Rearranging linear equations

What is an equation?

An equation is an expression of numbers and variables (represented by letters) linked together by an equals sign. Often equations need to be rearranged to isolate one of the variables. This variable is then called the subject of the equation.

Example:

Consider the equation $C = 4n$ where n is the number of boxes of apples purchased at $4 each and C is the total cost in dollars. Here, C is the subject of the equation. The equation could be rearranged to make n the subject.

Rearranging an equation

Since one side of an equation equals the other, it is in balance. This balance must be maintained; if a mathematical operation is applied to one side of the equation, the same operation must be applied to the other side.
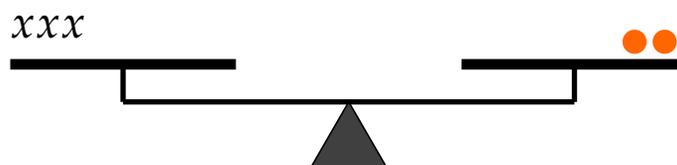
Example:

Consider equation $3x + 5 = 7$. This can be represented as a balance.
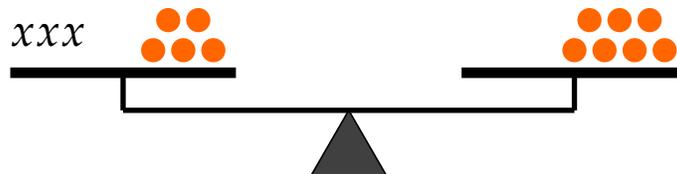
If we remove 5 from the left hand side of the balance, we must also remove 5 from the right hand side:

$$3x + 5 - 5 = 7 - 5$$
$$3x = 2.$$

Since we now have 3 times x on the left hand side, we isolate the x by dividing by 3 on both

sides of the equation:

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{2}{3} \qquad \text{the 3s cancel on the left hand side of the equation,}$$

$$x = \frac{2}{3}.$$

Order of operation

Each operation has an opposite operation. Some of these are summarised in the following table:

Operation	Opposite
$\times$	$\div$
$+$	$-$
$\sqrt{}$	$(\ )^2$

Generally, to rearrange an equation, work in **reverse order** to the original order of operation, **using the opposite operation**.

Example:

Make x the subject of the equation: $y = 4x + 3$.

Suppose the value of x was known. To find the value of y, multiply x by 4 and then add 3. Diagrammatically:

$$\boxed{x} \xrightarrow{\times 4} \boxed{4 \times x} \xrightarrow{+3} \boxed{4x + 3}$$

To rearrange the equation, you work from right to left. The operations are reversed.

$$\boxed{x} \xleftarrow{\div 4} \boxed{4 \times x} \xleftarrow{-3} \boxed{4x + 3}$$

Solving Algebraically:

$$
\begin{aligned}
y &= 4x + 3 \\
y - 3 &= 4x + 3 - 3 & \text{subtract 3 from both sides,} \\
y - 3 &= 4x & \text{since } 3 - 3 = 0 \text{ on the right hand side,} \\
\frac{y - 3}{4} &= \frac{4x}{4} & \text{divide both sides by 4,} \\
\frac{y - 3}{4} &= x & \text{the 4s cancel on the right hand side,} \\
x &= \frac{y - 3}{4} & \text{writing } x \text{ not he left hand side.}
\end{aligned}
$$

Resources

- Other QuickTips flyers;
- Online resources at Study Support, USQ Library;
- Make a consultation with a Mathematics Learning Advisor.