Module A5

REPRESENTING RELATIONSHIPS

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Introduction

Suppose you have just arrived in a strange town. You go to the local tourist bureau to seek directions to a particular local attraction. Think about how you would like to be given directions. Would you like to be given a map with the attraction clearly identified, or would you prefer the attendant to tell you the directions? Maybe you would prefer just to have the directions written down (without a map), or even to have someone take you to the attraction.

What if you were about to purchase a new stereo. Other than price, what would influence your decision? Maybe the salesperson telling you what you wanted to know, or reading the details in the brochure. Maybe you would base your decision on the way it looked or sounded.

This module is about representing relationships, and you may be wondering what this has to do with directions and stereos. In mathematics we look for the connection or relationship between many different things. We try to represent these relationships in different ways, catering for the different ways in which people prefer to receive information. In the above examples, different students would have had different preferences for receiving directions or deciding on which stereo to buy. For some people, a word statement of a relationship is enough to explain clearly the connection. For others, a picture or diagram may be the preferred method of viewing a relationship. Still others prefer a precise formula that clearly states the connection. You will see information in the newspaper presented in different ways for exactly these reasons. There may be a table of information, a graph to accompany the table and some text that will provide still further insight into the relationship.

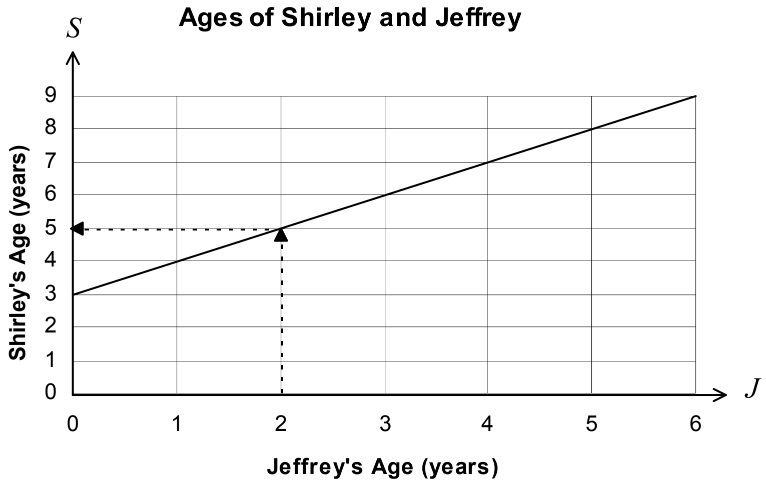
This module will look at different ways of representing relationships. It looks at representing relationships in words, graphs and formulas and goes on to look at ways of describing relationships.

When you have completed this module you should be able to:

- demonstrate an understanding of the nature of a formula;
- convert word statements into mathematical formulas;
- use the Cartesian Coordinate System to represent formulas;
- convert graphs and formulas into written descriptions; and
- describe more complex graphs in words.

5.1 Preview

A graph is a very convenient way to picture a relationship between two quantities. Consider the following graph showing the age relationship between a brother and sister, Shirley and Jeffrey.



Follow the arrows on the above graph to answer the following.

When Jeffrey was 2 years old, how old was Shirley? ....

Imagine similar arrows to answer the following.

When Jeffrey was 3 years old, how old was Shirley? ....

When Jeffrey was 4 years old, how old was Shirley? ....

When Jeffrey was 5 years old, how old was Shirley? ....

You should have found Shirley's ages to be 5, 6, 7 and 8 years respectively.

In fact we can find Shirley's age for any given age for Jeffrey.

Describe any relationship you can see between the ages of Jeffrey and Shirley

....

Did you say something about the fact that Shirley is always 3 years older than Jeffrey?

We could now go on and express this relationship in symbols as we have done in past modules.

If we let Shirley's age be represented by S and Jeffrey's age be represented by J we could say that Shirley's age is Jeffrey's age plus three.

That is, $S = J + 3$

We call this a **formula**. It allows us to find Shirley's age for whatever age of Jeffrey we might choose.

We have now expressed the relationship between the ages of Shirley and Jeffrey in three different ways:

- in words
- as a formula
- as a graph

Each method has its advantages. This module goes on now to look at each of these methods of representing a relationship in more detail.

5.2 Relationships in words

This is probably the method of expressing a relationship that you are most familiar with at this stage.

For example:

- Chris earns twice as much as David.
- At the restaurant there are 5 more women than men.
- The bank charges 8% interest on my loan.
- The grevillea was half the height of the palm tree.
- The adult weighed three times as much as the child.
- The perimeter of a square is four times the length of one side.
- The house was 15 metres longer than it was wide.

While it is important that we are able to express in words what we are trying to find, it is more convenient to use a formula when we need to do some calculations using these relationships. We will come back to relationships in words when we look at ways of describing relationships.

5.3 Relationships as formulas

We have already looked at expressing the relationship 'Shirley is 3 years older than Jeffrey' as a formula.

That is $S = J + 3$ where S represents Shirley's age in years,

and J represents Jeffrey's age in years.

It is important if we are going to express our relationship as a formula using letters, that we know exactly what these letters represent. In our case, we have let S equal Shirley's age in years and J equal Jeffrey's age in years. We could have chosen any letters we liked but these two clearly help us to identify the required relationship. It is also important to note that it is not just Shirley and Jeffrey that we are comparing, but their ages. We should also note that their ages are always measured in years.

In this case we call S and J **variables** because they can vary to take on any values for S and J that we give them.

Let's look at some of the word relationships from the previous section.

Example

Translate the following relationship into a formula.

Chris earns twice as much as David.

What are the variables in this case?

....

You could have let the variables be *C representing the amount of money that Chris earns* and *D representing the amount of money that David earns*. Note that it is the amount of money that each earns that is the variable, not the person.

Before you go on to write a formula, think about the question. You know that Chris is the person earning the most money. Remove from the question any unnecessary words.

Chris earns twice David

or, C earns 2 times D

Put the symbol × for times.

C earns $2 \times D$

Finally put in the equals sign to complete the formula.

$C = 2 \times D$ where C represents the amount of money that Chris earns in dollars,

and D represents the amount of money that David earns in dollars.

It is an acceptable shorthand to leave the multiplication sign out of expressions involving variables. We say the multiplication is **implied or understood**.

So we could write $C = 2D$ and we 'know' that we must multiply the 2 and the D.

Example

Translate the following relationship into a formula.

At the restaurant there are 5 more women than men.

We will firstly define the variables we will use.

Let W represent the number of women at the restaurant,

and M represent the number of men at the restaurant.

To write the formula you could use one of two methods.

The 'skeleton method' where you could set up the skeleton of the formula with lots of room.

$$W \qquad = \qquad M$$

There are more women than men so we must add the 5 to the number of men to get the number of women.

$$W = M + 5$$

The other method that you might use is to rearrange the question.

Five more women than men

or, Number of women is 5 more than number of men

Number of women is men plus 5

W is M plus 5

$W = M + 5$

Example

Translate the following relationship into a formula.

The bank charges 8% interest on my loan.

The formula using one of the above methods, could be written:

$I = 8\% \times A$ Where I represents the interest charged on the loan in dollars,

$I = \frac{8}{100} \times A$ and A represents the amount of the loan in dollars.

$I = 0.08 \times A$

We could also write this as $I = 0.08A$

Since it doesn't matter which letters are chosen for variables in a formula (provided that it is understood what the letters mean) this formula could also be written as:

$y = 0.08x$ Where y represents the interest charged on the loan in dollars,

and x represents the amount of the loan in dollars.

It is most important that the meaning of the variables be understood.

Formulas can involve two variables, just like the examples above, but you might also see a relationship which involves only one variable and sometimes ones involving 3, 4 or more variables.

Example

The number of people in Toowoomba is approximately 90 000. We could write this as the formula

N = 90 000 where N is the approximate population of Toowoomba.

This is an example of a formula with one variable.

Example

The formula for finding the perimeter of a rectangle is

$P = 2(L + W)$ where P represents the perimeter,

L represents the length of the rectangle,

and W represents the width of the rectangle.

This is an example of a formula with three variables.

You might also notice that the formula for the perimeter of a rectangle contains two operations, namely addition and multiplication. The multiplication sign is understood to be after the 2, that is, $P = 2 \times (L + W)$

In general, a formula can contain any number of different operations (addition, subtraction, multiplication, division, powers, square roots.......) depending on the relationship being described.

Activity 5.1

1. Express the following relationships as formulas.

   (a) The distance (d) travelled be a car is equal to its speed (s) multiplied by the time (t) of the journey.

   d =

   (b) Profit (P) equals the revenue income (R) less the costs (C).

   P =

   (c) Electrical voltage (V) equals electrical current (I) times electrical resistance (R).

   V =

2. Express the following relationships as formulas.

   (a) The grevillea was half the height of the palm tree.

   (b) The adult weighed three times as much as the child.

   (c) The perimeter of a square is four times the length of one side.

   (d) The house was 15 metres longer than it was wide.

3. A shop stocks four times as many cola softdrink bottles as soda water. Write a formula to calculate the number of cola softdrink bottles given the number of soda water bottles.

4. If taxable income is greater than $33 500 in the Chook Islands, tax is calculated by subtracting $9 500 from the taxable income, then finding 39% of this difference, then adding $9 500. Write a formula to calculate the tax to be paid from the taxable income.

5. A small company pays their salesperson Mae Ihelpyu $200 per week plus $50 for each item that she sells in the week. Write a formula to calculate Mae's weekly income for any number of items sold.

5.3.1 Substituting into formulas

Recall our formula relating the ages of Shirley and Jeffrey.

$S = J + 3$ where S represents Shirley's age in years,

and J represents Jeffrey's age in years.

By replacing the symbol for Jeffrey's age with a numerical value I am able to find Shirley's age.

When Jeffrey was 14 years old, how old was Shirley?

That is, when $J = 14$, what does S equal? We will **substitute** 14 for the J.

$$S = 14 + 3$$
$$= 17$$

So, when Jeffrey was 14 years old, Shirley was 17 years old. (Shirley is still 3 years older than Jeffrey as she always will be.)

Find Shirley's age when Jeffrey is 19 and when he is 35.

You should have found that when Jeffrey is 19, Shirley is 22 and when he is 35, Shirley is 38.

Example

The formula for the area of a square is

$A = s^2$ where A represents the area of the square,

and s represents the side length of the square.

When the length of the side of the square is 3 cm the area of the square will be

$$A = s^2$$
$$= s \times s$$
$$= 3\text{ cm} \times 3\text{ cm}$$ Remember to multiply the units. $\text{cm} \times \text{cm} = \text{cm}^2$
$$= 9\text{ cm}^2$$

Find the area of a square with a side length of 9 cm.

$$\begin{aligned} A &= s^2 \\ &= s \times s \\ &= \square \text{ cm} \times \square \text{ cm} \\ &= \square \text{ cm}^2 \end{aligned}$$

Did you get 81 cm^2?

Example

The formula for speed is

$$s = \frac{d}{t}$$

where s represents speed,

d represents the distance travelled,

and t represents the time taken.

A giant tortoise can cover 100 metres in 20 minutes, what is the speed of the tortoise?

We need to be very careful with units when calculating speeds. Recall the work on rates from the previous module if you need to.

Recall:
Module 4
Section 4.2.3

$$\begin{aligned} s &= \frac{d}{t} \\ s &= \frac{100 \text{ m}}{20 \text{ minutes}} \\ s &= 5 \text{ m/minute} \end{aligned}$$

If a human can walk at about 4 km/h how does this compare to the giant tortoise?

We cannot compare the two speeds at the moment because they are in different units.

Convert the tortoises speed to km/h. ....

Did you get 0.3 km/h? (5 m/minute = 0.005 km/minute = 0.005 × 60 km/hour

We can now see that the human can walk much faster than the tortoise. In fact, using our knowledge from the previous module, the human can walk more than 13 times faster than the tortoise.

Activity 5.2

1. The formula for the area of a rectangle is:

   $A = lw$ Where l represents the length of the rectangle,

   and w represents the width of the rectangle.

   Find the area of rectangles with the following lengths and widths.

   (a) $l = 7$ cm $w = 5$ cm

   (b) $l = 34$ cm $w = 21$ cm

   (c) $l = 3$ m $w = 67$ m

   (d) $l = 4$ cm $w = 82$ mm

2. Recall that in module 4 we talked about the ratio of the perimeter of a square to its side length. We said that the perimeter is always 4 times the length of the side.

   (a) Now write this as a formula.

   (b) Find the perimeter of squares with the following side lengths.

   (i) 4 cm

   (ii) 7 m

   (iii) 2.6 mm

   (iv) $\frac{3}{4}$ m

   (c) Suppose that you had a square chicken pen, of side length 3.2 metres. You wish to fence it with chicken wire costing $6.50 a metre. How much will it cost to fence the chicken pen?

3. Also in module 4 we talked about the ratio of the circumference to the diameter of a circle. We said that the ratio of the circumference of a circle to its diameter is equal to π. Another way to look at this is to say that the circumference is equal to π times the diameter.

   (a) Write a formula that will allow us to find the circumference of the circle, given the diameter.

   (b) Find the length of the circumference, rounded to one decimal place, for circles with the following diameters:

   (i) 3 cm

   (ii) 72 mm

   (iii) 6.5 m

   (iv) $\frac{3}{5}$ m

   (c) Suppose that you have built a circular rose garden with diameter 3.8 metres in your yard. You wish to purchase a log type edging that comes in rolls of three metres costing \$18.95 for roll. How many rolls of edging should you purchase, and what will be the cost?

4. If I stand on top of a cliff S metres high and drop a stone, it will take t seconds to reach the ground. The height of the cliff can be found using the following formula.

   $S = 4.9\,t^2$

   Use this formula to find the height of the cliff when it takes the following times for the stone to reach the ground.

   (a) 3 seconds

   (b) 8 seconds

   (c) 1.4 seconds

5. Observation towers are often built in forests to allow bushfires to be spotted quickly. The distance (D) in kilometres that can be seen from a tower of height h metres is given by:

   $$D = 8\sqrt{\frac{h}{5}}$$

   Find the distance that can be seen from towers with the following heights. Round your answers to the nearest kilometre.

   (a) 10 metres

   (b) 25 metres

   (c) 17.5 metres

6. We can quickly find the approximate area of skin on our body by using the following formula.

   $A = \frac{3h^2}{5}$ where A represents the area of skin,

   and h represents the person's height.

   (a) This formula only gives approximate answers for average people. Explain why it wouldn't be accurate for all people?

   (b) Calculate the area of skin covering people with the following heights.

      (i) 165 cm

      (ii) 175.5 cm

      (iii) 1.68 m

   (c) If each of these people suffered burns to 24% of their bodies, calculate the area of skin that is burnt.

7. To check whether a person is the correct weight for their height, health workers now calculate the Body Mass Index. It is found by using the following formula.

Body Mass Index = $\frac{w}{h^2}$ where w represents the weight in kilograms,

and h represents the height in metres.

For a person to be healthy their Body Mass Index should lie between 19 and 25 for a female and 20 to 26 for a male.

Calculate the Body Mass Index for the following people.

(a) Female, 165 cm, 65 kg

(b) Male, 178 cm, 85 kg

(c) Female, 150 cm, 75 kg

(d) you

Which of these people fall into the healthy category?

5.4 Relationships as graphs

Earlier in this module we looked at the relationship between Shirley and Jeffrey as a graph. We will now look at how you would go about graphing this and other relationships.

You have probably used a map at some stage to find the location of a particular place. Map makers commonly set up a **grid** to allow you to do this. They use a letter to label a column and a number for a row (this will sometimes vary).

Look at the map of the University of Southern Queensland on the next page. Suppose you were looking for the library and were given the grid reference J7. To find it you need to go across to column J and from there go up or down to row 7. The library will be found at this point. (It is actually labelled R on the map.)

