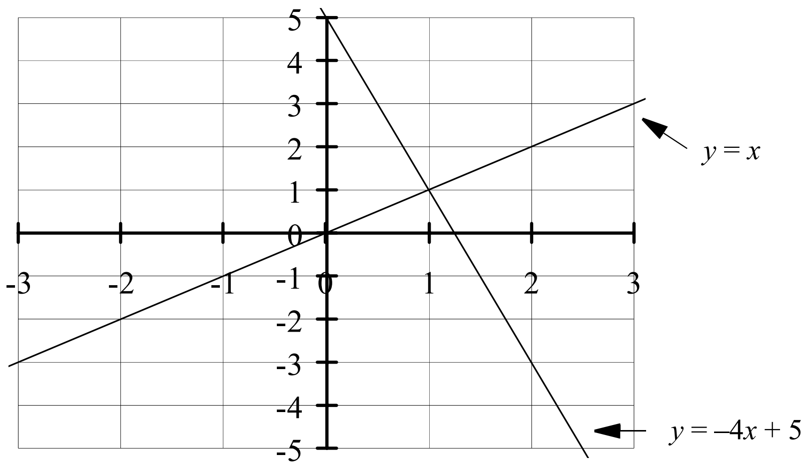
   (c) The equation is the same as you have found in the past. That is:

   $C = 50 + 0.15k$ where C represents the total daily cost of hire in dollars,

   and k represents the number of kilometres travelled.

Activity 8.7

1. (a)



The point of intersection of these two lines is (1,1)

Check: $y = x$ LHS $= y$ RHS $= x$

$= 1$ $= 1$

$= \text{LHS}$

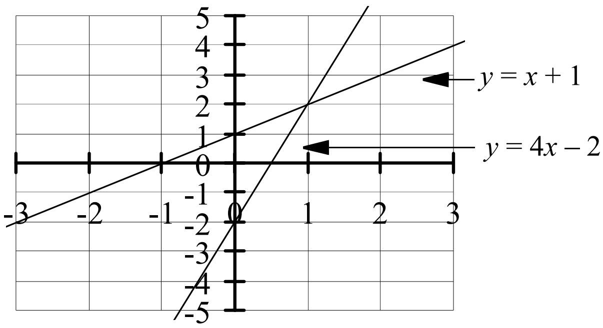
$y = -4x + 5$ LHS $= y$ RHS $= -4x + 5$

$= 1$ $= -4 \times 1 + 5$

$= 1$

$= \text{LHS}$

(b)

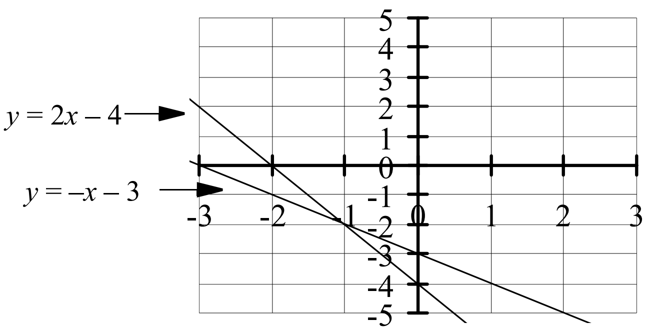


The point of intersection of these two lines is (1,2)

Check: $y = x + 1$ LHS = y = 2; RHS = $x + 1$ = 1 + 1 = 2 = LHS

$y = 4x - 2$ LHS = y = 2; RHS = $4x - 2$ = $4 \times 1 - 2$ = 2 = LHS

(c)



The point of intersection of these two lines is (–1,–2)

Check: $y = -x - 3$ LHS $= y$ RHS $= -x - 3$

$= -2$ $= -(-1) - 3$

$= 1 - 3$

$= -2$

$=$ LHS

$y = -2x - 4$ LHS $= y$ RHS $= -2x - 4$

$= -2$ $= -2 \times -1 - 4$

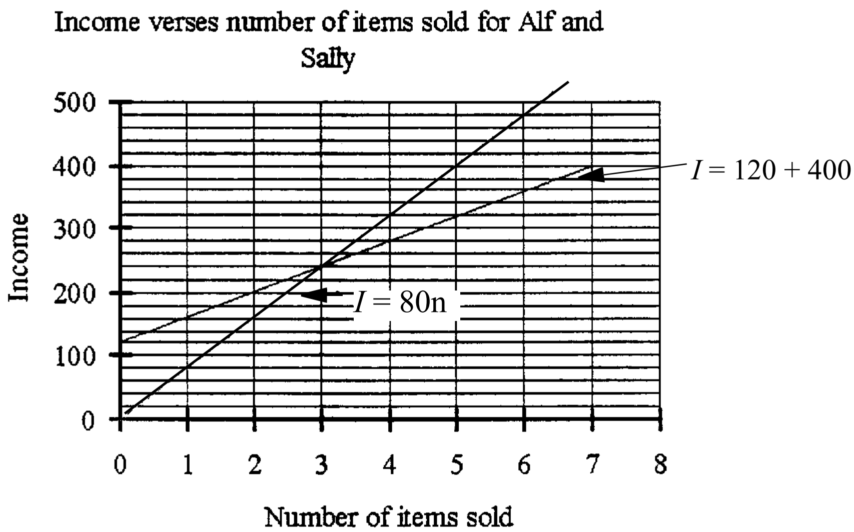
$= 2 - 4$

$= -2$

$=$ LHS

2. (a) For Alf: $I = 120 + 40n$ where I represents the income in dollars,

For Sally: $I = 80n$ and n represents the number of items sold.



