

## Solving logarithmic equations

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### Overview



This presentation will work through how to solve equations involving logarithms.

1. Solve for  $x$ :

$$6.78 = \ln\left(\frac{x}{234}\right)$$

2. Make  $P$  the subject:

$$t = \frac{\ln\left(\frac{1000}{P} - 1\right)}{-0.7}$$

3. Solve for  $x$ :

$$2^x = 8$$

4. Write  $x$  in terms of  $y$ :

$$y = \frac{2 \times 3^{2x-3} + 1}{5}$$

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### Example 1



$$6.78 = \ln\left(\frac{x}{234}\right)$$

$$6.78 = \log_e\left(\frac{x}{234}\right)$$

$$e^{6.78} = \frac{x}{234}$$

$$e^{6.78} \times 234 = \frac{x}{234} \times 234$$

$$234 \times e^{6.78} = x$$

$$x = 234 \times e^{6.78}$$

$$\approx 205\,936.$$

convert from logarithmic to exponential form:

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## Example 2



$$\begin{aligned}
 t &= \frac{\ln\left(\frac{1000}{P} - 1\right)}{-0.7} \\
 t \times -0.7 &= \ln\left(\frac{1000}{P} - 1\right) \\
 -0.7t &= \log_e\left(\frac{1000}{P} - 1\right) \\
 e^{-0.7t} &= \frac{1000}{P} - 1 \\
 e^{-0.7t} + 1 &= \frac{1000}{P} \\
 P \times (e^{-0.7t} + 1) &= 1000 \\
 P &= \frac{1000}{e^{-0.7t} + 1}.
 \end{aligned}$$

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## Example 3



There are a few ways to solve this equation, firstly,

$$\begin{aligned}
 2^x &= 8 \\
 &\downarrow \\
 2^x &= 2^3 \\
 x &= 3.
 \end{aligned}$$

Secondly, we can use our logarithm laws:

$$\begin{aligned}
 2^x &= 8 && \text{taking the logarithm of both sides,} \\
 \ln(2^x) &= \ln(8) \\
 x \ln 2 &= \ln 8 \\
 x &= \frac{\ln 8}{\ln 2} \\
 x &= 3.
 \end{aligned}$$

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## Example 4



$$\begin{aligned}
 y &= \frac{2 \times 3^{2x-3} + 1}{5} \\
 5y &= 2 \times 3^{2x-3} + 1 \\
 5y - 1 &= 2 \times 3^{2x-3} \\
 \frac{5y - 1}{2} &= 3^{2x-3} \\
 \log\left(\frac{5y - 1}{2}\right) &= \log(3^{2x-3}) \\
 \log\left(\frac{5y - 1}{2}\right) &= (2x - 3) \log(3) \\
 (2x - 3) \log(3) &= \log\left(\frac{5y - 1}{2}\right) \\
 2x - 3 &= \frac{\log\left(\frac{5y - 1}{2}\right)}{\log(3)} \\
 2x &= \frac{\log\left(\frac{5y - 1}{2}\right)}{\log(3)} + 3 \\
 x &= \frac{\frac{\log\left(\frac{5y - 1}{2}\right)}{\log(3)} + 3}{2} \\
 x &= \frac{1}{2} \left[ \frac{\log\left(\frac{5y - 1}{2}\right)}{\log(3)} + 3 \right]
 \end{aligned}$$

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