Now find the Office of Preparatory and Continuing Studies, the grid reference being L4. Go across to L and then up or down to 4. You should have found it labelled K5 on the map. If you are ever in Toowoomba call into K5 and say hello to us.

Activity 5.3

1. What is located at the following grid references.

   (a) I6

   (b) D5

   (c) N7

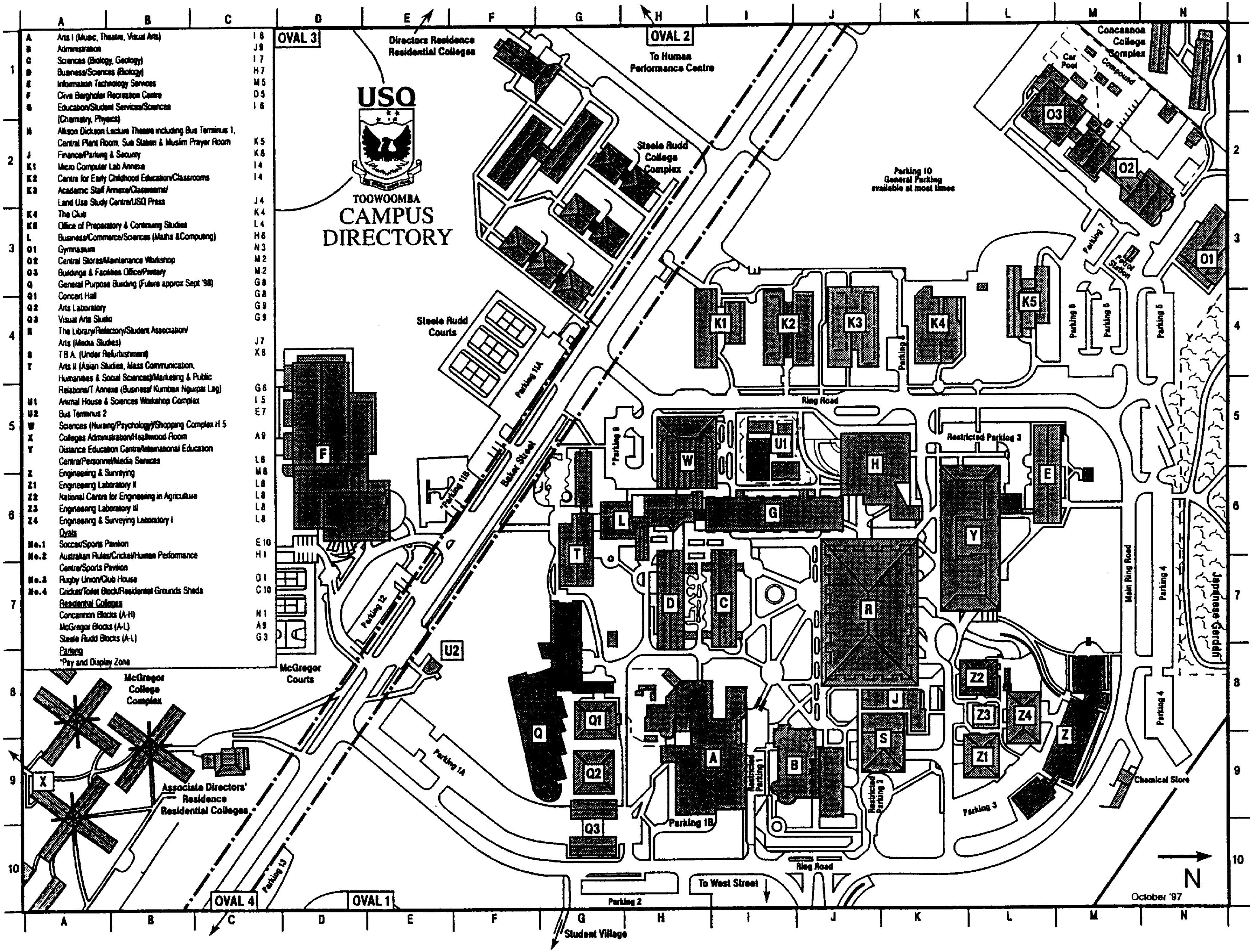
2. Find the grid reference for the following places.

   (a) Steele Rudd Courts.

   (b) W Block Sciences (nursing and psychology) and Shopping Complex.

   (c) Gymnasium

Obviously a grid system is a very useful tool because everyone agrees on the location of a particular point.

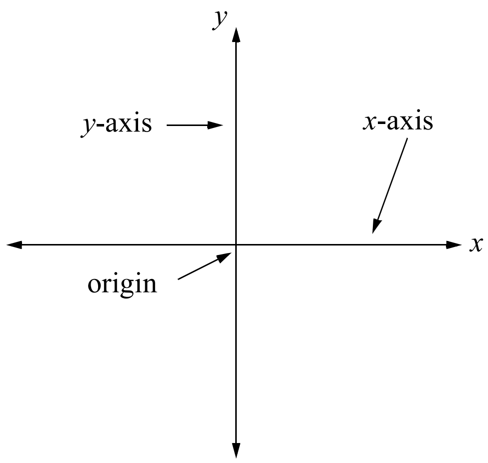

TOOWOOMBA
CAMPUS
DIRECTORY
USQ
A Arts I (Music, Theatre, Visual Arts) I 8
B Administration J 9
C Sciences (Biology, Geology) I 7
D Business/Sciences (Biology) H 7
E Information Technology Services M 5
F Clive Berghofer Recreation Centre D 5
G Education/Student Services/Sciences (Chemistry, Physics) I 6
H Alison Dickson Lecture Theatre including Bus Terminus 1, Central Plant Room, Sub Station & Muslim Prayer Room K 5
J Finance/Parking & Security K 8
K1 Micro Computer Lab Annexe I 4
K2 Centre for Early Childhood Education/Classrooms I 4
K3 Academic Staff Annexe/Classrooms/ Land Use Study Centre/USQ Press J 4
K4 The Club K 4
K5 Office of Preparatory & Continuing Studies L 4
L Business/Commerce/Sciences (Maths & Computing) H 6
O1 Gymnasium N 3
O2 Central Stores/Maintenance Workshop M 2
O3 Buildings & Facilities Office/Pottery M 2
Q General Purpose Building (Future approx Sept '98) G 8
Q1 Concert Hall G 8
Q2 Arts Laboratory G 9
Q3 Visual Arts Studio G 9
R The Library/Refectory/Student Association/ Arts (Media Studies) J 7
S T.B.A. (Under Refurbishment) K 8
T Arts II (Asian Studies, Mass Communication, Humanities & Social Sciences)/Marketing & Public Relations/T Annexe (Business/ Kumbari Ngurpai Lag) G 6
U1 Animal House & Sciences Workshop Complex I 5
U2 Bus Terminus 2 E 7
W Sciences (Nursing/Psychology)/Shopping Complex H 5
X Colleges Administration/Heallwood Room A 9
Y Distance Education Centre/International Education Centre/Personnel/Media Services L 6
Z Engineering & Surveying M 8
Z1 Engineering Laboratory II L 8
Z2 National Centre for Engineering in Agriculture L 8
Z3 Engineering Laboratory III L 8
Z4 Engineering & Surveying Laboratory I L 8
Ovals
No.1 Soccer/Sports Pavilion E 10
No.2 Australian Rules/Cricket/Human Performance Centre/Sports Pavilion H 1
No.3 Rugby Union/Club House O 1
No.4 Cricket/Toilet Block/Residential Grounds Sheds C 10
Residential Colleges
Concannon Blocks (A-H) N 1
McGregor Blocks (A-L) A 9
Steele Rudd Blocks (A-L) G 3
Parking
*Pay and Display Zone
OVAL 3
Directors Residence Residential Colleges
OVAL 2
To Human Performance Centre
Concannon College Complex
Car Pool
Compound
Steele Rudd College Complex
Parking 10 General Parking available at most times
Parking 7
Petrol Station
Steele Rudd Courts
Parking 11A
Parking 8
Parking 6
Parking 5
Ring Road
Parking 9
Restricted Parking 3
Parking 11B
Baker Street
Main Ring Road
Parking 4
Japanese Garden
Parking 12
McGregor Courts
McGregor College Complex
Associate Directors' Residence Residential Colleges
Parking 1A
Restricted Parking 1
Restricted Parking 2
Parking 1B
Parking 3
Chemical Store
Parking 13
OVAL 4
OVAL 1
Parking 2
To West Street
Ring Road
Student Village
N
October '97

Mathematicians use a similar system to refer to a point in a plane (a flat surface extending infinitely in all directions) and to distinguish it from any other points in that plane.

The system we use today was developed by a seventeenth century mathematician called René Descartes. Legend has it that the idea of coordinates came to him while he lay in bed and watched a fly crawling on the ceiling. Noting that each position of the fly could be expressed by two distances from the edges of the ceiling where the walls and ceiling met. He set up a **grid** using two number lines at right angles. The **horizontal** number line is usually called the ***x*-axis** and the **vertical** number line is usually called the ***y*-axis**. In fact these horizontal and vertical number lines can have any name as you will see as we move through this module. These two **axes** (the plural of axis and pronounced ax/ees) meet or **intersect** at a point called the **origin**. We call this grid system the **Cartesian Coordinate System** after its inventor.

5.4.1 The Cartesian plane

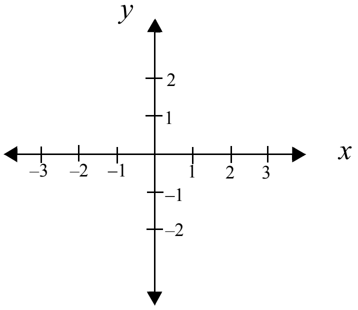
A Cartesian plane would look like this.



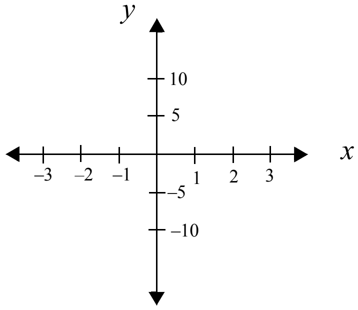
In this case I have labelled the axes x and y as you will often see, but I could also have labelled them J and S, or any other **variable** that I wish to represent.

As each axis is a number line it must have a **scale** drawn on it as we have done before when constructing a number line. The numbers extend out infinitely in either direction from the origin. All the way along the axis **one centimetre must always represent the same number of units**. It is not necessary to have the same scale on both axes although this will often be the case.

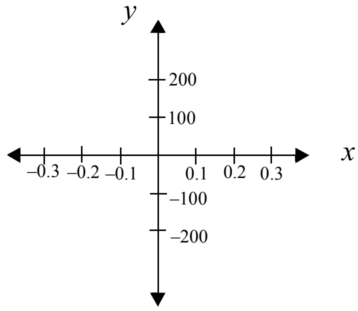
Look at the following Cartesian planes where different scales have been used. Note that only one scale has been used per axis. With a ruler, check that the scales are correct on each axis.



The same scale is used on each axis

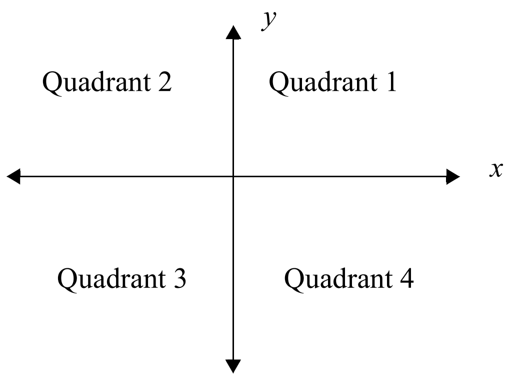


Different scales are used on each axis



Different scales are used on each axis

You will also have noticed that the two axes divide the plane into four regions. We call these regions **quadrants**. They are numbered as in the following diagram.



Any point can be represented on the Cartesian plane by using a pair of numbers that we call an **ordered pair**. To describe a point we give its horizontal or ***x*-coordinate** and its vertical or ***y*-coordinate**. This pair of numbers is always written in strict order, stating the x-coordinate first followed by the y-coordinate.

That is (*x*-coordinate , *y*-coordinate)

We abbreviate this to be (x, y)

Or, if we are not using x and y as our variables then in general we give the horizontal co-ordinate first and the vertical coordinate second.

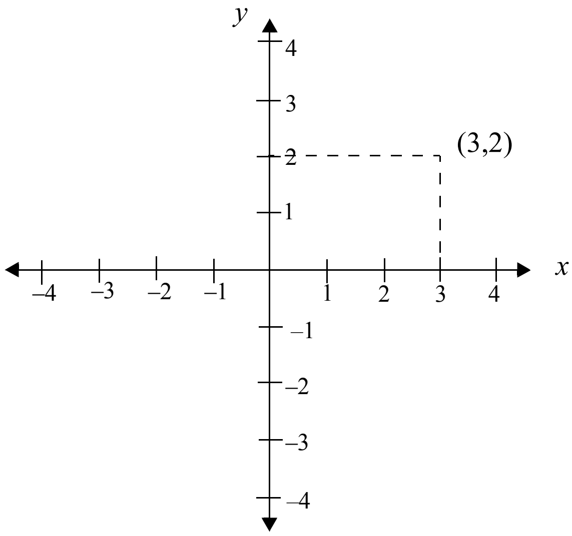
That is (horizontal coordinate, vertical coordinate)

Let's now look at using this scheme to locate some points on the Cartesian plane.

To locate the point (3,2) firstly establish which is the x-coordinate and which is the y-coordinate. As ordered pairs are always in the form (x,y) we know that the x-coordinate is 3 and the y-coordinate is 2.

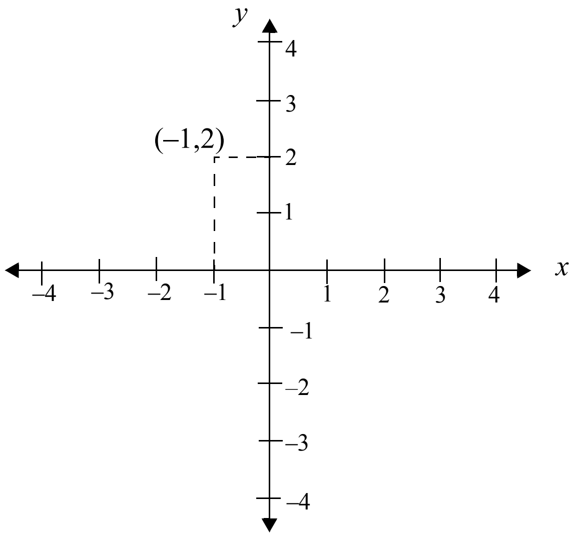
Having drawn a Cartesian plane, go across to 3 on the x-axis and up to 2 on the y-axis.

The dotted lines have been drawn to help you locate the point. These are only for guidance and would not normally be included on your diagram.

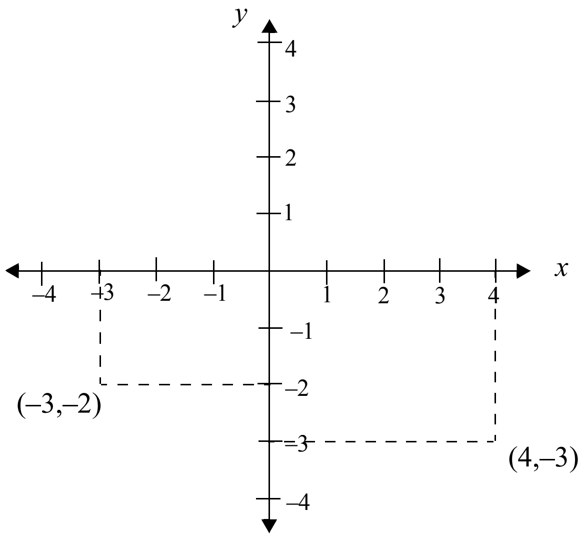


The point (3,2) lies in the first quadrant. We can locate points in other quadrants using the same technique.