Alf and Sally must sell three items to earn the same amount of money which is \$240

check: If Sally sells 3 items, she earns $3 \times \$80 = \$240$

If Alf sells three items he earns $\$120 + \$40 \times 3 = \$240$

Activity 8.8

1. (a) Graph (b) would be the most probable. At the beginning of the race the person is fast (the gradient is steep), at the end the person is slower. Graph (a) is possible but the person is running at a constant speed. Graph (c) is possible if the y axis is the distance from the start; so half way through the person turns around and is going home.

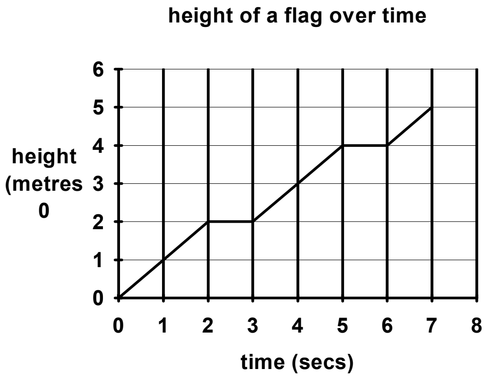
   (b) Graph (b) would be most probable since you are going up and down although it would probably be a curve.

   (c) Graph (c) might be appropriate. The chairlift has a constant speed for the first part of the graph. When a skier is going down a hill the speed could be increasing all the time. I presume she would have to stop at some stage but graph (c) does not show this. Graph (b) is possible as well, assuming the chairlift's speed is increasing and she comes to a very sudden stop maybe a tree?

   (d) People usually demand more bananas when the price is lower - as the price decreases the quantity increases - so the answer will be (b).

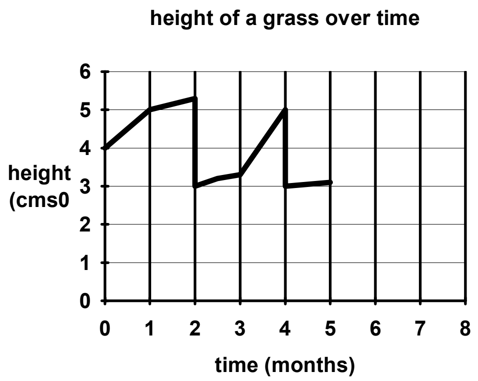
2. The following are only a sample of the solutions you could give.

(a)



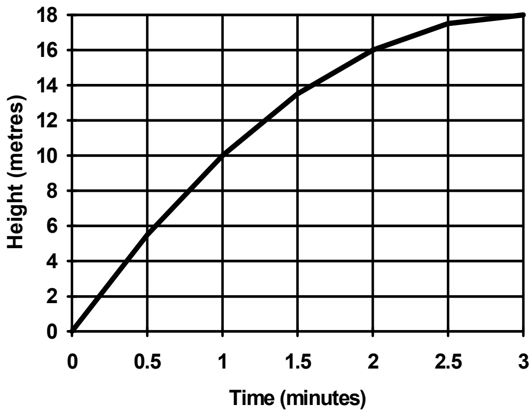
The flag is raised at a constant rate for two seconds then there is a rest for one second; the flag is again raised at the same speed for another two seconds followed by another one second rest. Finally the flag is raised one more metre in one second.

(b)



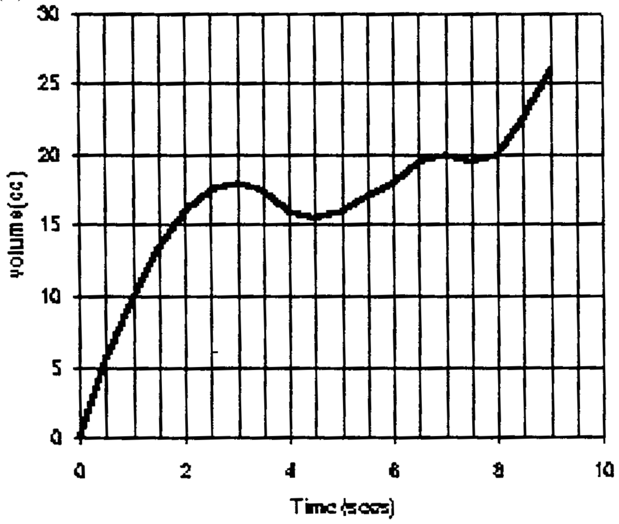
The grass grows quickly at first then more slowly in the second month. It is then cut to 3cm. The grass grows slowly in the third month but very quickly in the fourth month. When it reaches 5 cm it is again cut. In the fifth month the height appears not to change.

(c)



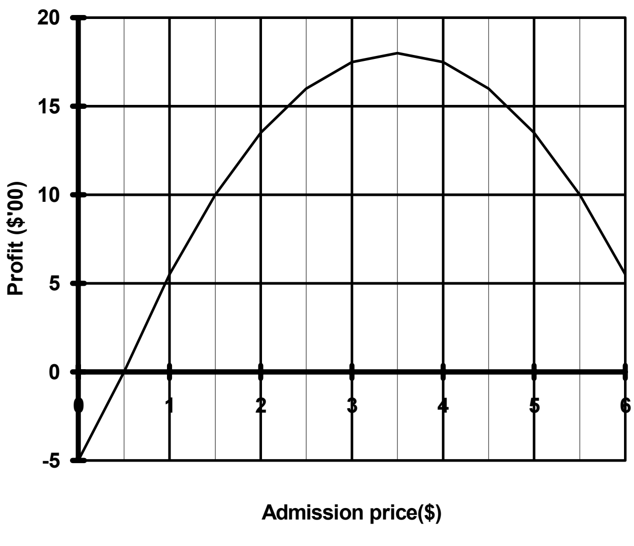
I start going up the hill quickly at first, then I slow down as I get closer to the top. In the first minute I have travelled 10 metres up, but in the second minute I have only travelled another 6 metres and in the third, I managed only 2 metres.

