



Limits Approaching

When you have a discontinuity of a function, we cannot directly calculate the limit at that discontinuity. When this happens, we need to check what value the function takes on as we get closer to the discontinuity.

For example, the function

$$f(x) = \frac{x^2 - 1}{x - 1}$$

does not exist when $x = 1$ because you would have $0 \div 0$.

Approaching the limit from the left

So instead of trying to work the value of the function for $x = 1$ try approaching values of x as they get closer 1 (from the left):

x	$f(x) = \frac{x^2 - 1}{x - 1}$
0.5	1.5
0.9	1.9
0.99	1.99
0.999	1.999
0.9999	1.9999
0.99999	1.99999

We can see that as x gets close to 1, from the left, then $f(x) = \frac{x^2 - 1}{x - 1}$ gets closer to 2. This can be written symbolically as:

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \frac{x^2 - 1}{x - 1} = 2.$$

Approaching the limit from the right

The limit can be also taken from the right. That is, check the function for x values approaching values as they get closer 1 (from the right):

x	$f(x) = \frac{x^2 - 1}{x - 1}$
1.5	2.5
1.1	2.1
1.01	2.01
1.001	2.001
1.0001	2.0001
1.00001	2.00001

Now it can be seen that as x gets close to 1, from the right, then $f(x) = \frac{x^2 - 1}{x - 1}$ gets close to 2. And it is written symbolically as:

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{x^2 - 1}{x - 1} = 2.$$

As both of these limits (from the left and right) are the same we can say that the limit as x approaches 1, of the function $f(x)$ exists and is equal to 2. Symbolically:

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = 2.$$

When the limit is the same approaching from the left and the right,

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$$

we say the limit exists.

When the limits are not the same

$$\lim_{x \rightarrow a^-} f(x) \neq \lim_{x \rightarrow a^+} f(x) \neq \lim_{x \rightarrow a} f(x)$$

we say that the limit as x approaches a does not exist.

Resources

- Other [QuickTips](#) flyers;
- Online resources at [Study Support](#);
- Make a consultation with a Mathematics Learning Advisor.