Let's plot the point (–1,2). Again we must firstly determine that the x-coordinate is –1 and the y-coordinate is 2. This time we must move left from the origin to get to –1 and then up to 2 on the y-axis. You will notice that this point is in the second quadrant.



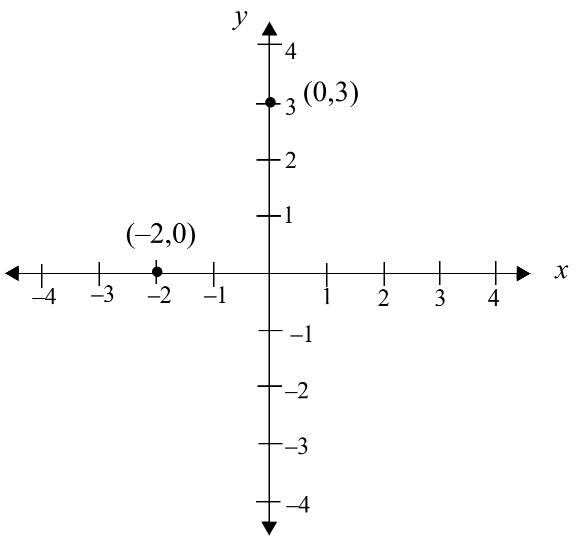
On the Cartesian plane below check the location of the points (–3,–2) and (4,–3)



You should have noticed that (–3,–2) is located in the third quadrant and (4,–3) in the fourth quadrant.

When you feel comfortable with locating points do not include the dotted lines.

What about points that lie on the axes rather than in one of the four quadrants. Take note of the location of the points (0,3) and (–2,0) below.



For the point (0,3) the x-coordinate is 0 which means we move neither left nor right from the origin, and then we move up to 3 on the y-axis. A point where the x-coordinate is 0 will always lie on the y-axis.

For the point (–2,0) the y-coordinate is 0. This means that after moving to –2 on the x-axis we move neither up nor down on the y-axis. A point where the y-coordinate is 0 will always lie on the x-axis.

Activity 5.4

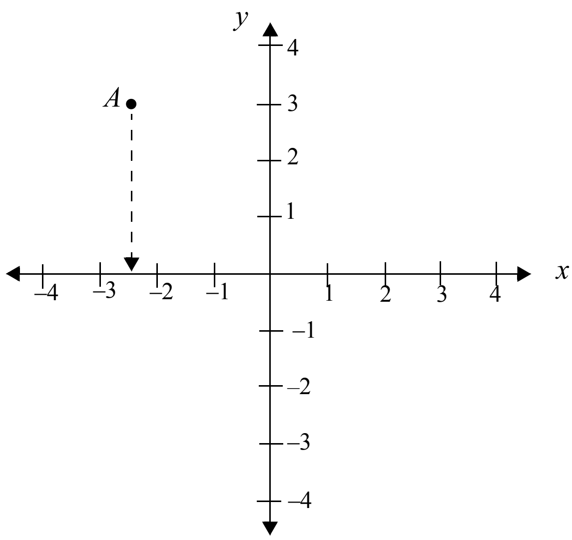
Set up a Cartesian plane on some 2 mm graph paper, with both axes showing from –5 to 5 using the scale 1 cm for every unit. Plot the following points on your Cartesian plane

(a)	(4,–3)	(f)	(–4,–5)
(b)	(2,3)	(g)	(0,0)
(c)	(–4,1)	(h)	(3,–5)
(d)	(1,0)	(i)	(0,–4)
(e)	(–1.5,4)	(j)	(–3,4)

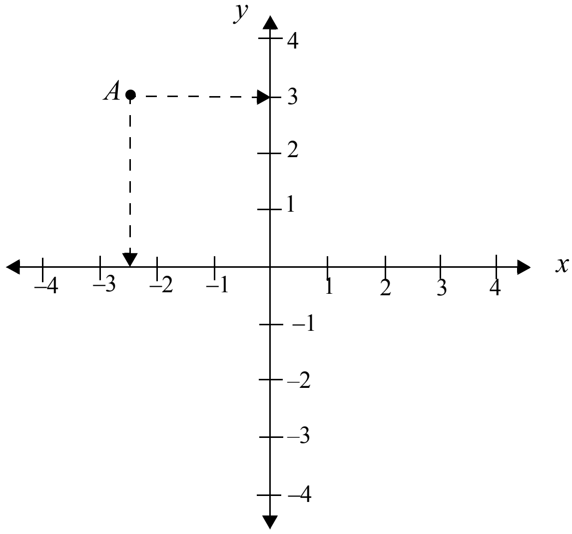
So far we have only looked at the case where you have been given an ordered pair and been required to locate it on the Cartesian plane. What if you were given the point located on the plane and needed to find its coordinates.

Consider the point A marked on the plane below. We determine the coordinates by finding the x-coordinate first.

Imagine a vertical line from the point to the x-axis. (We will draw these dotted lines in for convenience only.) The point where this line cuts the x-axis gives the x-coordinate. We find that it cuts at –2.5



The next step is to imagine a horizontal line from the point to the *y*-axis to find the *y*-coordinate. We find that it cuts at $y = 3$

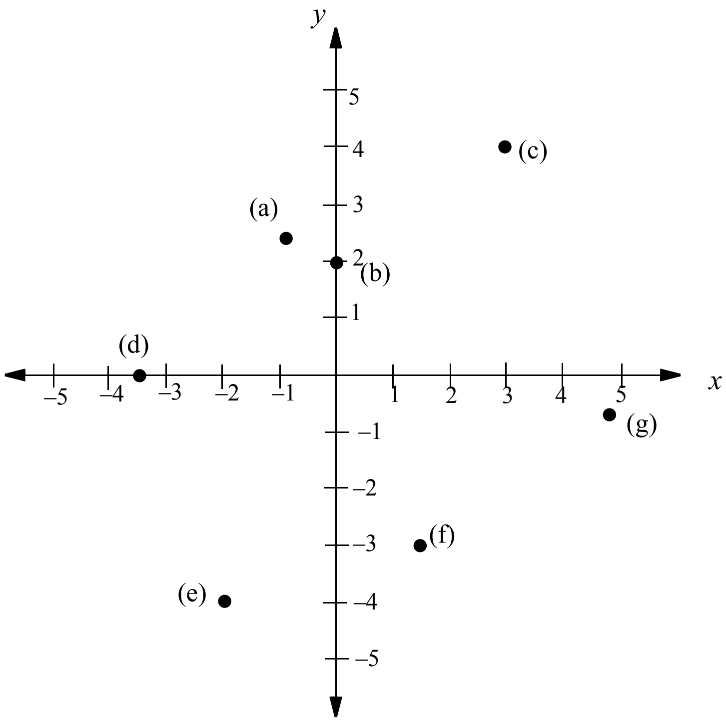


Therefore the coordinates of the point A are (–2.5,3)

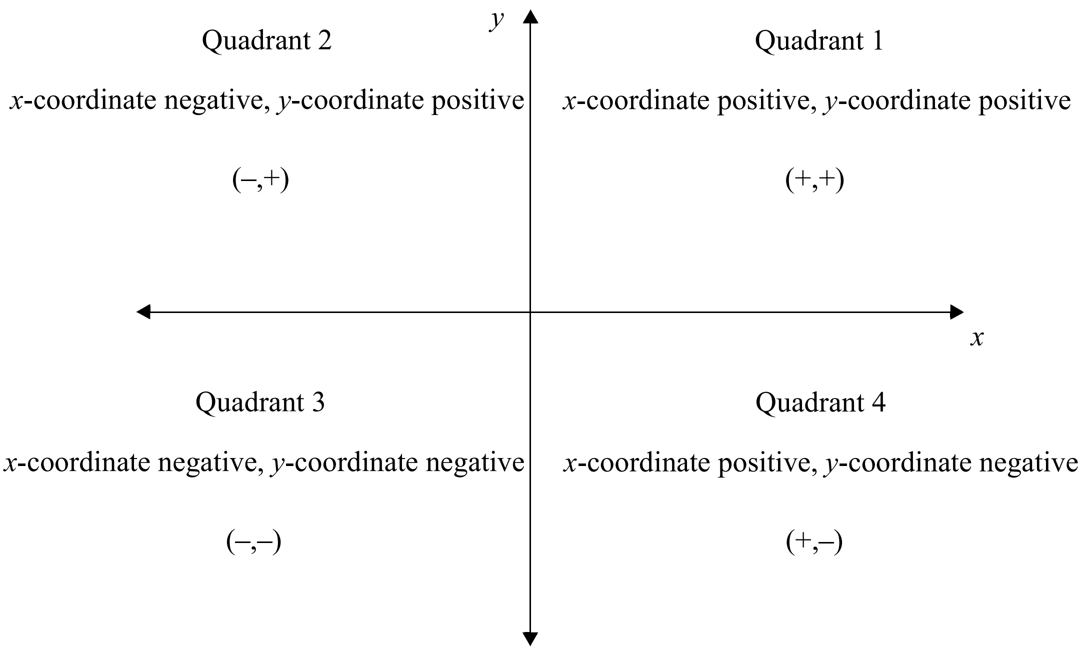
Practise, by finding the coordinates of the points below.

Activity 5.5

Find the coordinates of the points marked on the Cartesian plane below.



Let's summarize what we have learned about the points located in the various quadrants.



5.4.2 Drawing graphs

The general purpose of a graph is to give a visual representation of the relationship between two or more things. Graphs must contain clear, precise and meaningful information for the reader.

This section takes you step by step through the process of drawing a graph by plotting points. This is only one of many ways to develop a graph, but it is one that you can always fall back on, no matter what the shape of the graph. In this section we are only going to look at straight line graphs.

For this section you will need to have some 2 mm graph paper so that you can practise drawing graphs and reading off accurate answers. It is also helpful to have a well sharpened pencil to draw the graphs. You should draw the graphs on graph paper as we move through the steps.

Let's return to our brother and sister Shirley and Jeffrey. We know that Shirley is three years older than Jeffrey and that this can be represented by the formula

$S = J + 3$ where S represents Shirley's age in years,

and J represents Jeffrey's age in years.

Using the Cartesian coordinate system, if we could find a number of ordered pairs, we could locate these on the Cartesian plane and join them up to get a graph of this relationship.

In fact you have already found these ordered pairs in an earlier activity.

Recall:
Module 5
Section 5.3.1

When Jeffrey was 14, Shirley was 17

When Jeffrey was 19, Shirley was 22

When Jeffrey was 35, Shirley was 38

A convenient way to represent this information is to enter it in a **table of values**.

Jeffrey's age (years)	14	19	35
Shirley's age (years)	17	22	38

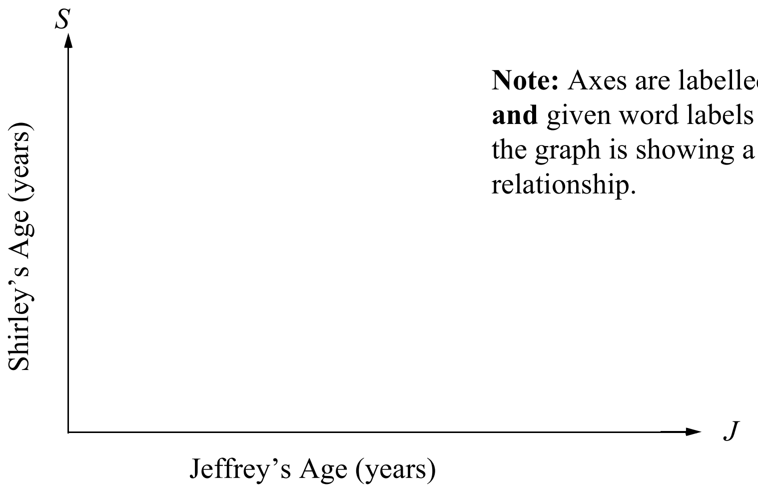
From our table of values we can write a series of ordered pairs that satisfy the formula.

(14,17) (19,22) (35,38)

In each case Jeffrey's age (horizontal axis) comes first and Shirley's age (vertical axis) is second.

The series of ordered pairs can now be plotted on a Cartesian plane so that we can see the relationship between Jeffrey's age and Shirley's age.

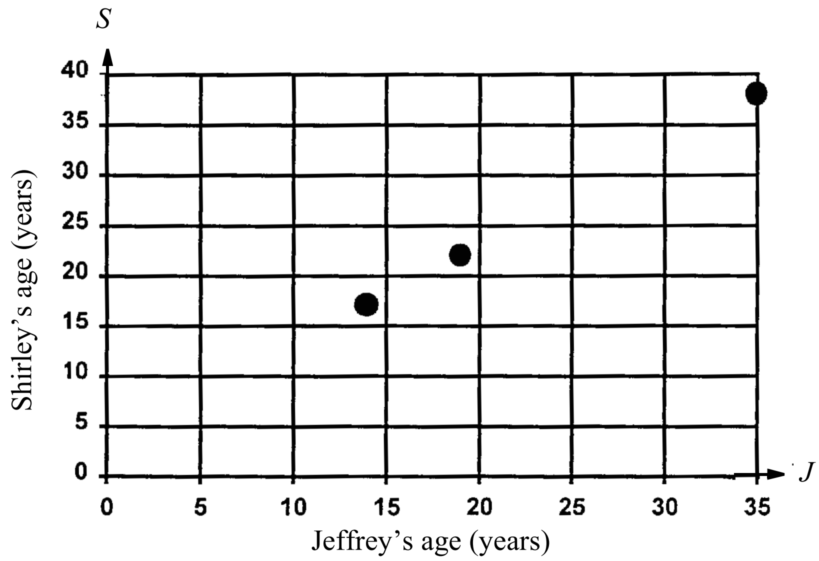
Before we draw our axes let's think about the possible values for the ages. Since neither Jeffrey's age nor Shirley's age can be negative we are only interested in the first quadrant.



Note: Axes are labelled *J* and *S* **and** given word labels because the graph is showing a specific relationship.

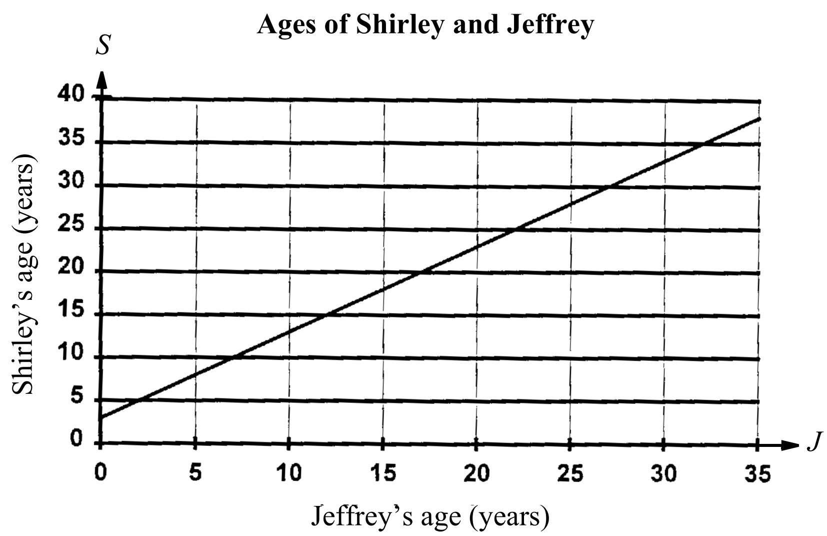
Now choose an appropriate scale to suit the ordered pairs. In our table of values, the maximum value for Shirley's age is 38 and for Jeffrey, 35. Let's then make the axis for Shirley's age go up to 40 and for Jeffrey's to 35.

Now plot the ordered pairs on the Cartesian plane drawn.



