Algebra: Factorisation

Overview

This presentation will cover:

- Simple factorisation
- Factorising quadratics

Factorisation is the reverse process to expansion.

Common factors

Consider, the expression $ax + ab$.

Each term in the expression contains the same factor (a).

In this case, a is called a common factor of ax and ab.

In the same way we could say that 3 is a common factor of 18 and 33, since we could write $18 = 3 \times 6$, and $33 = 3 \times 11$.

For example, to factorise the expression $3x + 6$ we note that 3 is a common factor of both parts. We can then say

$$3x + 6 = 3\times x + 3\times 2 = 3\times(x + 2) = 3(x + 2)\,.$$

Now you try!

Factorise:

1. $n^2 - 5n$
2. $-2y - 8$
3. $2xy^2 - 4xy$
4. $2ab - 10a + 3b - 15$

Solutions

1. $n^2 - 5n = n \times n - n \times 5 = n(n-5)$
2. $-2y - 8 = (-2) \times y + (-2) \times 4 = -2(y+4)$
3. $2xy^2 - 4xy = 2xy \times y - 2xy \times 2 = 2xy(y-2)$
4. $2ab - 10a + 3b - 15 = 2a(b-5) + 3(b-5) = (b-5)(2a+3)$

Note: always check you have factorised correctly by expanding back out to make sure you get the original question.

Factoring quadratic expressions

Expansion and factorisation are actually reverse procedures. Examining the expansion to see how it can help us to factorise.

$$\begin{aligned}(p+1)(p+3) &= p(p+3) + 1(p+3) \\ &= p^2 + 3p + 1p + 1 \times 3 \\ &= p^2 + (3+1)p + 3 \times 1\,.\end{aligned}$$

$3+1=4$ $\quad$ $3\times 1 = 3$

We have two numbers (3 and 1), which add to give 4 and multiply to give 3.

So if we must factorise $p^2 + 4p + 3$, we try to find two numbers that add to give $+4$ and multiply to give $+3$.

Factorising quadratic expression (continued)

Numbers to try,

factors of $+3$	Sum	Product	
-3 and -1	$-3 + -1 = -4$	$-3 \times -1 = 3$	
3 and 1	$3 + 1 = 4$	$3 \times 1 = 3$	✓

The correct factors 3 and 1.

Factorising quadratic expression (continued)

The correct factors are 3 and 1.

$$\begin{aligned}p^2 + 4p + 3 &= p^2 + (3+1)p + 3 \times 1 \\ &= p^2 + 3p + p + 3 \\ &= p(p+3) + (p+3) \\ &= p(p+3) + 1(p+3) \\ &= (p+3)(p+1)\,.\end{aligned}$$

Exercise

Factorise:

1. $x^2 + 7x - 30$
2. $6x^2 + 7x + 2$

Solution $x^2 + 7x - 30$

We need two numbers that multiply to give -30 and add to give 7.

Factors of -30	Sum	Product	
3 and -10	$3 + -10 = -7$	$3 \times -10 = -30$	
-3 and 10	$-3 + 10 = 7$	$-3 \times 10 = -30$	✓
2 and -15	$2 + -15 = -13$	$2 \times -15 = -30$	
-2 and 15	$-2 + 15 = 13$	$-2 \times 15 = -30$	
5 and -6	$5 + -6 = -1$	$5 \times -6 = -30$	
-5 and 6	$-5 + 6 = 1$	$-5\times = -30$	

Solution $x^2 + 7x - 30$ (continued)

The correct factors are -3 and 10, so

$$\begin{aligned} & x^2 + 7x - 30 \\ = \; & x^2 + (10 - 3)x - 3 \times 10 \\ = \; & x^2 + 10x - 3x + -3 \times 10 \\ = \; & x(x + 10) - 3x + -3 \times 10 \\ = \; & x(x + 10) - 3(x + 10) \\ = \; & (x + 10)(x - 3)\,. \end{aligned}$$

Check your answer by expanding.

Solution $6x^2 + 7x + 2$

We need two numbers that multiply to give $+12$ and add to give $+7$.

Factors of 12	Sum	Product	
6 and 2	$6 + 2 = 8$	$6 \times 2 = 12$	
-6 and -2	$-6 + -2 = -8$	$-6 \times -2 = 12$	
3 and 4	$3 + 4 = 7$	$3 \times 4 = 12$	✓
-3 and -4	$-3 + -4 = -7$	$-3 \times -4 = 12$	

Correct factors are 3 and 4.

Solution $6x^2 + 7x + 2$ continued

$$
\begin{aligned}
& 6x^2 + 7x + 2 \\
= \; & 6x^2 + (3 + 4)x + 2 \\
= \; & 6x^2 + 3x + 4x + 2 \\
= \; & 3x(2x + 1) + 2(2x + 1) \\
= \; & (2x + 1)(3x + 2)\,.
\end{aligned}
$$

Check your answer by expanding.

Therefore the final answer is $6x^2 + 7x + 2 = (2x + 1)(3x + 2)\,.$

Summary

This presentation covered:

- Factorising (adding one set of brackets)
- Factorising quadratics (adding two set of brackets)