(d)



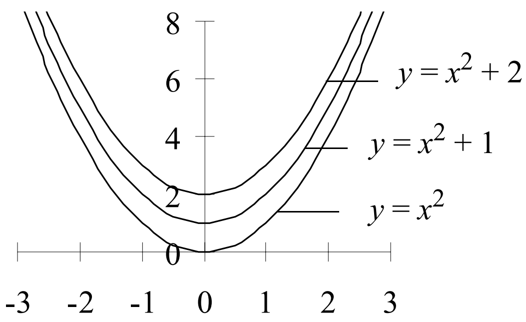
The balloon is blown one breath at a time. After each breath a little volume is lost. The second breath inflates the balloon at a slower rate.

(e)



If you don't charge enough you will make a loss. The maximum profit will be about $1800 if you charge $3.50. If you charge too much the profits will again decrease.

Activity 8.9



Activity 8.10

1.

(a) $y = 3x$ line

(b) $y = 2x^2$ parabola

(c) $y = -4x + 1$ line

(d) $y = 2 - 3x^2$ parabola

(e) $y = 2x^2 + x + 1$ parabola

2.

(a) The graph of $y = 4x^2 - 3x + 7$ will be a parabola because the highest power of x is two. The parabola will open upwards since the coefficient of x^2 is positive. The parabola will cut the y-axis at 7 because this is the constant term in the equation.

(b) The graph of $y = -3x^2 - 4$ will be a parabola because the highest power of x is two. The parabola will open downwards since the coefficient of x^2 is negative. The parabola will cut the y-axis at –4 because this is the constant term in the equation.

(c) The graph of $y = 5x + 6$ will be a straight line since both variables are to power one. The gradient of this line will be 5 and the line will cut the y-axis at 6.

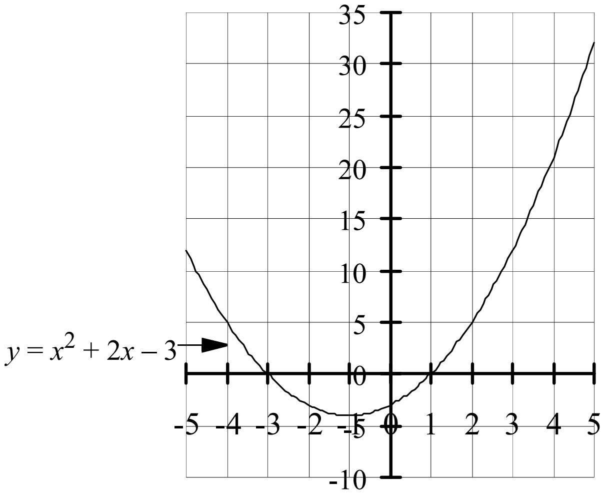
(d) The graph of $y = 5 + 4x - 3x^2$ will be a parabola because the highest power of x is two. The parabola will open downwards since the coefficient of x^2 is negative. The parabola will cut the y-axis at 5 because this is the constant term in the equation.

(e) The graph of $y = 7 - 5x$ will be a straight line since both variables are to power one. The gradient of this line will be –5 and the line will cut the y-axis at 7.

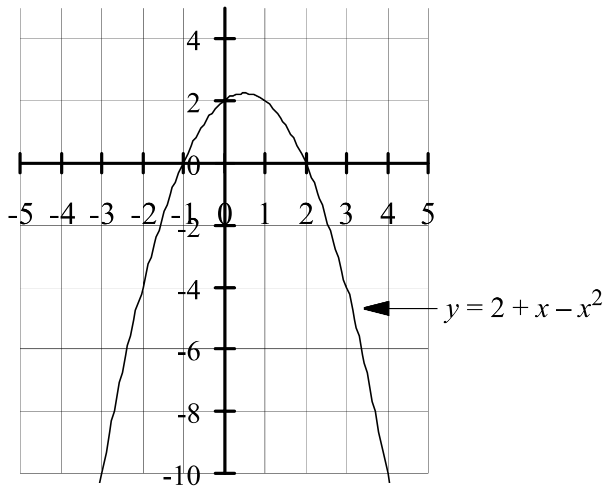
(f) The graph of $y = 4 - 5x + 2x^2$ will be a parabola because the highest power of x is two. The parabola will open upwards since the coefficient of x^2 is positive. The parabola will cut the y-axis at 4 because this is the constant term in the equation.

3.

(a)

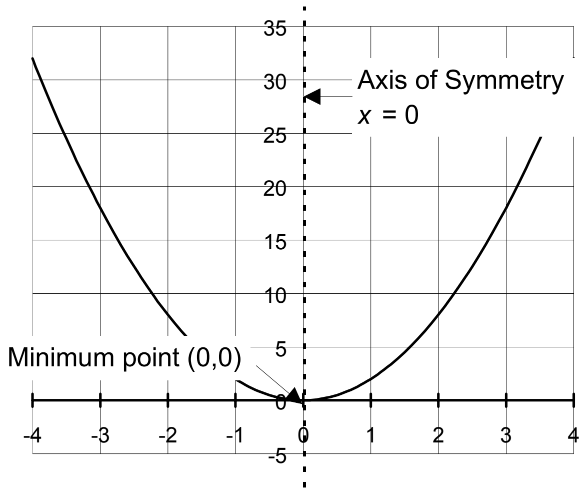


(b)