To illustrate the **trend in the graph** we draw a straight line through all of the plotted points. When drawing a trend line we need to think about whether all values on the x-axis are possible. In our case we are talking about ages in years. We can have 5 years, 10 years and so on, as marked on the graph, but we could also have $5\frac{1}{2}$ years or 6 years 3 months and 4 days, so yes all values are possible on the x-axis. Since all values are possible we will draw a solid line when joining the plotted points.

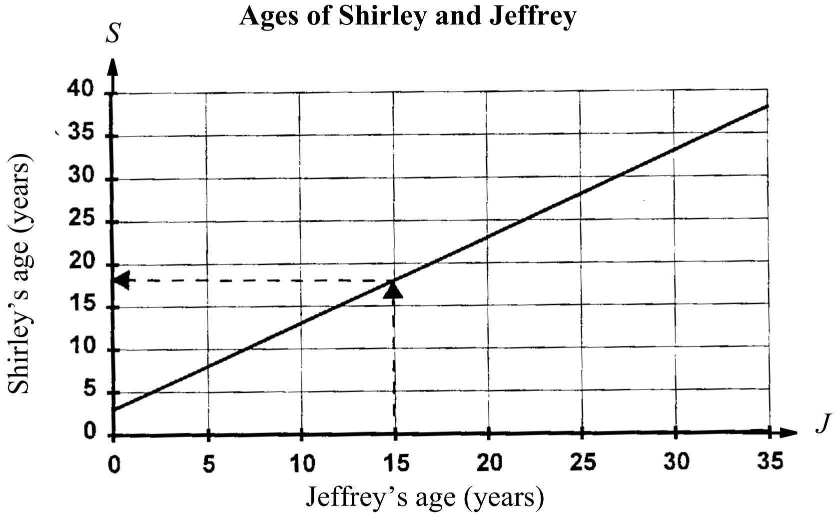
Finally give the graph a title.



Notice that we do not stop at the points we have plotted when drawing in the trend line. The points we have plotted were only three of an infinite number of possibilities.

Once the graph is drawn it is easy to read off Shirley's age for any age of Jeffrey.

Ages of Shirley and Jeffrey



When Jeffrey was 15, Shirley was 18. This is not easy to find on the above graph, but having drawn yours on 2 mm graph paper you should be able to read off your answer fairly accurately.

Let's now summarize the steps we have taken to draw a graph.

- From the formula complete a table of values.
- Draw up a Cartesian plane and label the axes (don't forget the units if appropriate).
- Choose a suitable scale.
- Plot the ordered pairs from the table of values.
- Draw a trend line extending through and beyond the plotted points.
- Give your graph a title.

Let's now look at another example, following the same steps as last time.

Example

Recall: Activity 5.1 Question 5

Recall this question for which you wrote a formula earlier in this module.

A small company pays their salesperson Mae Ihelpyu \$200 per week plus \$50 for each item that she sells in the week. Write a formula to calculate Mae's weekly income for any number of items sold.

You would have determined that the formula required was:

$I = 200 + 50n$ where I represents the total income in dollars,

and n represents the number of items sold.

If you were to graph this formula you could find the total income for Mae selling any number of items in a given week. Or, given a set amount of money to be earned in one week, we could calculate the number of items to be sold.

Can you see that the amount of money Mae earns **depends** on the number of items she sells?

We call Mae's income the **dependent variable** because it depends on the number of items she sells, and we call the number of items sold the **independent variable** because this can be any number of items.

It is usual to put the independent variable first in the table of values.

Complete the following table of values.

Number of Items sold (*n*)	0	5	10
Total Income in \$ (*I*)			

When $n = 0$

$I = 200 + 50n$

$= 200 + 50 \times 0$

$= 200 + 0$

$= 200$

When $n = 5$

$I = 200 + 50n$

$= 200 + 50 \times 5$

$= 200 + 250$

$= 450$

When $n = 10$

$I = 200 + 50n$

$= 200 + 50 \times 10$

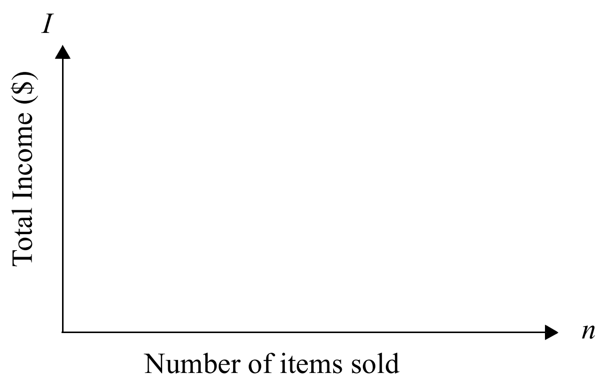
$= 200 + 500$

$= 700$

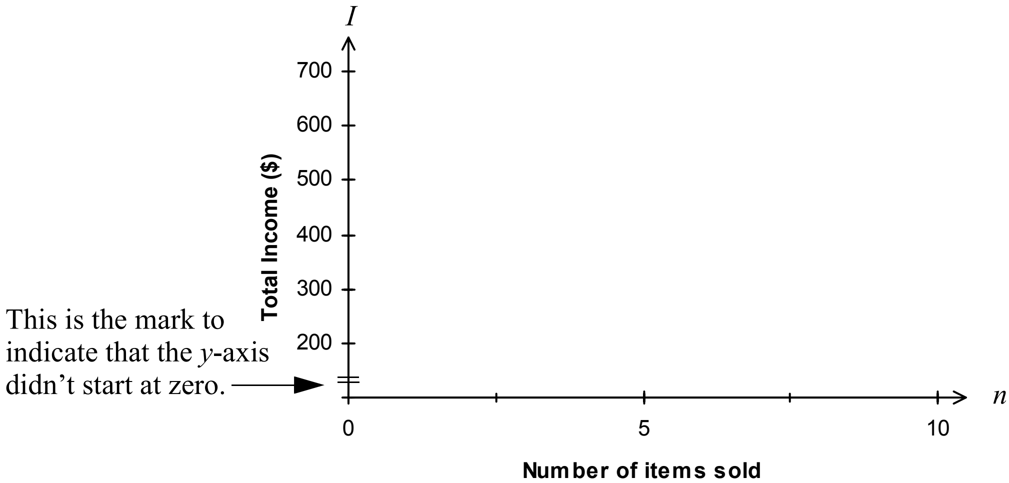
Completing the table of values we have:

Number of Items sold (*n*)	0	5	10
Total Income in \$ (*I*)	200	450	700

Now draw up the Cartesian plane and label the axes. Again we only require the first quadrant as the number of items sold can only be positive and the total income must also be positive.



Now choose a suitable scale. The horizontal axis (number of items sold) will need to start at zero, the minimum number of items that could be sold. The vertical axis though, could start at $100, a number just smaller than the least amount earned, which is $200. If we do not start the scale on an axis at zero, we should indicate that we have not started at zero.



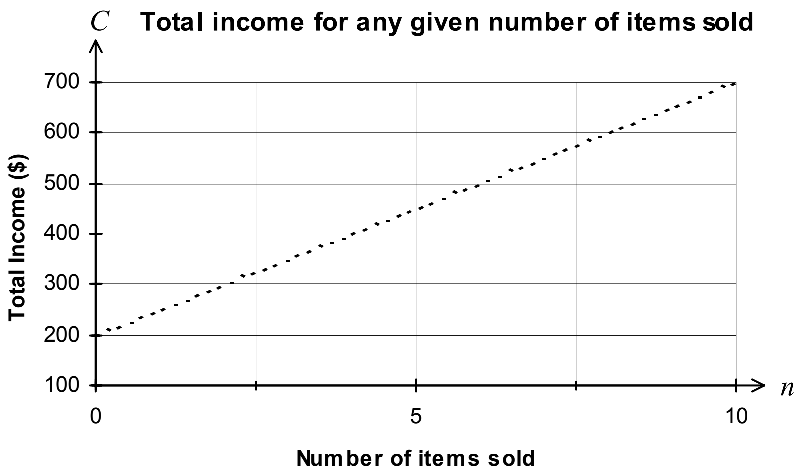
Now plot the ordered pairs from the table of values

(0,200) (5,450) (10,700)

Again the **independent variable** comes first and is the one **plotted on the** ***x*****-axis.**

Before drawing a trend line through the points, we need to think about whether all values on the x-axis are possible. In this case we are talking about number of items sold. We can have 5 items sold, 10 items sold and so on, as marked on the graph, but in this case we could not have $5\frac{1}{2}$ sold (who wants to buy, say $\frac{1}{2}$ a TV?) so this time, all values are **not** possible on the x-axis. Since all values are not possible we will draw a dotted or broken line when joining the plotted points.

Finally give your graph a title.



Let's now read off some values from your graph. These are not easy to find on the above graph, but having drawn yours on 2 mm graph paper you should be able to read off your answers fairly accurately.

What would Mae earn if she sold 7 items in one week? ....

How many items would Mae have to sell to earn $600 in a week? ....

You should have found that Mae earned $550 for selling 7 items, and to earn $600 would need to sell 8 items.

You could **check** these answers by **substituting** them into the formula.

When $n = 7$	When $n = 8$
$I = 200 + 50n$	$I = 200 + 50n$
$= 200 + 50 \times 7$	$= 200 + 50 \times 8$
$= 200 + 350$	$= 200 + 400$
$= 550$	$= 600$

Example

Let's finally look at a graph from a mathematical equation. An equation is just another name for a formula. It is looking at the relationship between two variables as before.

Sketch the graph of the formula $y = 2x - 1$

We follow the same steps as before.

The first thing to calculate is a table of values. You will need to decide what x values you will choose to put into your table. In the above two examples the values on the x-axis have been restricted to only positive numbers. There is no such restriction here, so we will choose a positive and a negative value for x.

x	–2	0	2
y			

When $x = –2$

$$\begin{aligned} y &= 2x - 1 \\ &= 2 \times -2 - 1 \\ &= -4 - 1 \\ &= -5 \end{aligned}$$

When $x = 0$

$$\begin{aligned} y &= 2x - 1 \\ &= 2 \times 0 - 1 \\ &= 0 - 1 \\ &= -1 \end{aligned}$$

When $x = 2$

$$\begin{aligned} y &= 2x - 1 \\ &= 2 \times 2 - 1 \\ &= 4 - 1 \\ &= 3 \end{aligned}$$

Now complete the table of values.

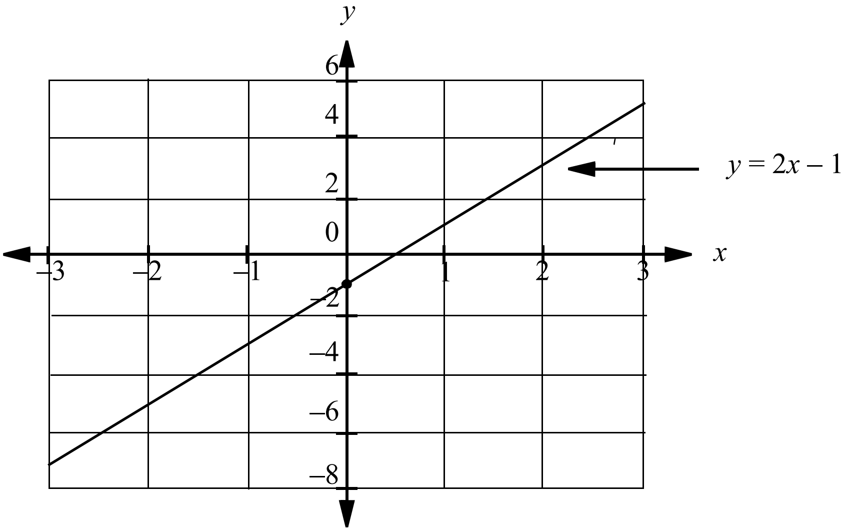
x	–2	0	2
y	–5	–1	3

The ordered pairs that we will plot are:

(–2,–5) (0,–1) (2,3)

When graphing an equation like this it is usual to graph over all four quadrants.

Plot the points, draw a line through and beyond the points. Finally label your graph. Note that this time our label or title looks slightly different.



Activity 5.6

To complete this activity you will need graph paper.

1. Before the invention of mechanical clocks, candles were sometimes used to measure the passage of time. A formula for the height of such a candle related to time is given below.

   $h = 10 - 2t$

   where h equals the height of the candle in centimetres, and t equals the time in hours that the candle has been burning.

   (a) Use this formula to complete the following table.

t	0	1	2	3	4	5
h						

   (b) Graph this formula, letting the horizontal axis represent time.

   (c) What was the height of the candle before it was lit?

   (d) What does the 10 represent in the formula?

   (e) For how many hours will the candle burn?

2. Suppose that you were planning a holiday in the United States. You wanted to know what clothes you should take so you looked up the expected temperatures in the newspaper. Problem is the temperatures were in degrees Fahrenheit and you never knew or have long forgotten what this means. A **conversion graph** will be useful to convert between degrees Fahrenheit and degrees Celsius.

(a) Draw a graph for the following information. You will need a full page graph with careful choice of scales to be able to read off accurate answers.

Degrees Celsius	0	10	20	40	80	100
Degrees Fahrenheit	32	50	68	104	176	212

(b) If you had found that the average temperature in the month you were planning to visit the USA was 59°F, what would this be in degrees Celsius?

(c) If it was 35°C in Australia, what would this be in degrees Fahrenheit?

3. The Leaky Boat Company hires out paddle boats for $7.50 an hour.

(a) Develop a formula relating the time you have the boat to the total cost of the hire. Don't forget to define your variables.

(b) Complete the following table of values.

Number of Hours	1	2	3	5	8
Total Cost ($)					

(c) Draw a graph showing the time you have the boat against the cost of the hire.

(d) If a family decides to hire the boat for 4 hours in the morning, what will be the cost of the boat?

(e) If another family comes to hire the boat for the afternoon but only has $24, for how many hours will they be able to hire the boat?

4. A car rental company charges an initial fee of $50 per day plus 15 cents per kilometre.

(a) Write a formula relating the kilometres travelled to the total cost for a days rental. Don't forget to define the variables.

(b) Draw up a table of values and construct a graph to represent this relationship.

(c) From your graph, determine the cost of renting the car for one day and driving 260 kilometres.

(d) If you had $110 to spend, how many kilometres could you travel in one day?

5. The amount of time that you can spend in the sun without burning is related to the sun protection factor (SPF) of the sunscreen lotion you use. The table below is for a person who can stay in the sun without any lotion for only 15 minutes without burning.

Sunscreen Factor	8	10	12	14
Time in minutes	120	150	180	210

(a) How does the increasing of the sunscreen factor by 2 change the time that can be spent in the sun?

(b) Draw a graph of this relationship.

(c) If this person wanted to stay in the sun for 1 hour without burning, what is the minimum sunscreen factor that would be required?

6. The following table shows the weight fluctuations for Ima Bitheavy who was on a weight loss program.

Week	1	2	3	4	5	6	7	8
loss/gain (kg)	–2.6	–1.2	+0.4	–0.9	+1.5	–2.7	–0.1	+0.4

(a) If Ima started the program weighing 94.7 kilograms, draw a graph showing the weight fluctuations over the eight weeks. Think carefully about the scale you will use on the vertical axis.

(b) What was the lightest weight that Ima reached?

5.5 Describing relationships

So far in this module we have been given a relationship in words and then been asked to write a formula for this relationship, or we have been given a formula and been asked to represent that relationship as a graph. More often, in your future studies you are going to be given a formula or graph and asked to interpret its meaning. This part of the module will be giving you practise in recognising the relationship depicted.

5.5.1 Describing relationships from a formula

Given that $P = F + 5$ without any indication of what P and F represent, try to describe a situation that fits this formula.

....

You could have chosen many answers. Some possibilities include:

- Pauline is 5 years older than Frederick
- In the fridge there are 5 more pies than fish fingers

- Peter gets \$5 more pocket money than Freda
- The paint shop is 5 metres taller than the fruit shop

The possibilities are endless. Remember that P and F can represent anything. A formula is not complete until the variables are defined.

Activity 5.7

Describe in words what the following formulas are representing.

1. $J = C + 2$ where J represents Joseph’s age in years, and C represents Chris’ age in years.

2. $C = J - 2$ where C represents Cathy’s age in years, and J represents Jenny’s age in years.

3. $A = L \times W$ where A represents the area of a rectangle, L represents the length of the rectangle, and W represents the width of the rectangle.

4. The following formula is Ohm’s law:
$I = \frac{E}{R}$ where I represents the current,
E represents the electromotive force,
mand R represents the resistance.

5. $A = \pi r^2$ where A represents the area of a circle, and r represents the radius of the circle.

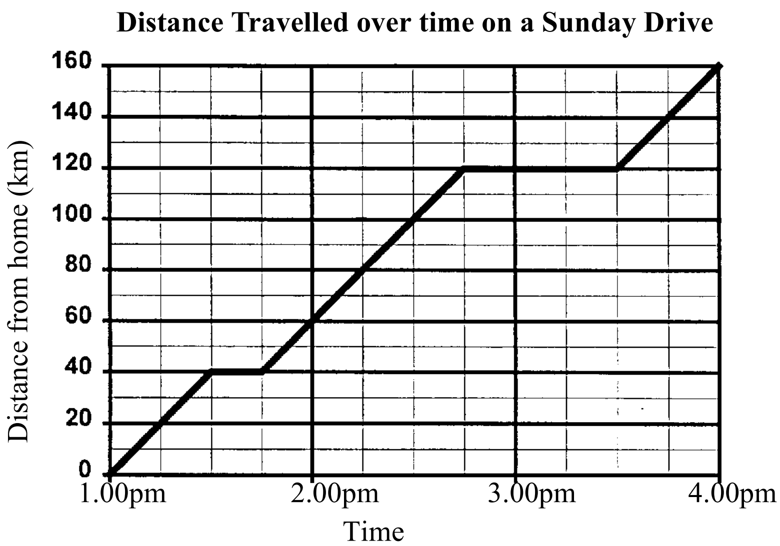
5.5.2 Describing a relationship from a graph

This is a situation that you will come across in your everyday life, whether reading the newspaper or a magazine, looking at advertising material or studying a textbook in the future. Graphs are a very convenient way to display information and it is up to the individual to interpret what the graph is ‘saying’. A well presented graph will be clear and easy to follow. It will have many of the features that we have previously discussed:

- a heading describing what the graph is about;
- clearly labelled horizontal and vertical axes;
- scales on the axes; and
- lines or points plotted to indicate the relationship represented.

Example

Consider the following graph showing details of a Sunday drive.



When interpreting a graph we would firstly look at three items.

- The title is the first step. This should clearly tell us what the graph is about.
- Next look at the horizontal axis. In this case the axis represents time, from 1:00 pm to 4:00 pm.
- Finally look at the vertical axis. In this case the axis represents the distance from home in kilometres.

We should now have a clear picture on what the graph is about.

Next we look to the graph itself.

A series of questions might help you to clarify the meaning of this graph.

(a) At what time did the Sunday traveller leave home? ....

Did you say 1:00 pm? The distance from home at 1:00 pm was zero, so this is the time the traveller left home.

(b) At what time did the traveller reach their destination? ....

You should have said 4:00 pm.

(c) How far was the traveller from home when they reached their destination? ....

They were 160 km from home.

(d) How far did the traveller go in the first hour? ....

The first hour went from 1:00 pm to 2:00 pm and the distance from home at these times was 0 kilometres at 1:00 pm and 60 km at 2:00 pm. The distance travelled in the first hour was 60 kilometres.

(e) How far did the traveller go in the last hour? ....

Did you say 40 kilometres from 3:00 pm to 4:00 pm?

(f) What do you think happened between 1:30 and 1:45 pm? ....

What are your reasons for giving this answer? ....

....

You should have indicated that the traveller was stopped over this time period (or the traveller was lost and travelling in circles) because there was no change in the distance from home, it remained at 40 km.

(g) Over what other time period was the traveller stopped? ....

The graph indicates that the traveller was stopped between 2:45 and 3:30 pm.

(h) What was the total time spent travelling? ....

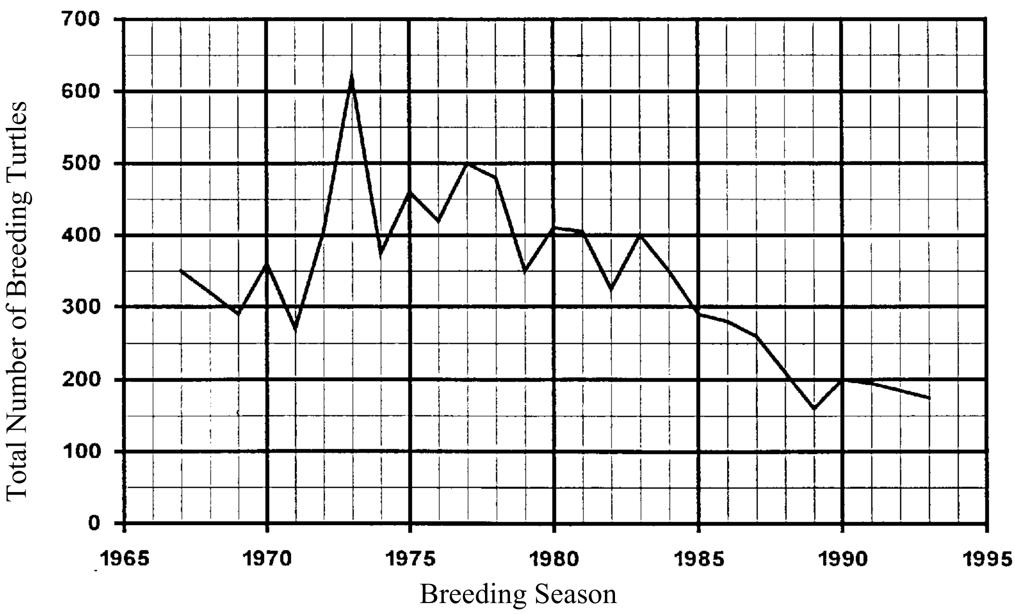
This question requires you to do some calculations. The Sunday drive lasted for 3 hours but some of this time was spent not travelling. The total time spent travelling was 2 hours.

As you can see, there is much information to be read from this graph. The following activity takes you through the interpretation of a number of different line graphs.

Activity 5.8

1. The following graph shows the annual breeding numbers of loggerhead turtles at Mon Repos on the Bundaberg coast over about 20 years.

Annual Breeding Numbers of Loggerhead Turtles between 1967 and 1993



(a) In what year did the most turtles nest at Mon Repos?

(b) In what year was there the least number of turtles?

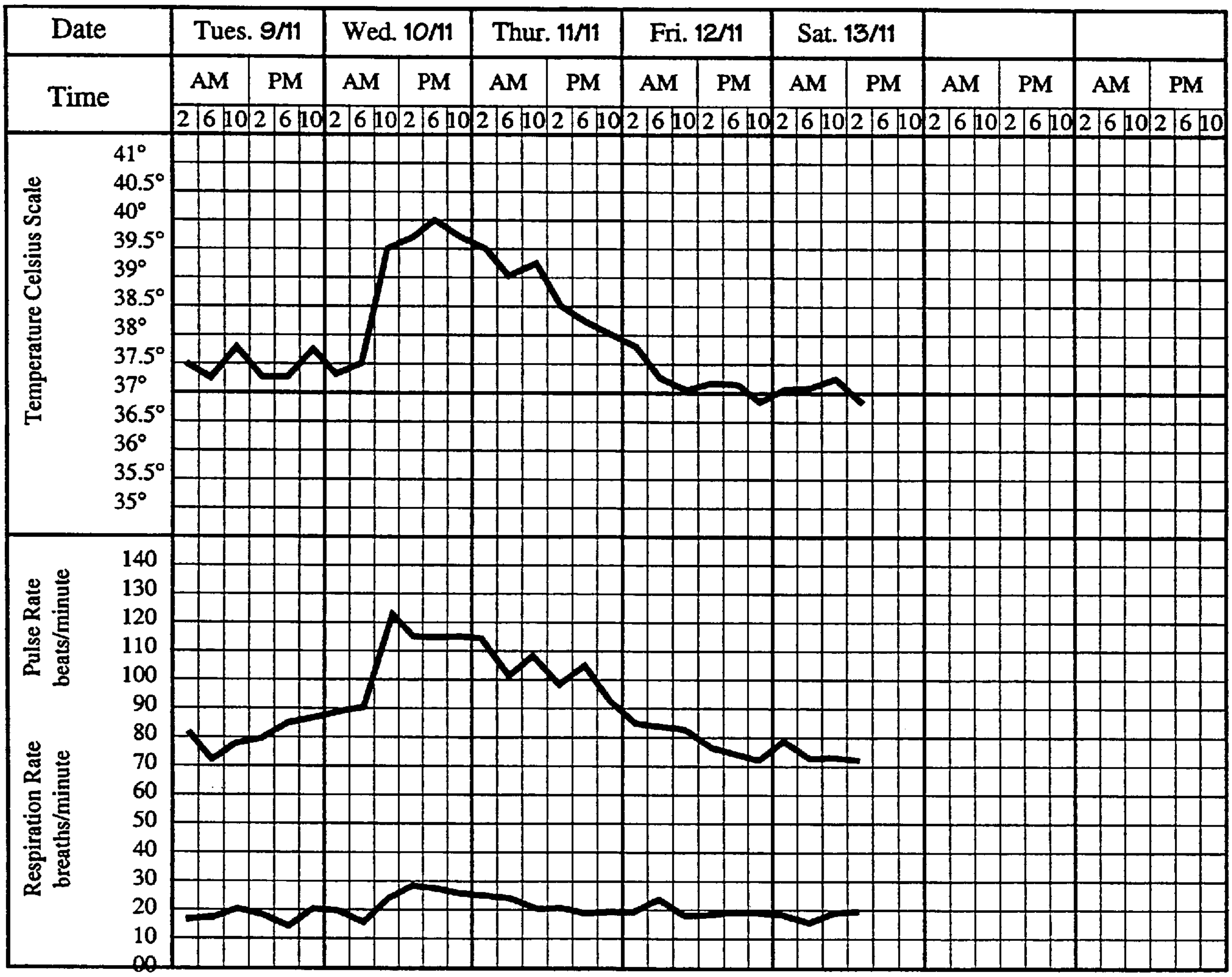
(c) Between which two years was there the greatest increase in turtle numbers?

(d) Between which two years was there the greatest decrease in turtle numbers?

(e) Scientists working in the Mon Repos area have predicted that this nesting sight will only last another 10 years if action is not taken. How do you think they might have come to this conclusion?

2. When observing a patient the temperature, pulse and respiration (breathing) rates are measured at regular intervals. The values are then plotted on a running chart which shows the patient's progress over a period of days.

   The chart below belongs to Mr Smith who is recovering from an appendectomy.



(a) At 2 pm on Thursday, 11th November, what was Mr Smith's:

   (i) temperature?

   (ii) pulse rate?

   (iii) respiration rate?

(b) The start of an infection is often signalled by a sudden rise in temperature. As a result, any rapid rise in temperature must be reported to a doctor for further investigation.

   (i) On which day and over what time period, did Mr Smith's temperature rise rapidly?

   (ii) By how much did it rise?

(c)

(i) At 6 pm on Saturday, Nurse Cameron finds that Mr Smith's temperature is 37.6°C. Plot this value on his chart.

(ii) At the same time, Nurse Watts determines Mr Smith's pulse and respiration rates by counting for a period of 15 seconds and obtains the following values:

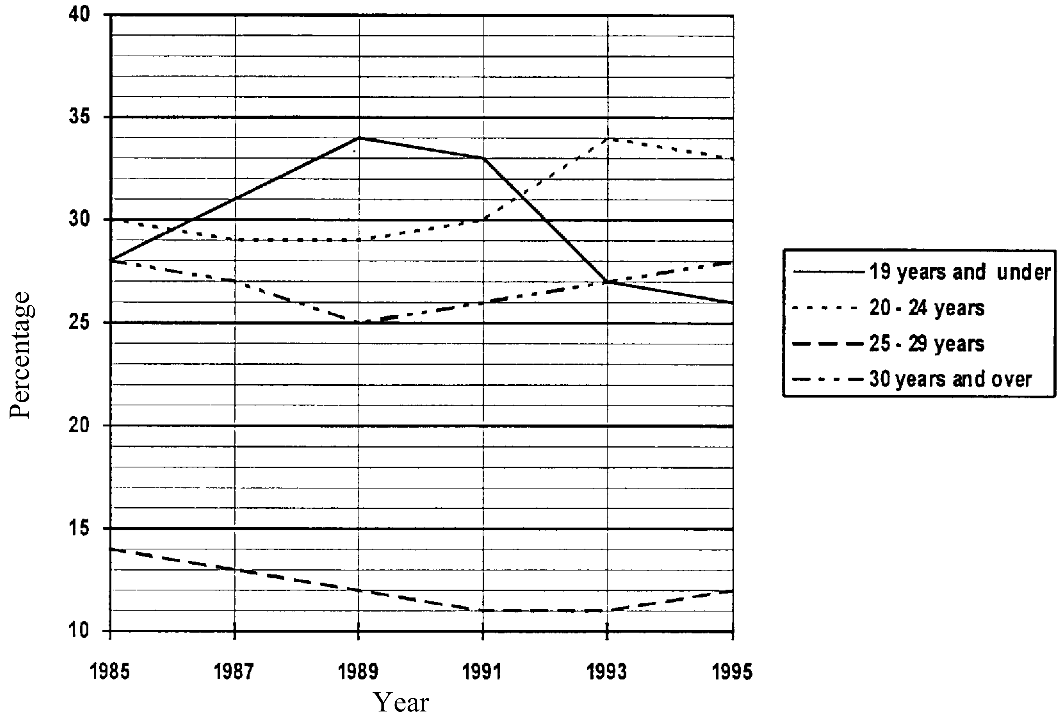
Pulse: 19 counts/15 seconds

respiration: 5 counts/15 seconds

Convert these values to the appropriate unit/minute and plot them on the chart.

3. The following graph shows the age distribution of higher education students.

Age distribution of Higher Education Students 1985 - 1995



(Source: *Selected Higher Education Student Statistics*, DEET.)

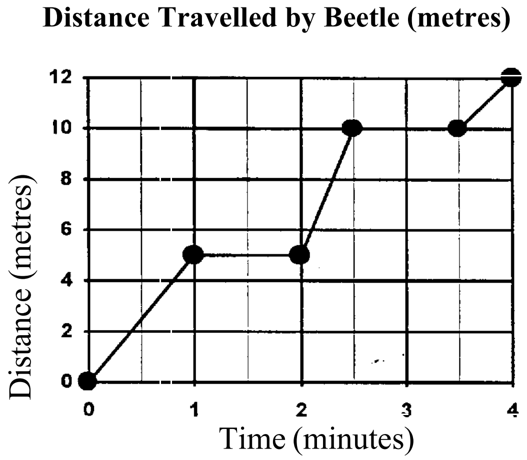
(a) What percentage of students in higher education in 1991 were aged 20 – 24 years?

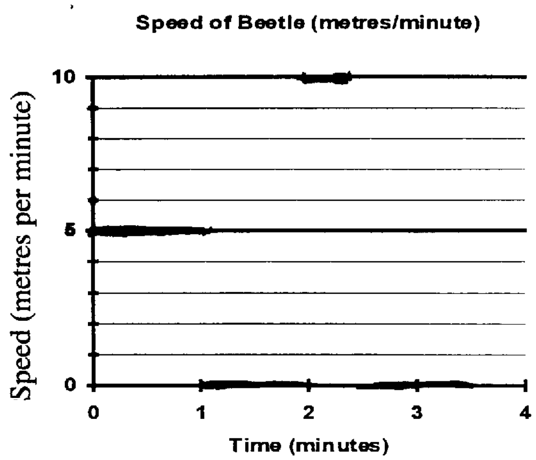
(b) In which year did the 19 years and under group have the greatest representation in higher education?