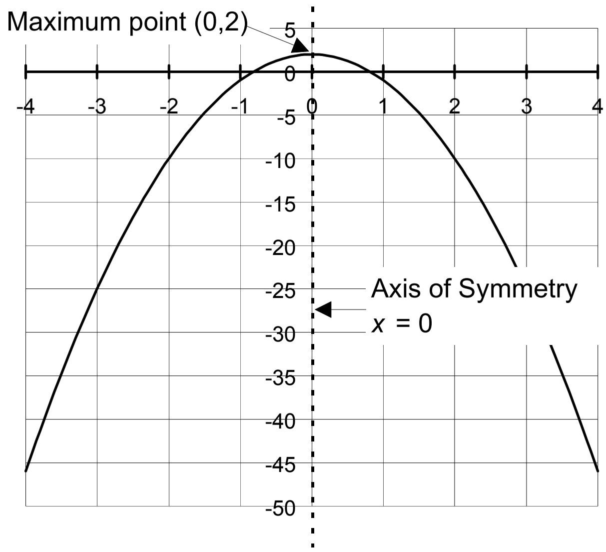


Activity 8.11

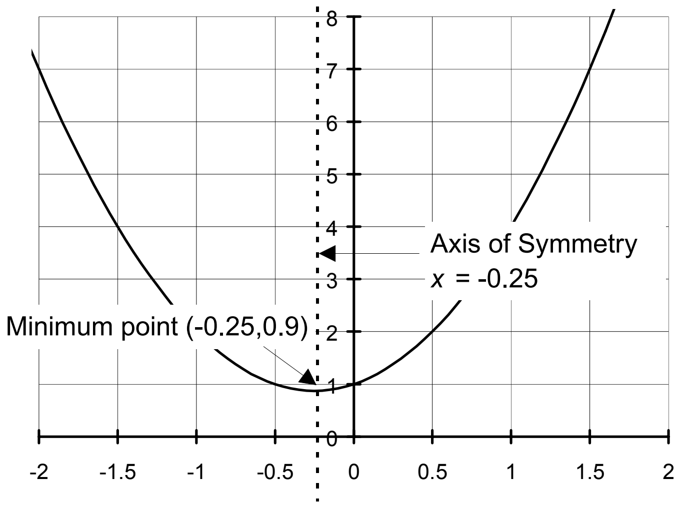
1 (a)



(b)



(c)



Note that these values should be read from your graph. With this in mind your answers could be slightly different to those given.

2. (a) When $x = -1.5$, $y = 4$; when $x = 3$, $y = 18$

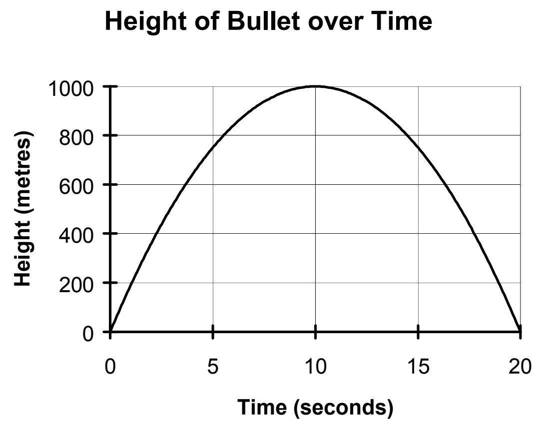
(b) When $x = -1.5$, $y = -3$; when $x = 3$, $y = -25$

(c) when $x = -1.5$ $y = 4$; when $x = 3$, $y = 22$

3.(a)

t	0	5	8	10	12	15
H	0	750	960	1000	960	750

(b)

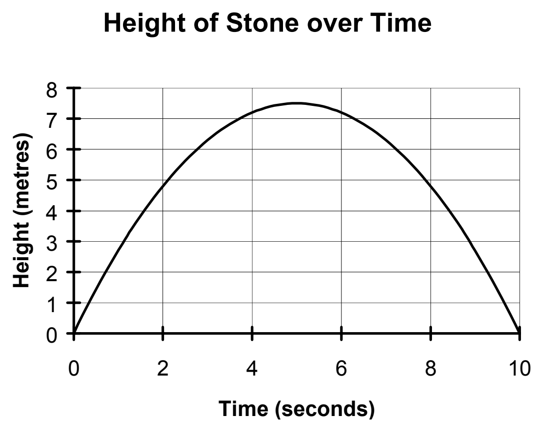


(c) Maximum height = 1 000 metres

(d) Time: about 6 seconds and again at about 14 seconds

4.

t	0	1	2	3	4	5	6	10
H	0	2.7	4.8	6.3	7.2	7.5	7.2	0



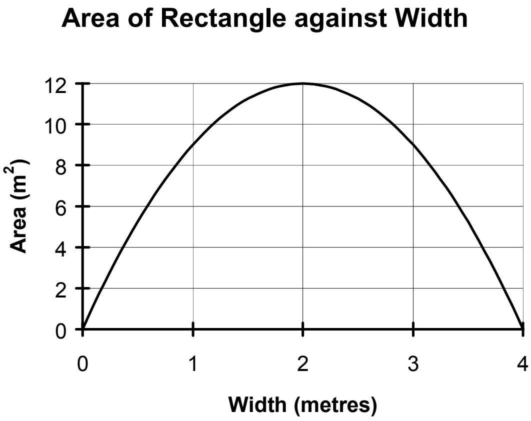
(a) the greatest height reached = 7.5 metres

(b) the time is 5 seconds

(c) after 2.5 seconds the stone is about 6 metres from the ground

5.