(c) Which age group has the lowest representation in the higher education sector?

4. Following are two graphs showing how a beetle travelled over a set distance.

The first graph shows the distance the beetle walked over a given time. The second graph shows the speed at which the beetle was travelling at any particular time.





(a) Between which times is the beetle not moving? Explain how this is reflected in both graphs.

(b) From the first graph determine how far the beetle travelled between 0 and 1 minutes.

(c) From the second graph determine how fast the beetle travelled between 0 and 1 minutes.

Can you see how this speed was calculated?

Recall from earlier in this module we gave the formula for speed as:

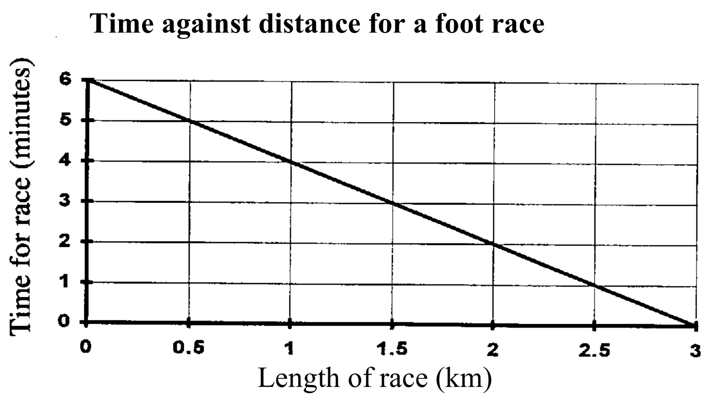
$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

From the first graph you found that the beetle covered 5 metres in the first minute.

That is: $\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{5\text{ m}}{1\text{ minute}} = 5$ m/minute, which is the speed you found from the second graph.

(d) From the first graph, calculate the speed at which the beetle travelled between 3.5 and 4 minutes. Mark this on the second graph.

5. The following graph shows the time it takes to run a race plotted against the distance over which the race was run.



Look at this graph carefully until you understand what relationship the graph is describing. Write a sentence explaining what is wrong with this graph.

5.6 A taste of things to come

1. With some medications, especially for children, it is important to know a person's body surface area (BSA). This is of particular importance in critical care, where loss of heat and moisture from the body needs to be monitored.

   There are two formulas that are used to calculate body surface area (in m^2)

   Formula 1 $\quad \text{BSA} = 0.007\,184 \times W^{0.425} \times H^{0.725}$

   Formula 2 $\quad \text{BSA} = \sqrt{\dfrac{W \times H}{3600}}$ This formula is used for the calculation of pediatric doses of chemotherapeutic drugs.

   where W represent the weight in kilograms,

   and H represents the height in centimetres.

   Complete the table rounding your answers to 3 decimal places for the following people:

   (i) Mr Tallzo height: 185 cm weight: 83 kg

   (ii) Ms Smallzo height: 155 cm weight: 65 kg

   (iii) Miss Childers height: 130 cm weight: 24 kg

	Body Surface Area	
Patient	**Formula 1**	**Formula 2**
Mr Tallzo		
Ms Smallzo		
Miss Childers		

2. If you have an opportunity to study Australian history in your degree, you may use as a reference text Robert Hughes' *The Fatal Shore* (Collins Harvill, Great Britain, 1987). An extract from the book follows:

   A graph of transportation to Australia would run fairly flat (though uphill) from 1788 to 1816, then climb more steeply, shoot to a peak in the mid 1830's and then flatten again.

(a) Knowing that transportation went from 1788 through to 1853, draw a **sketch** of what you think this graph will look like from the description given above. A sketch is just a quick diagram, showing the main features of the graph. It is not necessary for this to be done on graph paper.

(b) The exact figures for the number of people transported in various years are given below. Draw a graph, showing the number of convicts coming to Australia between 1787 and 1853. Does this graph look like the one you sketched above?

Years	**Year to Plot**	**Number of Convicts**
1787 – 9	1787	1 010
1790 – 4	1792	4 417
1795 – 9	1797	1 662
1800 – 4	1802	2 603
1805 – 9	1807	1 734
1810 – 14	1812	4 222
1815 – 19	1817	10 672
1820 – 4	1822	13 347
1825 – 9	1827	18 331
1830 – 4	1832	27 697
1835 – 9	1837	24 433
1840 – 4	1842	18 757
1845 – 9	1847	8 206
1850 – 3	1852	8 379

(c) Hughes, in his book, goes on to describe four distinct phases of the transportation of convicts to Australia. Can you distinguish four phases from the graph you have drawn in part (b) above? Write a few sentences describing these four phases.

3. On a multiple choice test where the teacher thought that students might be guessing a lot of the answers, a *correction for guessing* formula might be used. One such formula is:

$$\text{Adjusted Score} = R - \frac{W}{n-1}$$

where R represents the number of correct responses,

W represents the number of incorrect responses,

and n represents the number of alternatives on each question.

(a) Calculate the adjusted score for students with the following scores on a multiple choice test with 5 alternatives for each question. Round your answer to one decimal place if necessary.

Name	Correct Responses	Incorrect Responses	Adjusted Score
T. Cruise	20	10	
D. Moore	22	5	
V. Kilmer	18	12	
M. Griffiths	24	4	

(b) Did adjusting the scores for guessing have any effect on the ranking of the students?

You should now be ready to attempt questions 1–4 of Assignment 3 A (see your Introductory Book for details). If you have any questions, please refer them to your course tutor.

5.7 Post-test

1. A familiar sound in the country on a warm summer evening is the chirping of crickets. The rate at which crickets chirp depends on the temperature: the warmer it is, the more they chirp in any given time. In fact the relationship can be represented by the following formula.

$$n = \frac{C-4}{0.6}$$

where n represents the number of chirps in 15 seconds,

and C represents the temperature in degrees Celsius.

Calculate the number of cricket chirps if the temperature is:

(a) 28°C

(b) 37°C

(c) 43°C

2. The following table gives the distances covered in certain times for a variety of travellers.

Who's Speeding?	***d***	***t***	$s = \frac{d}{t}$	**Speed km/h**
Brisk walk	30 kilometres	5 hours	$s = \frac{30 \text{ km}}{5 \text{ h}}$ $= 6$ km/h	
Giant Tortoise	400 metres	2 hours		
Fastest running speed of man	100 metres	8.4 seconds		
Landspeed record	1 kilometre	3.6 seconds		
Snail	10 metres	2 hours		

(a) Complete the table for the given values. Be very careful with units. Recall the work on rates if you need to.

(b) Convert each of the speeds you have calculated to km/h

(c) Rank the travellers in order from slowest to fastest.

Recall:
Module 4
Section 4.2.3

3. The Feedumwell Catering Company caters for functions. It charges $180 for the use of the function room, and then $22 per person for the meal.

(a) Write a formula relating the cost of a function to the number of guests. Don't forget to define the variables you use.

(b) Complete the following table of values for the costs of functions with the indicated numbers of guests.

Number of Guests	10	20	45	50	70	100
Cost of Function ($)						

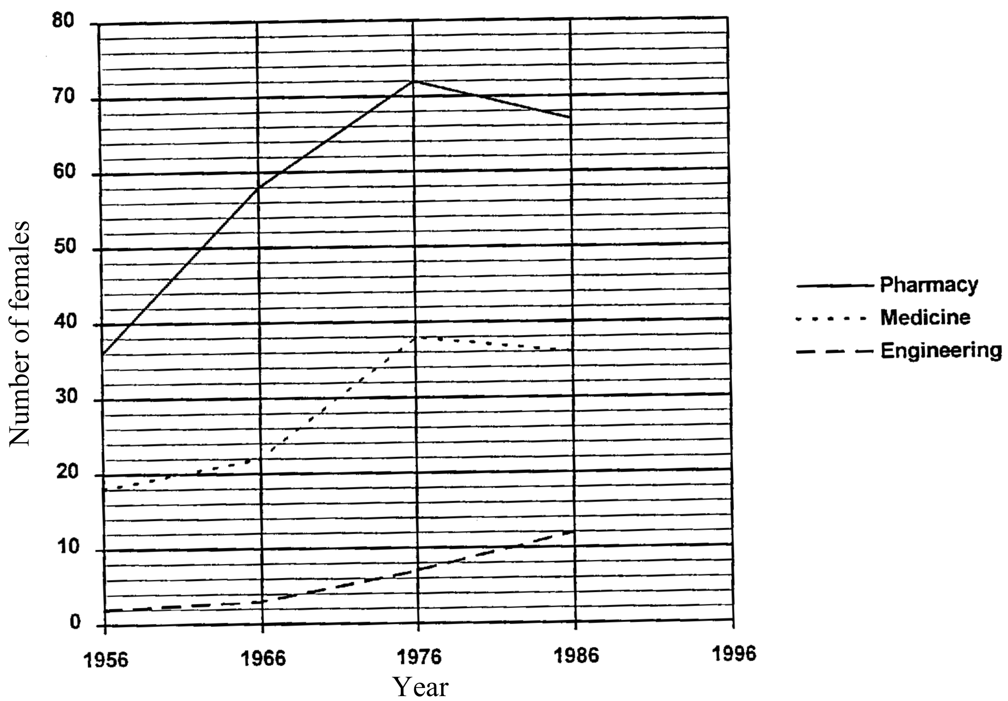
(c) Plot these values on a graph and draw a trend line.

From your graph:

(d) Read off the cost of a function at which 48 guests were present.

(e) Find the greatest number of possible guests at a wedding if the budget was $1 500 for the reception.

4. The following questions refer to this graph.



(a) Give a suitable title to this graph.

(b) (i) For what period are some enrolments decreasing?

(ii) In which courses are they decreasing in this period?

(c) Calculate the overall percentage increase in enrolments since 1956 for women in:

(i) medicine.

(ii) engineering.

(d) Write a short paragraph comparing the changes in enrolments for women in medicine and engineering.

5. For the following question, no data is given. You are to make up the data to suit your situation.

(a) You and your family or friends decide to spend a few days travelling around in the car. For one of these days, covering a 24 hour period, draw a graph showing the time spent travelling against the distance covered. (Do this on some graph paper).

(b) Write a paragraph to explain how your graph represents what has happened throughout your day travelling.

5.8 Solutions

Solutions to activities

Activity 5.1

1.

(a) $d = s \times t$ where d represents the distance travelled,

$d = s\,t$ s represents the speed of the car,

and t represents the time of the journey.

(b) $P = R - C$ where P represents the profit,

R represents the revenue,

and C represents the costs.

(c) $V = I \times R$ where V represents the electrical voltage,

$V = IR$ I represents the electrical current,

and R represents the electrical resistance.

2. You may have different letters to those given here, but the formula should look the same and you should have defined the variables clearly and fully.

(a) The grevillea was half the height of the palm tree.

$G = \frac{1}{2} P$ where G represents the **height** of the grevillea,

and P represents the **height** of the palm tree.

(b) The adult weighed three times as much as the child.

$A = 3 \times C$ where A represents the **weight** of the adult,

$A = 3C$ and C represents the **weight** of the child.

(c) The perimeter of a square is four times the length of one side.

$P = 4 \times s$ where P represents the perimeter,

$P = 4s$ and s represents the **length** of one side.

(d) The house was 15 metres longer than it was wide.

$L = W + 15$ where L represents the **length** of the house, in metres,

and W represents the **width** of the house, in metres.

3. $C = 4 \times S$ where C represents the **number** of cola bottles,

$C = 4S$ and S represents the **number** of soda bottles.

4. $T = \frac{39}{100} \times (I - \$9\ 500) + \$9\ 500$ where T represents the tax payable ($),

$T = 0.39 \times (I - \$9\ 500) + \$9\ 500$and I represents the taxable income ($)

5. $I = 200 + 50 \times n$ where I represents the total income in dollars,

and n represents the number of items sold.

Activity 5.2

1.

(a) $\mathrm{A} = \mathrm{lw}$

$= 7\text{ cm} \times 5\text{ cm}$

$= 35\text{ cm}^2$

(b) $A = lw$

$= 34\text{ cm} \times 21\text{ cm}$

$= 714\text{ cm}^2$

(c) $A = lw$

$= 3\text{ m} \times 67\text{ m}$

$= 201\text{ m}^2$

(d) $A = lw$

$= 4\text{ cm} \times 82\text{ mm}$ Convert to same unit.

$= 4\text{ cm} \times 8.2\text{ cm}$

$= 32.8\text{ cm}^2$

2.

(a) $P = 4s$ where P represents the perimeter,

and s represents the side length.

(b)

(i) $P = 4s$

$= 4 \times 4$ cm

$= 16$ cm

(ii) $P = 4s$

$= 4 \times 7$ m

$= 28$ m

(iii) $P = 4s$

$= 4 \times 2.6$ mm

$= 10.4$ mm

(iv) $P = 4s$

$= 4 \times \frac{3}{4}$ m

$= 3$ m

(c) Perimeter of the chicken pen $= 4 \times 3.2$ m $= 12.8$ m

Cost of the chicken wire $= 12.8 \times \$6.50 = \$83.20$

The cost to fence the chicken pen will be \$83.20

3.

$C = \pi d$ where C represents the circumference of the circle, and d represents the diameter of the circle.

(i) $C = \pi d$

$= \pi \times 3$ cm

≈ 9.4 cm

(ii) $C = \pi d$

$= \pi \times 72$ mm

≈ 226.2 mm

(iii) $C = \pi d$

$= \pi \times 6.5$ m

≈ 20.4 m

(iv) $C = \pi d$

$= \pi \times \frac{3}{5}$ m

≈ 1.9 m

(c) Circumference of the garden $= \pi \times 3.8 \text{ m} \approx 11.9 \text{ m}$

If log edging comes in rolls of 3 metres, then 4 of these will be sufficient to edge the garden.

Cost of edging = 4 × \$18.95 = \$75.80

4.

(a) $S = 4.9\,t^2$

$= 4.9 \times 3^2$

$= 4.9 \times 9$

$= 44.1$ Cliff is 44.1 metres high.

(b) $S = 4.9\,t^2$

$= 4.9 \times 8^2$

$= 4.9 \times 64$

$= 313.6$ Cliff is 313.6 metres high.

(c) $S = 4.9\,t^2$

$= 4.9 \times (1.4)^2$

$= 4.9 \times 1.96$

$= 9.604$ Cliff is 9.604 metres high.

5.

(a) $D = 8\sqrt{\frac{h}{5}}$

$= 8\sqrt{\frac{10}{5}}$

$= 8 \times \sqrt{2}$

≈ 11 From this tower you could see about 11 kilometres.

(b) $D = 8\sqrt{\frac{h}{5}}$

$= 8\sqrt{\frac{25}{5}}$

$= 8 \times \sqrt{5}$

≈ 18 From this tower you could see about 18 kilometres.

(c) $D = 8\sqrt{\frac{h}{5}}$

$= 8\sqrt{\frac{17.5}{5}}$

$= 8 \times \sqrt{3.5}$

≈ 15 From this tower you could see about 15 kilometres.

6.

(a) This formula only relies on height. It has not taken into account a person's weight which could effect the amount of skin area.

(b)

(i) $A = \frac{3}{5}h^2$

$= \frac{3}{5} \times 165^2$

$= \frac{3}{5} \times 27\,225$

$= 16\,335$ Area of skin is 16 335 cm^2 (1.6335 m^2)

(ii) $A = \frac{3}{5}h^2$

$= \frac{3}{5} \times 175.5^2$

$= \frac{3}{5} \times 30\,800.25$

$= 18\,480.15$ Area of skin is 18 480.15 cm^2 (1.848 015 m^2)

(iii) $A = \frac{3}{5}h^2$

$= \frac{3}{5} \times 1.68^2$

$= \frac{3}{5} \times 2.822\,4$

$= 1.693\,44$ Area of skin is 1.693 44 m^2

(c) (i) $24\% \times 16\ 335\ \text{cm}^2 = 0.24 \times 16\ 335\ ,\text{cm}^2 = 3\ 920.4\ \text{cm}^2$ was burnt.

(ii) $24\% \times 18\ 480.15\ \text{cm}^2 = 0.24 \times 18\ 480.15\ \text{cm}^2 = 4\ 435.236\ \text{cm}^2$ was burnt.

(iii) $24\% \times 1.693\ 44\ \text{m}^2 = 0.24 \times 1.693\ 44\ \text{m}^2 = 0.406\ 4256\ \text{m}^2$ was burnt.

7.

(a) Body Mass Index $= \frac{w}{h^2}$

$= \frac{65}{1.65^2}$

$= \frac{65}{2.7225}$

≈ 23.9 This is a healthy Body Mass Index.

(b) Body Mass Index $= \frac{w}{h^2}$

$= \frac{85}{1.78^2}$

$= \frac{85}{3.1684}$

≈ 26.8 This is not a healthy Body Mass Index.

(c) Body Mass Index $= \frac{w}{h^2}$

$= \frac{75}{1.5^2}$

$= \frac{75}{2.25}$

≈ 33.3 This is not a healthy Body Mass Index.

(d) Your answer will depend on your height and weight.

Activity 5.3

1.

(a) I6 G block, Education, Student Services, Sciences.

(b) D5 F block, Indoor Recreation Centre.

(c) N7 The Japanese Gardens.

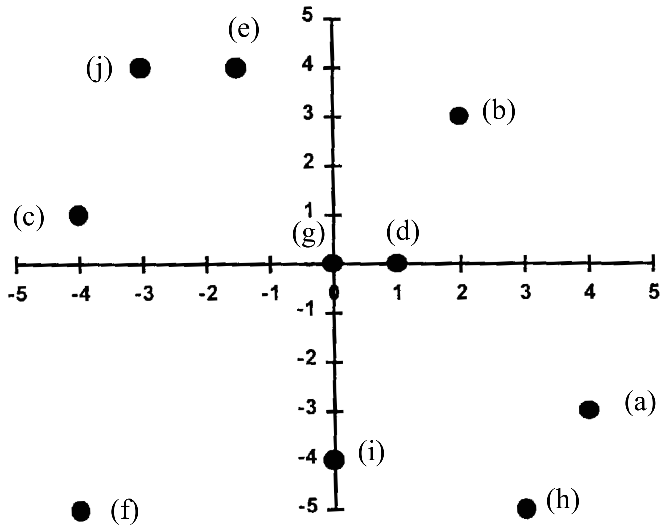
2.

(a) Steele Rudd Courts. F4

(b) W Block Sciences (nursing and psychology) and Shopping Complex.H5 or H6

(c) Gymnasium N3

Activity 5.4



Activity 5.5

(a) (–1,2.5)

(b) (0,2)

(c) (3,4)

(d) (–3.5,0)

(e) (–2,–4)

(f) (1.5,–3)

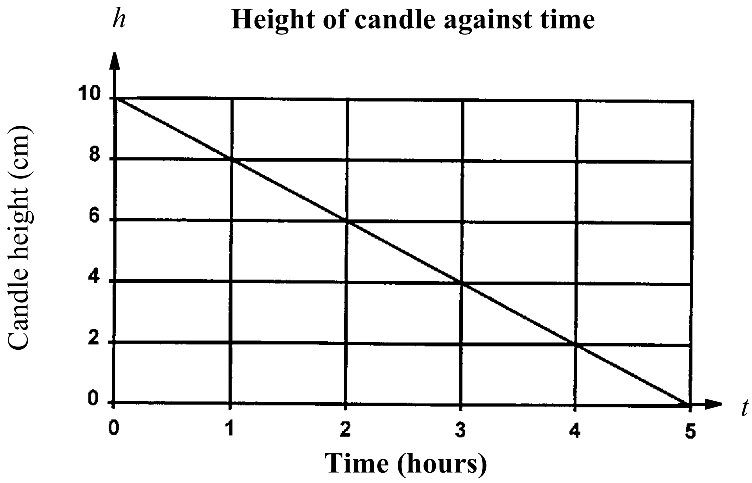
(g) (4.8, –0.7)

Activity 5.6

1. (a)

t	0	1	2	3	4	5
h	10	8	6	4	2	0

(b)

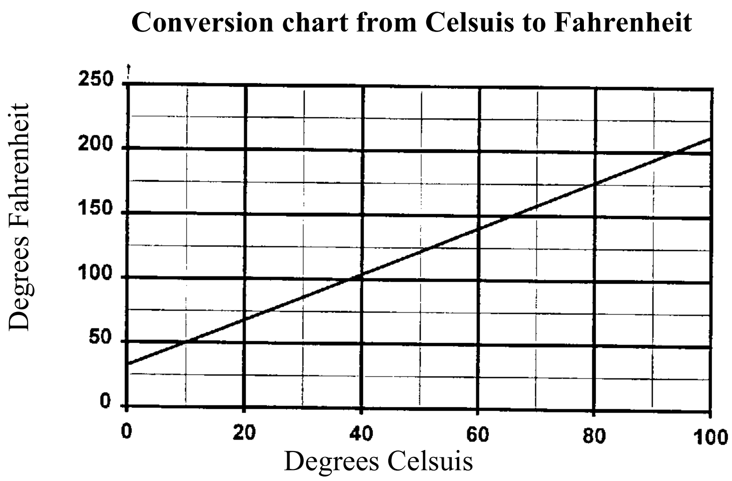


(c) At time 0 hours (before the candle was lit) the height of the candle was 10 cm.

(d) The 10 in the formula represents the height of the candle before it was lit.

(e) After 5 hours the candle has a height of 0 cm so it will not burn further. The candle burns for 5 hours.

2. (a)



(b) From your graph you should be able to read that 59°F is the same as 15°C. Warm clothing will be needed for this trip.

(c) 35°C is equivalent to 95°F.

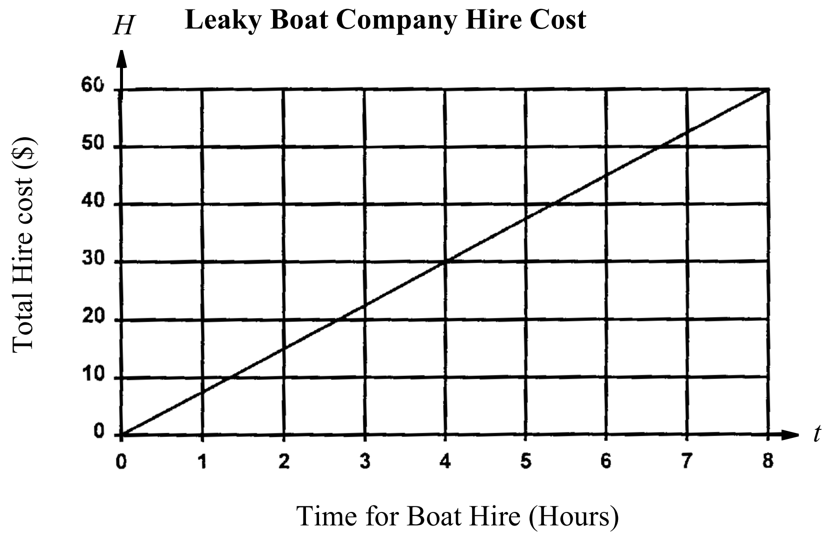
3.

(a) $H = 7.50t$ where H represents the cost of hire in dollars, and t represents the time (in hours) that you have the boat.

(b)

Number of Hours	1	2	3	5	8
Total Cost ($)	7.50	15.00	22.50	37.5	60.00

(c)



(d) Cost for 4 hours hire will be $30.00

(e) For $24 you would be able to hire the boat for 3.2 hours. This is 3 hours and 12 minutes, since 0.2 of 1 hour is 12 minutes.

4.

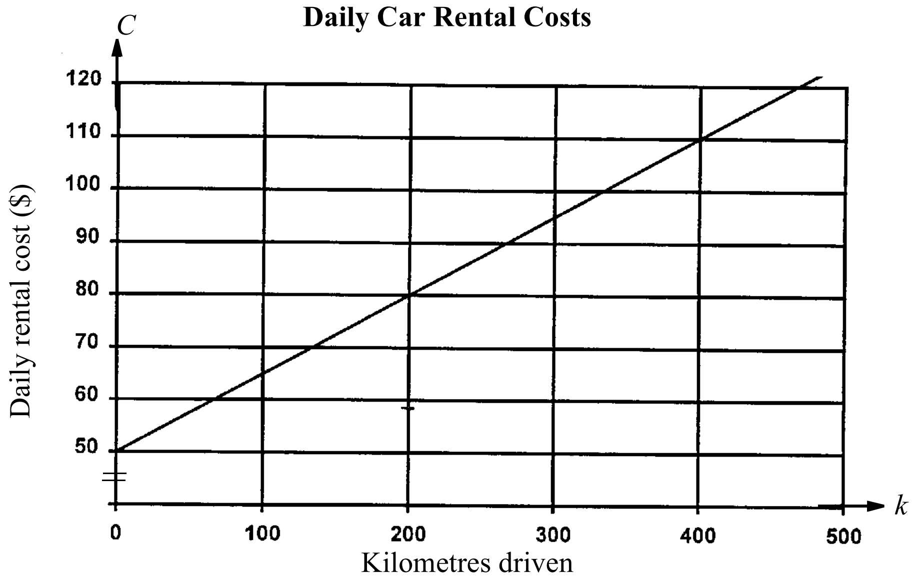
(a) $C = 50 + 0.15k$ where C represents the total daily cost of hire in dollars,

and k represents the number of kilometres travelled.

Note that since the 50 is in dollars the 15 cents must also be written in dollars.

(b)

Number of kilometres travelled	0	100	200	300
Total cost of hire ($)	50	65	80	95



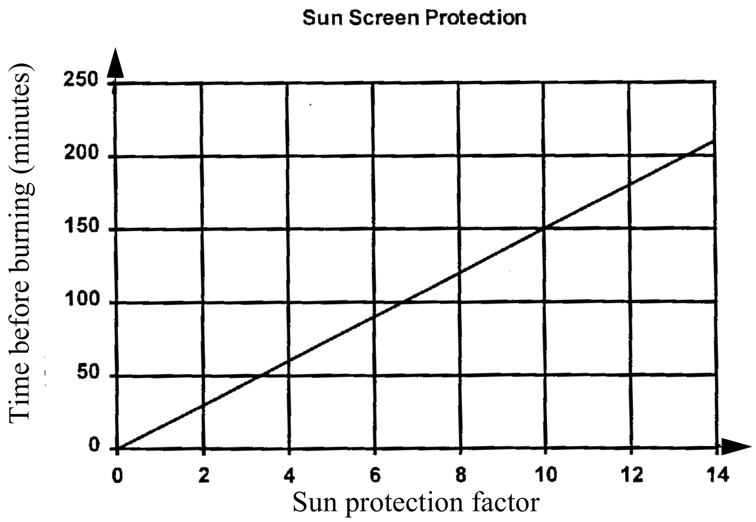
(c) The cost for the day, having driven 260 kilometres is $89.

(d) If you had $110 you could travel 400 kilometres.

5.

(a) For each increase of 2 in the sun protection factor, 30 extra minutes can be spent in the sun before burning.

(b)

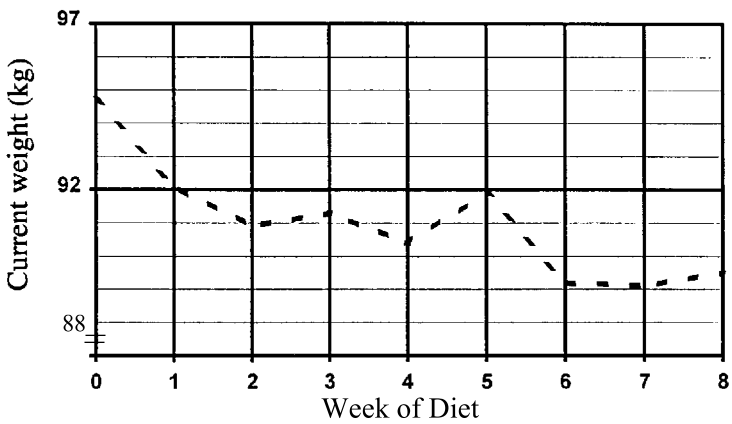


(c) If the person wishes to stay in the sun for one hour they should use at least a SPF 4.

6.

(a)

Chart showing Ima Bitheavy's Weight Loss



(b) The lightest weight that Ima reached was 89.1 kilograms.

Activity 5.7

1. This formula is representing the fact that Joseph's age is equal to Chris' age plus 2 years. In other words Joseph is two years older than Chris.
2. This formula is representing the fact that Cathy's age is equal to Jenny's age minus 2 years. In other words Cathy is two years younger than Jenny.
3. The area of a rectangle is equal to the length of the rectangle multiplied by the width.
4. Ohm's law states that the current is equal to the electromotive force divided by the resistance.
5. This formula allows us to find the area of a circle. It states that the area is equal to pi multiplied by the radius squared.

Activity 5.8

1.

(a) 1973 had the greatest number of breeding turtles.

(b) The least number of turtles was in 1989.

(c) Between 1972 and 1973 there was the greatest increase in turtle numbers.

(d) Between 1973 and 1974 saw the greatest decrease in turtle numbers.

(e) If you were to continue the graph following the same path as from 1991 to 1993 this line would cut the horizontal axis at about the year 2005.

2.

(a) (i) Mr Smith's temperature would be 38.5°C

(ii) Mr Smith's pulse rate would be 98 beats per minute.

(iii) Mr Smith's respiration rate would be 21 or 22 breaths per minute.

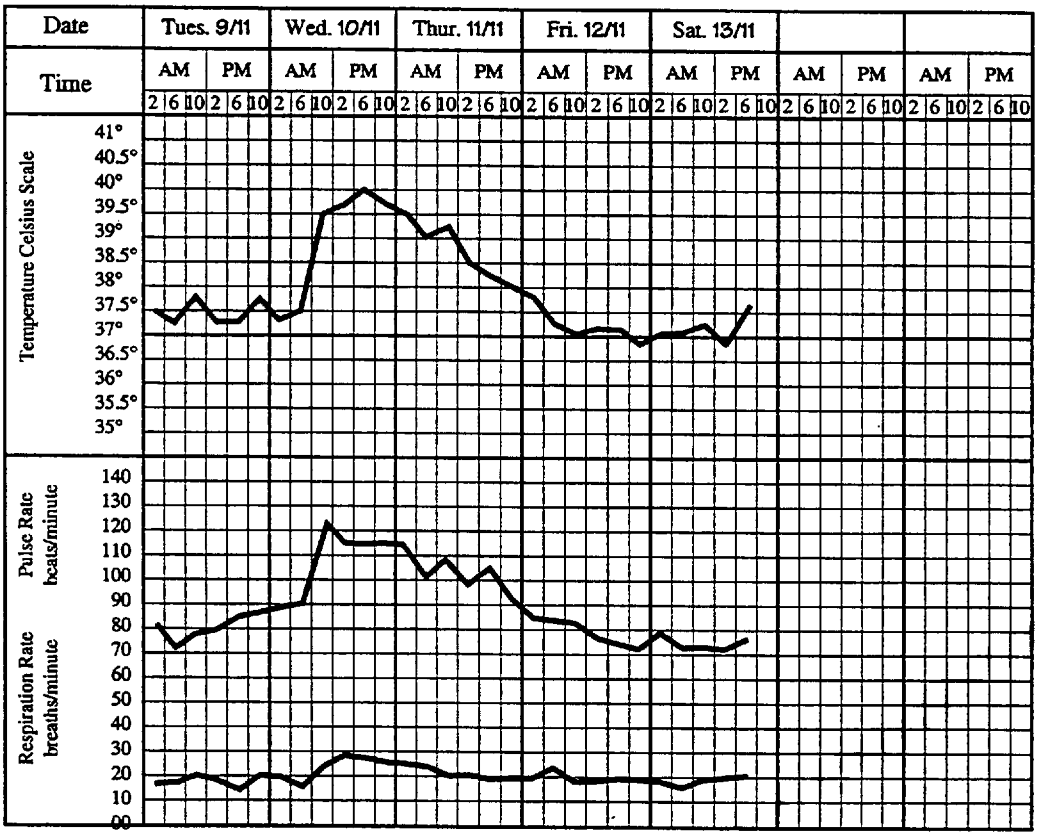
(b) (i) Wednesday the 10th November from 6 am to 10 am saw a rapid rise in temperature.