w	0	1	2	3	4
A	0	9	12	9	0

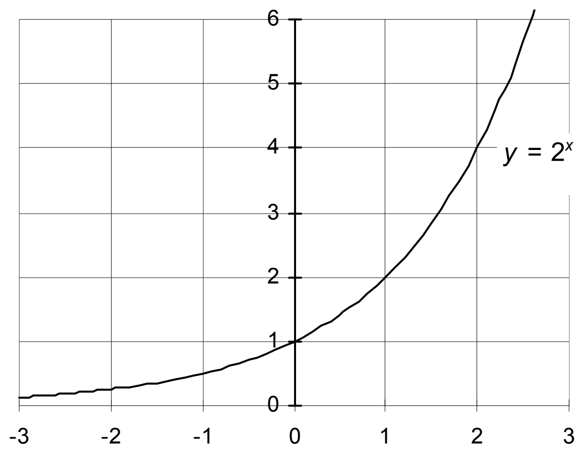


(a) maximum area = 12 m^2

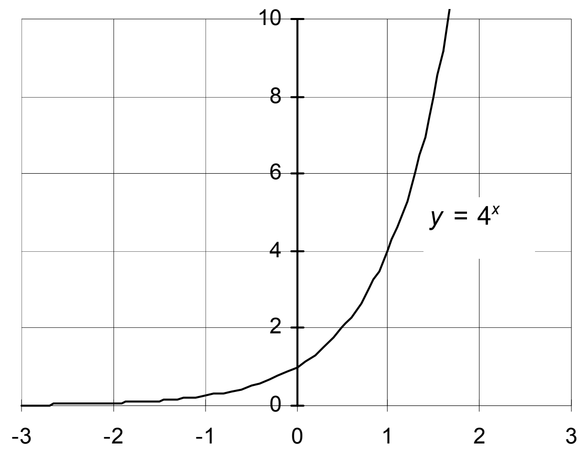
(b) A width of 2 gives a maximum area of 12.

Activity 8.12

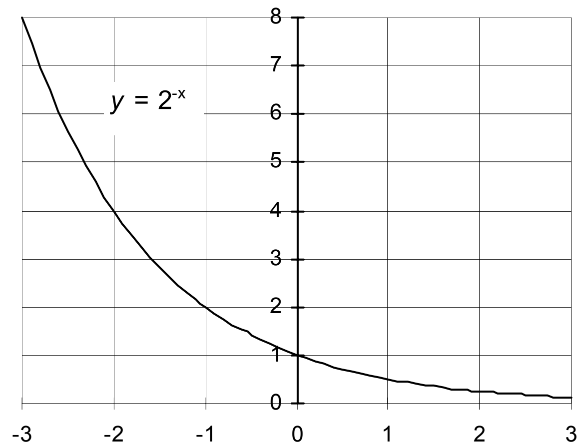
1.



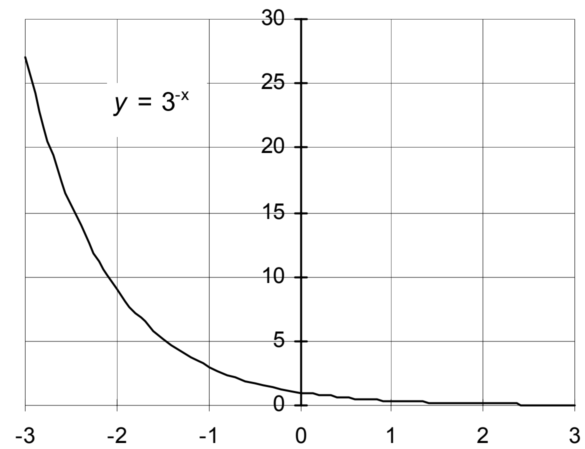
2.


y = 4^x
10
8
6
4
2
0
-3
-2
-1
0
1
2
3

3.

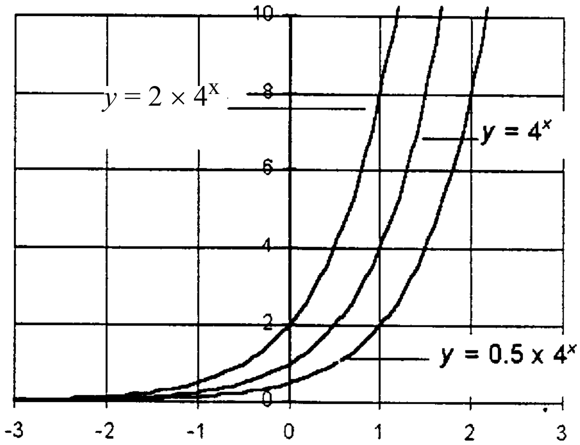

y = 2^-x
8
7
6
5
4
3
2
1
0
-3
-2
-1
0
1
2
3

4.

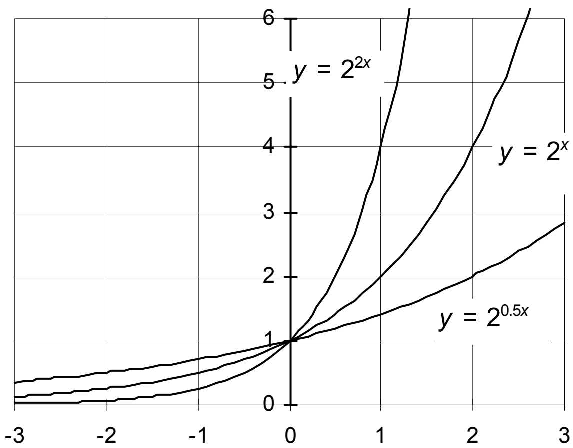

y = 3^-x
30
25
20
15
10
5
0
-3
-2
-1
0
1
2
3

Activity 8.13

1.



2.