(ii) Mr Smith's temperature rose by about 2°C

(c) (i) See chart below

(ii) Pulse rate: 19 counts/15 seconds gives 76 beats per minute.

(iii) Respiration rate: 5 counts/15 seconds gives 20 breaths per minute.



3.

(a) 30% of students in higher education in 1991 were aged 20 – 24 years.

(b) 1989 saw the greatest representation of 19 years and under students.

(c) Students aged 25 – 29 are the least represented in the higher education sector for these years.

4.

(a) For the first graph, from 1 to 2 minutes and from 2.5 to 3.5 minutes, there is no change in distance so the beetle is not moving.

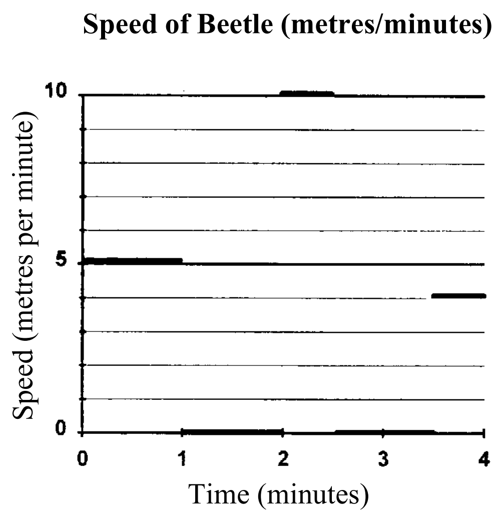
For the second graph, from 1 to 2 minutes and from 2.5 to 3.5 minutes, the speed of the beetle is 0 so the beetle is not moving.

(b) From 0 to 1 minute the beetle travelled 5 metres.

(c) From the second graph the beetle is travelling at 5 metres per minute from 0 to 1 minutes.

(d) Between 3.5 and 4 minutes the beetle travelled 2 metres.

The speed will be $\dfrac{2 \text{ metres}}{0.5 \text{ minutes}} = 4\text{metres/minute}$



5. This graph is showing time taken against distance travelled. The problem with the graph is that the longer the race, the less time it takes. A 1 km race will take about 4 minutes while a 3 km race takes no time at all.

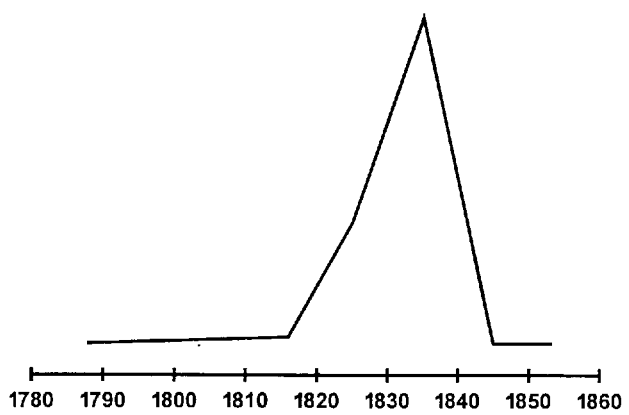
Solutions to a taste of things to come

1.

		Body Surface Area	
	Patient	Formula 1	Formula 2
(i)	Mr Tallzo	2.069 m^2	2.065 m^2
(ii)	Ms Smallzo	1.640 m^2	1.673 m^2
(iii)	Miss Childers	0.945 m^2	0.931 m^2

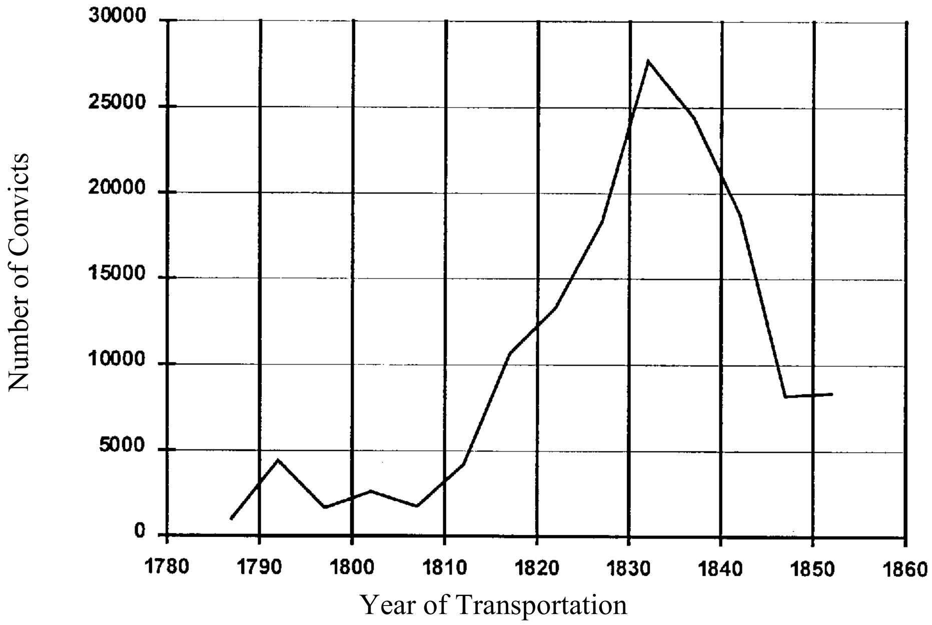
2.

(a) One possible graph might look like the following. After 1816 the graph was to rise then shoot to a peak in the mid 1830's. I have taken mid 1830s to be about 1835 and the first rise to be about halfway between these two (about 1825)



(b) The graph drawn from the exact data is given below.

Number of Convicts coming to Australia 1787 to 1853



This graph does look a bit like the one drawn in part (a)

(c) Hughes identifies the first phase as going from 1788 to 1810. It started with Britain's desire to clear its jails and have a presence in the Pacific. Over this period approximately 11 800 people were transported. This was no more than 7% of the total numbers transported.

Sun screen protection

The second phase is identified as going from 1811 to 1830. The system of transportation had been well established and Britain wanted to continue to use it. During this period the Napoleonic wars finished and this combined with internal crises led to a dramatic increase in numbers transported. Over this time about 50 200 people, some 31% of the total number of transportees, came to Australia. (The graph we have drawn looks more like this phase began in 1822 or even 1828.)

The third phase, from 1831 to 1840, saw the system peak and begin its decline. In these years, 51 200 people sailed for Australia.

The final phase recognised by Hughes was from 1841 to 1853 when all transportation was abolished. From 1840 all transportation to NSW was abolished but about 26 000 convicts were poured into Van Diemen's Land (now Tasmania).

3.

(a)

Name	Correct Responses	Incorrect responses	Adjusted Score
T. Cruise	20	10	17.5
D. Moore	22	5	20.8
V. Kilmer	18	12	15
M. Griffiths	24	4	23

(b) After adjusting the scores, the students are still ranked in the same order as before the correction for guessing.

Solutions to post-test

1.

(a) $n = \frac{C-4}{0.6}$

$= \frac{28-4}{0.6}$

$= 40$ At 28°C the cricket would make 40 chirps every 15 seconds.

(b) $n = \frac{C-4}{0.6}$

$= \frac{37-4}{0.6}$

$= 55$ At 37°C the cricket would make 55 chirps every 15 seconds.

(c) $n = \frac{C-4}{0.6}$

$= \frac{43-4}{0.6}$

$= 65$ At 43°C the cricket would make 65 chirps every 15 seconds.

2.

(a)

Who's Speeding?	***d***	***t***	$S = \frac{d}{t}$	**Speed km/h**
Brisk walk	30 kilometres	5 hours	$S = \frac{30 \text{ km}}{5 \text{ h}} = 6 \text{ km/h}$	6 km/h
Giant Tortoise	400 metres	2 hours	$S = \frac{400 \text{ m}}{2 \text{ h}} = 200 \text{ m/h}$	0.2 km/h
Fastest running speed of man	100 metres	8.4 seconds	$S = \frac{100 \text{ m}}{8.4 \text{ s}} \approx 11.9 \text{ m/s}$	42.84 km/h
Landspeed record	1 kilometre	3.6 seconds	$S = \frac{1000 \text{ m}}{3.6 \text{ s}} \approx 277.8 \text{ m/s}$	1 000 km/h
Snail	10 metres	2 hours	$5 = \frac{10\text{m}}{2\text{h}} = 5\text{m/h}$	0.005 km/h

(b) See table.

(c) From slowest to fastest: Snail, Giant Tortoise, Brisk walk, Fastest running speed of man and Landspeed record

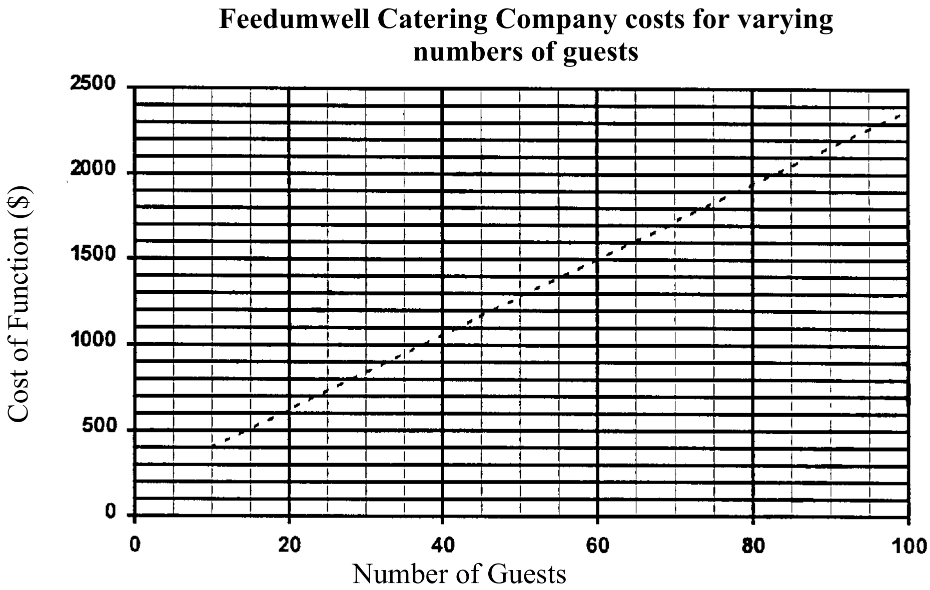
3.

(a) $C = 180 + 22n$ where C represents the cost of the function in dollars, and n represents the number of guests.

(b)

Number of Guests	10	20	45	50	70	100
Cost of Function ($)	400	620	1 170	1 280	1 720	2 380

(c)



(d) From the graph the cost for 48 guests looks like about $1 250.

Checking the exact value which is $1 236, this looks about right. You were asked to read the value from your graph so you must give the answer that you can read from your graph.

(e) If the budget for the wedding reception was $1 500 there could be up to 60 guests.

4.

(a) The Number of Females in Pharmacy, Medicine and Engineering at University, between 1956 and 1986.

(b) (i) Between 1976 and 1986

(ii) In Pharmacy and Medicine the numbers are decreasing.

(c) (i) Numbers in medicine went from 18 to 36 an increase of 18

$$\begin{aligned}\text{Percentage increase} &= \frac{\text{amount of increase}}{\text{original amount}} \times 100\% \\ &= \frac{18}{18} \times 100\% \\ &= 100\%\end{aligned}$$

There has been a 100% increase in the number of women studying medicine.

(ii) Numbers in engineering went from 2 to 12 an increase of 10

$$\begin{aligned}\text{Percentage increase} &= \frac{\text{amount of increase}}{\text{original amount}} \times 100\% \\ &= \frac{10}{2} \times 100\% \\ &= 500\%\end{aligned}$$

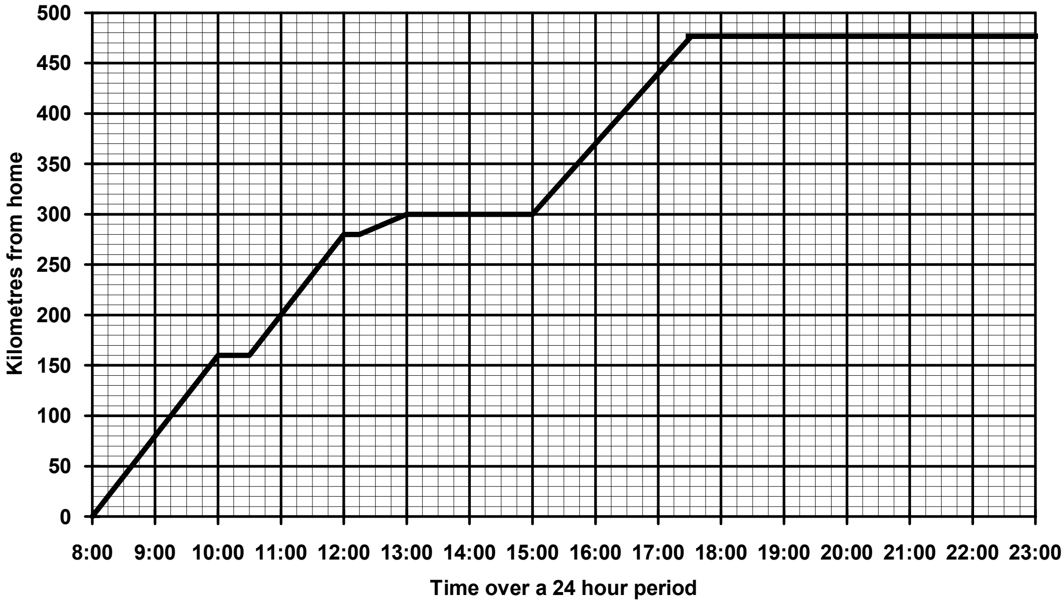
There has been a 500% increase in the number of women studying engineering.

(d) Overall both medicine and engineering have had increased enrolments although engineering increased at a much faster rate (from 2 to 12 compared to 18 to 36). Between 1976 and 1986 medicine numbers have actually decreased by 2 while those in engineering have increased by 7.

5.

(a)

Details of Distance travelled over a given time on a Travelling Holiday



(b) At 8:00 am we left home to begin our travels. We travelled until 10:00 am, covering a distance of 160 km. At 10:00 am we stopped for half an hour to have morning tea. There was nothing to look at at this spot so we continued with our journey. We travelled for a further 120 km until at 12 noon we stopped to get some petrol and use the facilities at the petrol station. This took us 15 minutes. The going was slow to our next stop, 45 minutes away. We only covered 20 kilometres in this time due to the poor condition of the road.

At 1:00 pm we finally arrived at the waterfall where we planned to have lunch. It was a beautiful day and we enjoyed swimming and eating. At 3:00 pm we left the waterfall so that we could move on to our accommodation for the night. We made good time and arrived at our destination 475 kilometres from home at 5:30 pm. We stayed here for the remaining time in this 24 hour period.