Activity 8.14

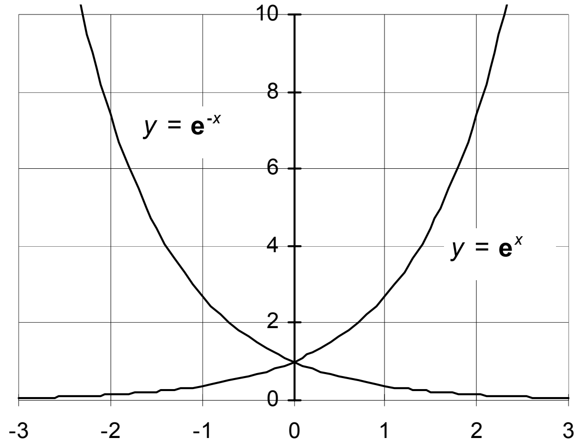
1.

x	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2
$y=\mathbf{e}^x$	0.14	0.22	0.37	0.61	1	1.65	2.72	4.48	7.39

2.

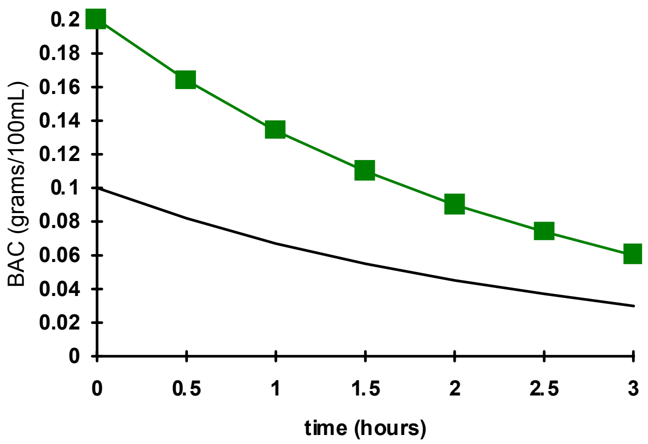
x	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2
$y=\mathbf{e}^{-x}$	7.39	4.48	2.72	1.65	1	0.61	0.37	0.22	0.14

3.



Activity 8.15

1. Your graph should look something like this.

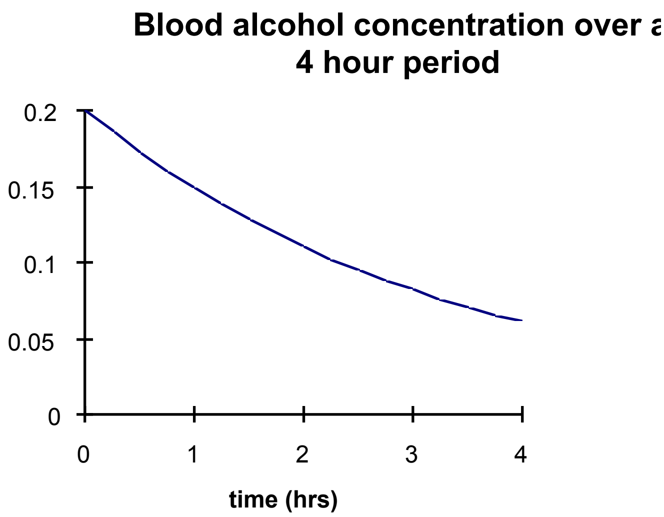


2. (a) If they got rid of it more slowly, the curve would start at 0.1 but would go down much slower.

   (b) This answer will depend on you. It should end up looking something like the graph you will draw in part (c)

   (c)

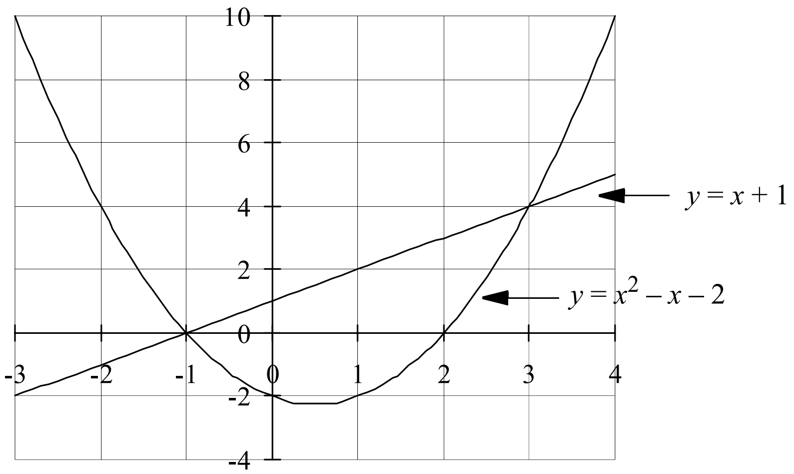
x	0	0.5	1	1.5	2	2.5	3	3.5	4
$y=0.2\mathbf{e}^{-0.3t}$	0.2	0.17	0.15	0.13	0.11	0.09	0.08	0.07	0.06



As the value of the coefficient of the exponent gets smaller the curve is less steep.

Activity 8. 16

1.



The points of intersection are (–1,0) and (3,4)

We should check these two results by substituting them into the original equations.

When $x = -1$ $\quad y = x^2 - x - 2$ $\quad\quad y = x + 1$

$y = (-1)^2 - (-1) - 2$ $\quad\quad y = -1 + 1$

$y = 1 + 1 - 2$ $\quad\quad y = 0$

$y = 0$

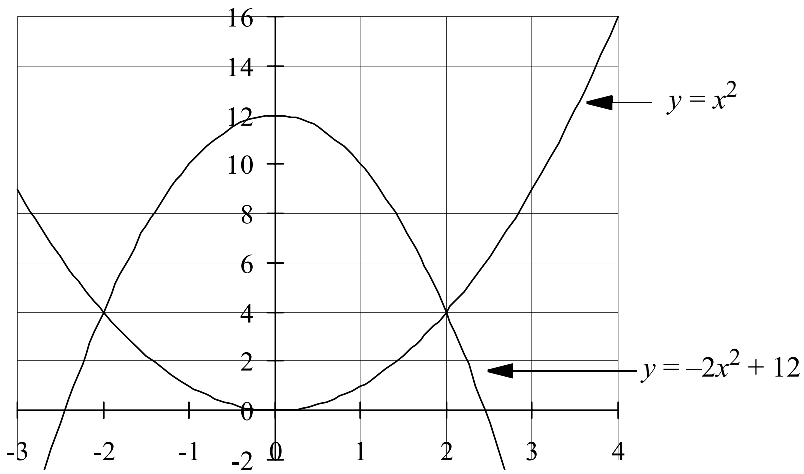
When $x = 3$ $\quad y = x^2 - x - 2$ $\quad\quad y = x + 1$

$y = 3^2 - 3 - 2$ $\qquad$ $y = 3 + 1$

$y = 9 - 3 - 2$ $\qquad$ $y = 4$

$y = 4$

2.



The points of intersection are (–2,4) and (2,4)

We should check these two results by substituting them into the original equations.

When $x = -2$ $\qquad$ $y = x^2$ $\qquad$ $y = -2x^2 + 12$

$y = (-2)^2$ $\qquad$ $y = -2 \times (-2)^2 + 12$

$y = 4$ $\qquad$ $y = -2 \times 4 + 12$

$y = -8 + 12$

$y = 4$

When $x = 2$ $\qquad$ $y = x^2$ $\qquad$ $y = -2x^2 + 12$

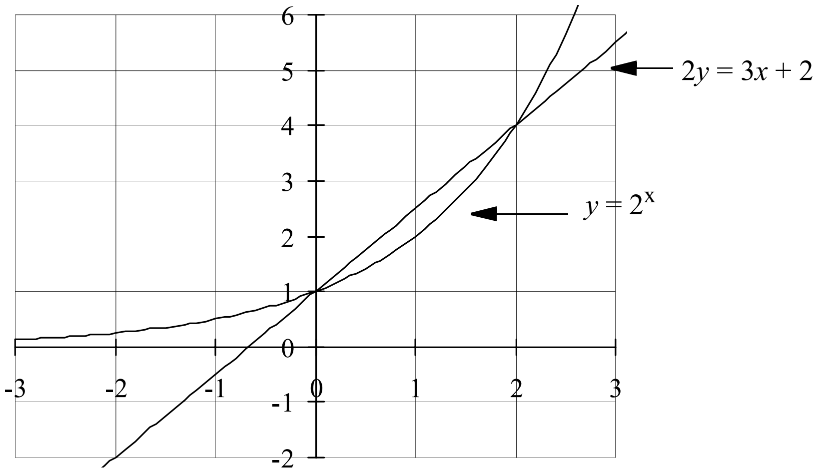
$y = 2^2$ $\qquad$ $y = -2 \times 2^2 + 12$

$y = 4$ $\qquad$ $y = -2 \times 4 + 12$

$y = -8 + 12$

$y = 4$

3.

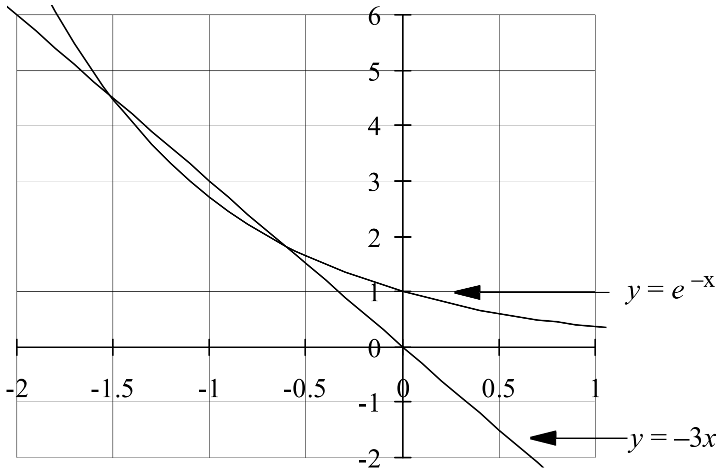


The points of intersection are (0,1) and (2,4)

We should check these two results by substituting them into the original equations.

When $x = 0$	$y = 2^x$	$2y = 3x + 2$
	$y = 2^0$	$y = 1.5x + 1$
	$y = 1$	$y = 1.5 \times 0 + 1$
		$y = 0 + 1$
		$y = 1$
When $x = 2$	$y = 2^x$	$2y = 3x + 2$
	$y = 2^2$	$y = 1.5x + 1$
	$y = 4$	$y = 1.5 \times 2 + 1$
		$y = 3 + 1$
		$y = 4$

4.



It is hard to get an accurate picture of the points of intersection from this graph, but on your grid paper you should have obtained a good approximation.

The points of intersection are approximately (–1.5,4.5) and (–0.6,1.8)

We should check these two results by substituting them into the original equations.

When $x = –1.5$	$y = \mathbf{e}^x$	$y = – 3x$
	$y = \mathbf{e}^{–1.5}$.	$y = – 3 \times – 1.5$
	$y \approx 4.5$	$y = 4.5$

When $x = –0.6$	$y = \mathbf{e}^x$	$y = – 3x$
	$y = \mathbf{e}^{–0.6}$	$y = – 3 \times – 0.6$
	$y \approx 1.8$	$y = 1.8$

Solutions to a taste of things to come

1. (a) $N = 10\mathbf{e}^{3\times 0}$

$= 10\mathbf{e}^{0}$

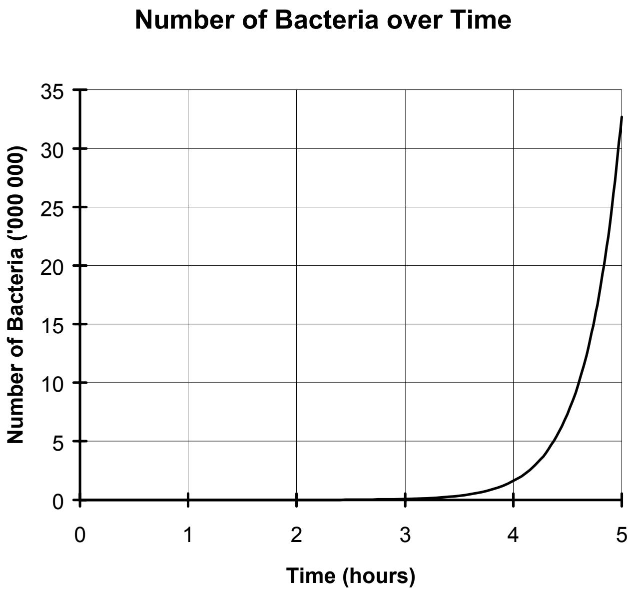
$= 10$ So there are 10 bacteria to start.

(b) $N = 10\mathbf{e}^{3\times 4}$

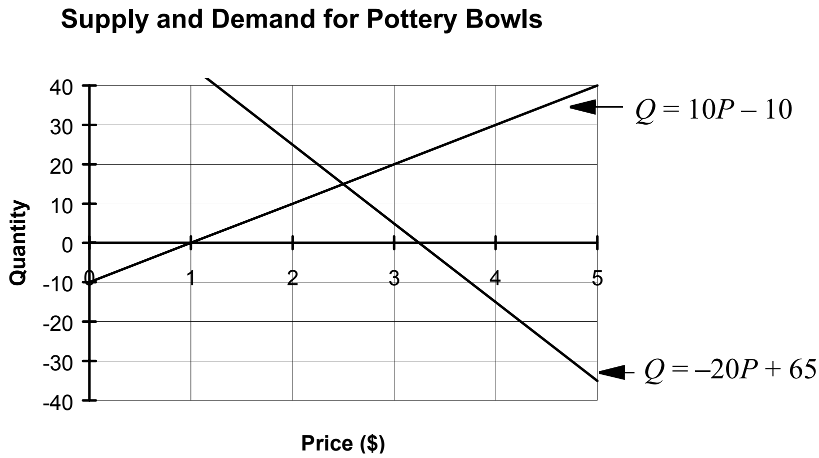
$= 10\mathbf{e}^{12}$

$\approx 1\ 627\ 548$

(c)



2.



You should have found the point of equilibrium to be \$2.50 and 15 bowls. You should of course check this in the original equations as you did in module 7.

Solutions to post-test

1 (a) intercept = – 1; gradient = $\frac{4}{2}$ = 2. Equation is $y = 2x - 1$

(b) intercept = 200; gradient = $\frac{-200}{2}$ = –100. Equation is $y = -100x + 200$

2. (a) linear graph; gradient = 3; y intercept = –2

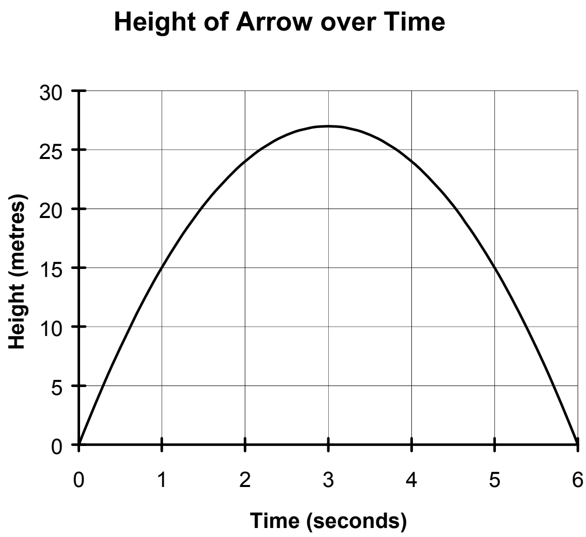
(b) parabolic graph; opening upwards with y-intercept of 3

(c) parabolic graph; opening downwards with y-intercept of 5

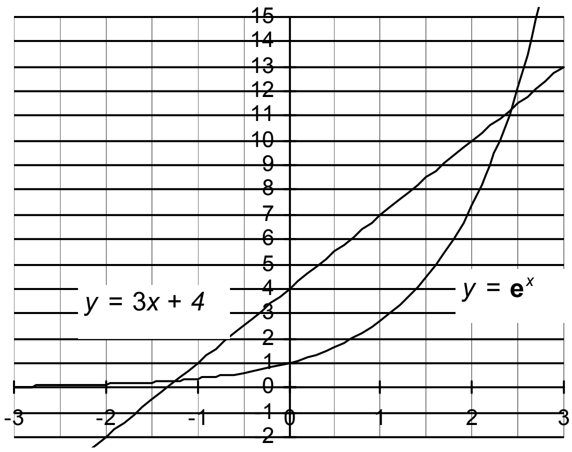
(d) linear graph; gradient = 5; y-intercept = 0

3. (a) maximum height is about 27 metres.

(b) It takes about 3 seconds to reach this height.



4.



The points of intersection will be about (2.4,11) and (–1.25,0.3) These answers will be very difficult to read from your graph, so these are just approximations.